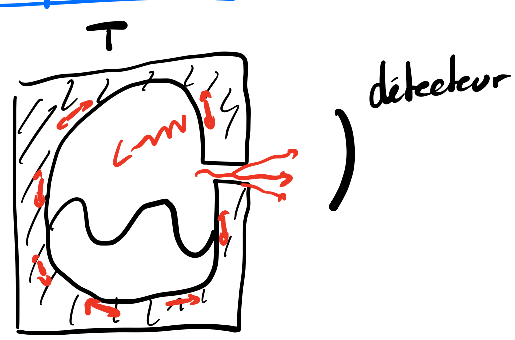
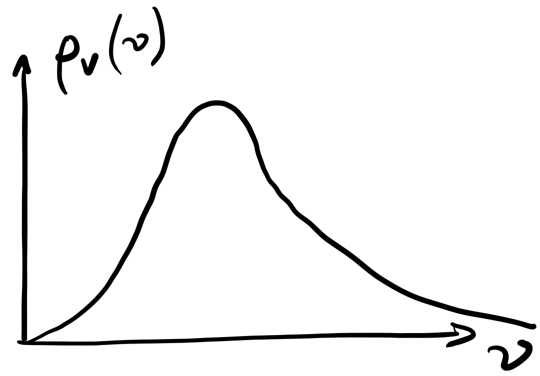
Physique atomique

Ref : Mecanique Quantique Jean Dalibard.
→ Ecole X.
→ en ligne 2003.

I et II Cohen

I Corps Noir





$\rho(\nu)$ densité d'energie par unité de volume.

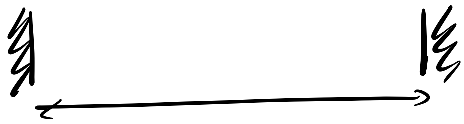
$$= \frac{1}{V}\frac{dE}{d\nu}$$

$$= \underbrace{\frac{1}{V}\frac{dn}{d\nu}}_{\text{densité de mode par unité de volume}} \times \underbrace{\frac{dE}{dn}}_{\text{Energie d'un mode } \bar{E}(\nu)}$$

Boltzman → $\bar{E} = \dfrac{\int_0^\infty E \times e^{-\frac{E}{kT}} dE}{\int_0^\infty e^{-\frac{E}{kT}} dE} = \dfrac{(kT)^2}{kT} = kT$

$D(\nu) = \dfrac{1}{V}\dfrac{\partial n}{\partial \nu}$ $\quad n(\nu)$: nombre de mode dans l'interval $[0, \nu]$.

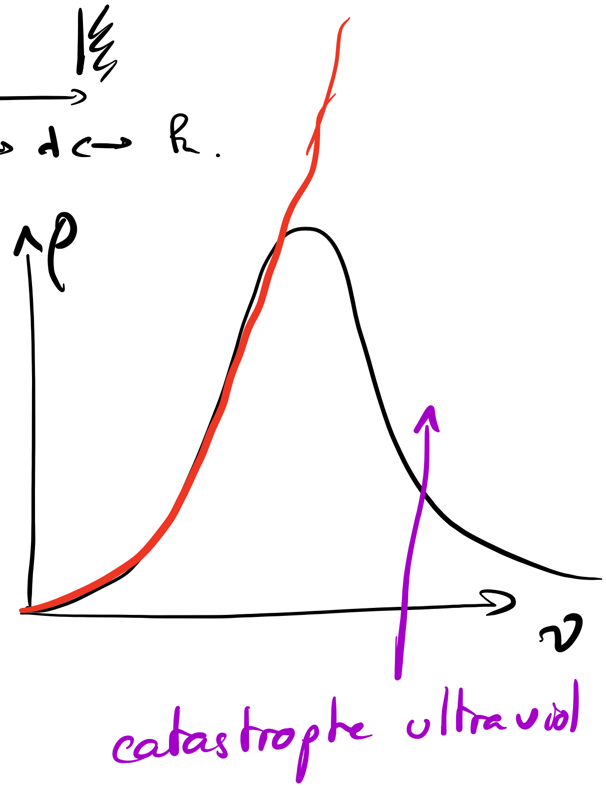
$= \frac{k^2}{\pi^2} \times \frac{\pi}{c}$ exo $\rightarrow$



$L \leftrightarrow d \leftrightarrow k.$

$= \frac{8\pi^2}{c^3} \nu^2$

$$\rho(\nu) = \frac{8\pi^2}{c^3} \nu^2 k T$$



comportement $\nu^3 e^{-\frac{h\nu}{kT}}$

hypothèse de Planck.

dans un mode ν : nombre entier de

$h\nu$

$\uparrow$ constante de Planck

Relation entre

$E \leftrightarrow \nu$.

$$\bar{E}(\nu) = \frac{\int E e^{-E/kT} dE}{\int e^{-\frac{E}{kT}}}$$

$$\hookrightarrow \sum_{n=0}^{\infty} E_n \times p_n$$

$$E_n = n h\nu$$

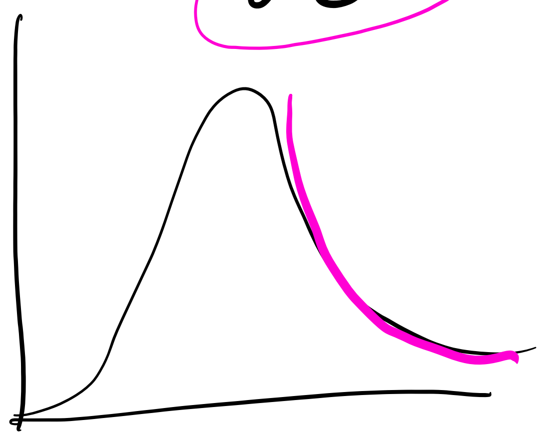
$$p_n = \frac{e^{-\frac{E_n}{kT}}}{\sum_{p=0}^{\infty} e^{-E_p/kT}}$$

$$= \frac{h\nu \, e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}}$$

$$\rho = D(\nu) \times \bar{E}(\nu) = \frac{8\pi}{c^3} h \nu^3 \frac{e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}}$$

$$\nu \to 0 : \quad \propto \nu^2$$

$$\nu \to \infty : \quad \nu^3 e^{-\frac{h\nu}{kT}}$$

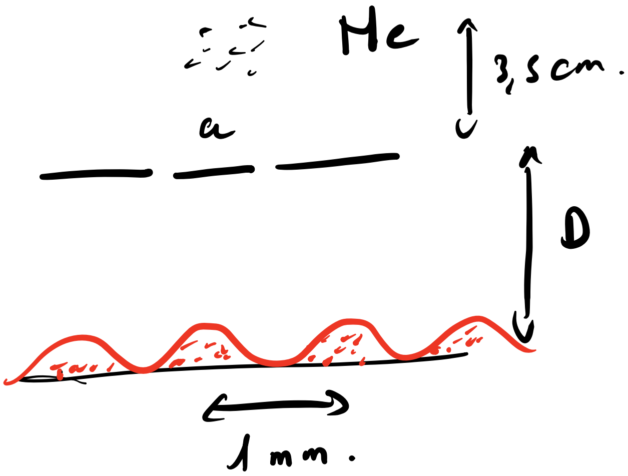


Le champ est quantifié.

$h = 6.6 \; 10^{-34}$ J.s

$\hbar = 1.05 \; 10^{-34}$ J.s

II Matière est une onde



$a = 6 \mu m$.

$D = 85$ cm.

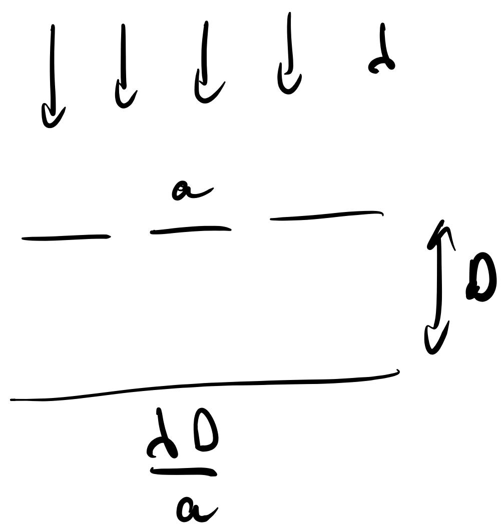
PRA 46 R17 1992

comportement ondulatoire.

Relation de DeBroglie $\vec{p} = m\vec{v} = \hbar\vec{k}$.

<u>AN</u> $m_{He} = 3.3\ 10^{-26}$ kg.

$v = \sqrt{2gx}$ → $x = 45$ cm. (position moyenne)



$\frac{\lambda D}{a} = 0.94$ mm.

↳ experience pour $\hbar$

$p = mv = \frac{h}{\lambda}$

<u>CONC 2</u> Interference → ∃ un champ $\mathbb{C}$

Meca Q

$$A = A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2}$$

"presence d'atomes" (de lumiere) $I \propto |A|^2$

Fonction d'onde $\Psi(\vec{r})$: amplitude de probabilité de presence $d^3P(\vec{r},t) = |\Psi(\vec{r},t)|^2 d^3\vec{r}$

Pté : $\iiint_{\Sigma} |\Psi(r,t)|^2 \, d^3\vec{x} = 1$.

* $\langle x \rangle = \iiint x \, d^3P(r,t)$

$$= \iiint x \, |\Psi(\vec{r},t)|^2 \, d^3\vec{x}$$

$$\Delta x = \langle x^2 \rangle - \langle x \rangle^2$$

$$= \iint x^2 \, |\Psi(\vec{r},t)|^2 \, d^3\vec{x} - \langle x \rangle^2$$

* Interference → superposition des fonction d'onde

$$\Psi = \alpha \Psi_1 + \beta \Psi_2$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Analogie avec Optique de Fourier

$$\Psi(\vec{x},t) = \frac{1}{(2\pi\hbar)^{3/2}} \int \varphi(\vec{p}) \, e^{\frac{i(\vec{p}.\vec{r} - Et)}{\hbar}} d^3p.$$

avec $E = \frac{p^2}{2m}$.

1) Onde plane en PA : onde de De Broglie

$$\Psi(\vec{x},t) = \Psi_0 \, e^{i(\vec{k}\vec{x} - \omega t)}$$

$E = \frac{p^2}{2m}$

$|\vec{p}| = |\hbar \vec{k}| = \frac{h}{\lambda}$

2) Principe de superposition :

a) Onde de De Broglie vérifie eq° de schrodinger libre (SL) $i\hbar \frac{\partial \Psi(\vec{x},t)}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi(\vec{x},t)$.

b) Ψ verifie SL

Theoreme de Plancherel :

$$\iiint |\varphi(\vec{p})|^2 d^3\vec{p} = \iiint |\Psi(x)|^2 d^3x = 1.$$

$\varphi(\vec{p})$ | fonction d'ond.
| amplitude de proba d'impulsion p.

$$\boxed{d^3 P(\vec{p}) = |\varphi(\vec{p})|^2 d^3\vec{p}}$$

$$\langle p_x \rangle = \int p_x |\varphi(p)|^2 d^3p.$$

$$\underbrace{p_x\,\varphi(p)}_{-i\hbar\, TF\left(\frac{\partial \Psi}{\partial x}\right)} . \overline{\varphi(p)} \leftarrow TF(\bar{\Psi})$$

$$\frac{\partial \Psi}{\partial x} = \frac{1}{(2\pi\hbar)^{3/2}} \int \frac{i p_x}{\hbar} \varphi(p) . e^{i\frac{\vec{p}.\vec{x}}{\hbar}} d^3\vec{p}$$

$$\frac{i p_x \varphi(p)}{\hbar} = TF\left(\frac{\partial \Psi}{\partial x}\right)$$

$$= \int \overline{\Psi}(\vec{x}) \Big[\underbrace{-i\hbar \frac{\partial}{\partial x}}_{\text{operateur d'impulsion dans la direction } x} \left(\Psi(\vec{x}) \right) \Big] d^3x$$

III Grandeur physique : generalisation de l'impulsion

Pour toute grandeur physique "a" ont associe une observable $\hat{A}$: operateur lineaire hermitien ($\hat{A}^* = \hat{A}$)

et $\langle a \rangle = \int \Psi^*(r,t) \left(\hat{A}\, \Psi(r,t) \right) d^3\vec{r}$

Impulsion $\langle p_x \rangle \longleftrightarrow \hat{p}_x = -i\hbar \frac{\partial}{\partial x}$

$\langle \vec{p} \rangle \longleftrightarrow \hat{\vec{p}} = -i\hbar \vec{\nabla}$

Position $\langle x \rangle \longleftrightarrow x. = \hat{x}$

Energie cinetique $\langle E_c = \frac{p^2}{2m} \rangle \longleftrightarrow \hat{H}_c = -\frac{\hbar^2}{2m}\Delta$

Energie potentielle $\langle E_p = V(\vec{r}) \rangle \quad \hat{V} = V(\vec{r})$.

tot $\langle E = E_c + E_p \rangle \quad \hat{H} = \hat{H}_c + \hat{V}$

$= -\frac{\hbar^2}{2m}\Delta + V(r)$

Moment cinetique $\vec{L} = \vec{r} \times \vec{p} \quad \hat{L} = -i\hbar\, \vec{r} \times \vec{\nabla}$

SL

$\Downarrow$

Schrodinger $i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\Delta + V(r) \right] . \Psi$

$= \hat{H}\, \Psi(\vec{r},t)$