AN INTRODUCTION TO DIGITAL IMAGE CORRELATION

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Part 1/2

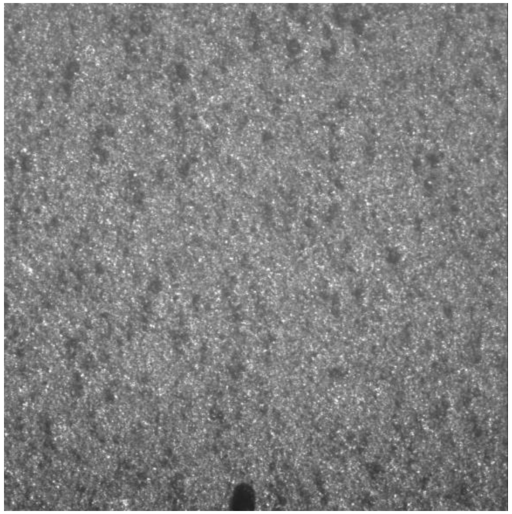


Image 1

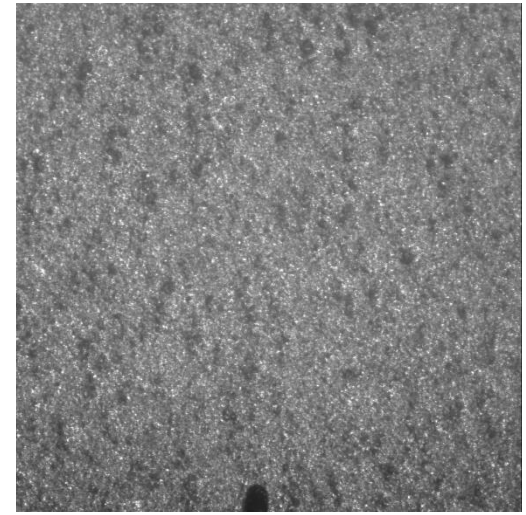


Image 2

Relative displacement field?

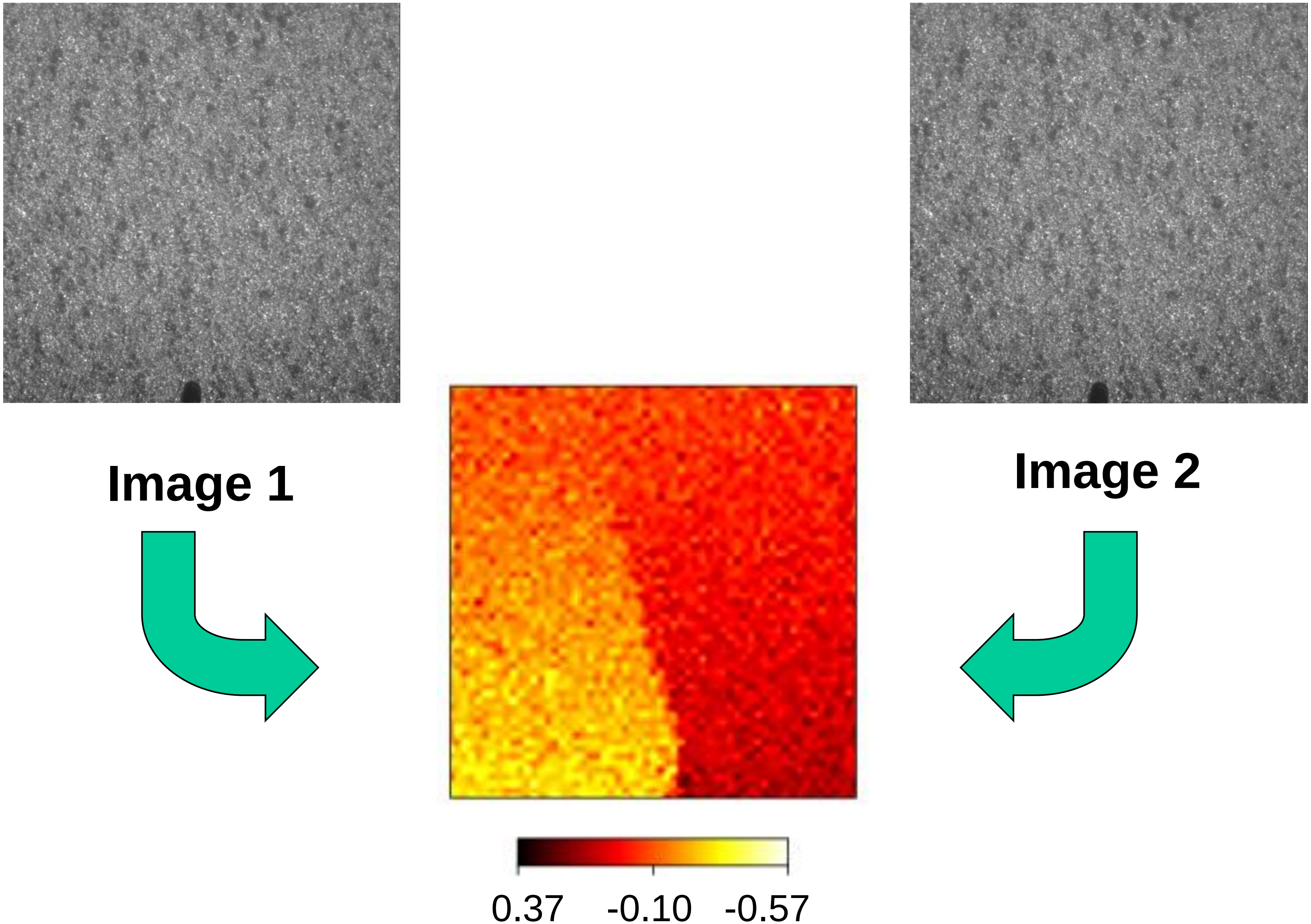
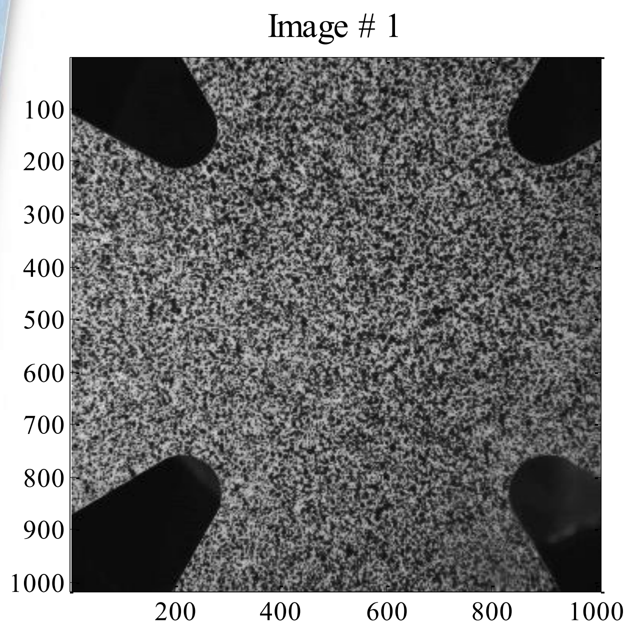
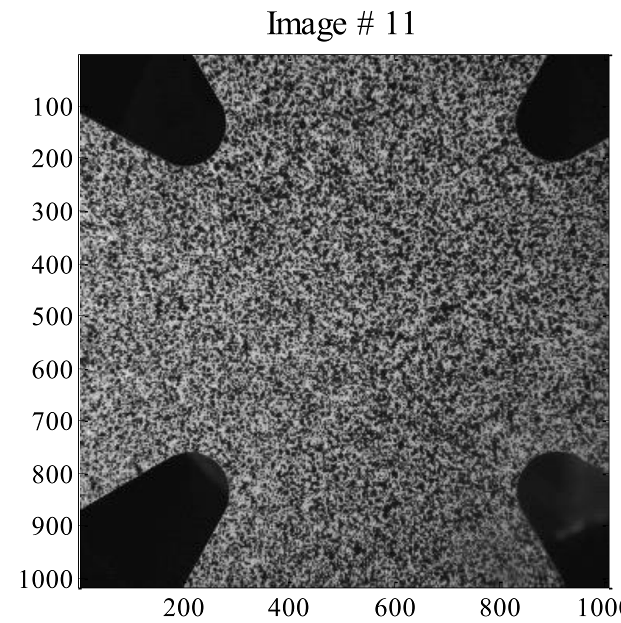


Image 1
Image 2
0.37
-0.10
-0.57

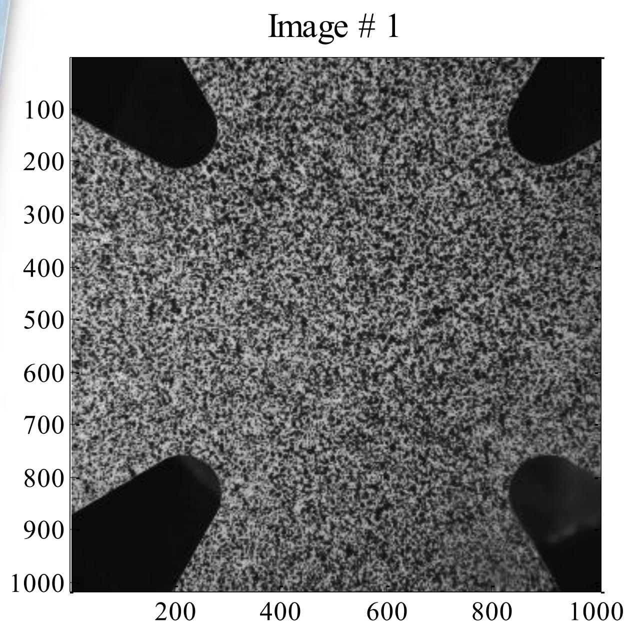


Reference image



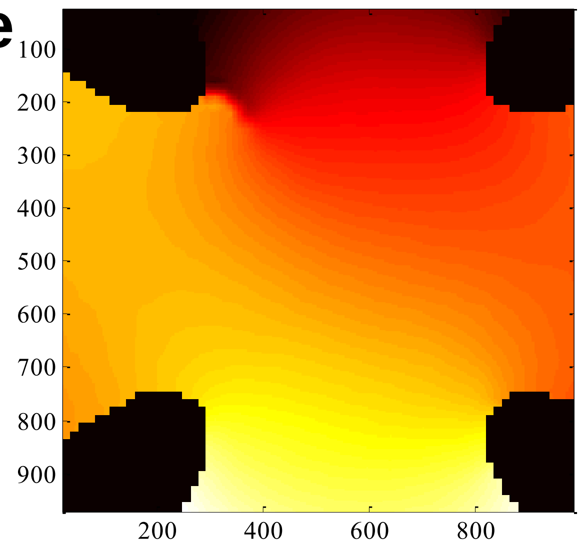
Deformed image

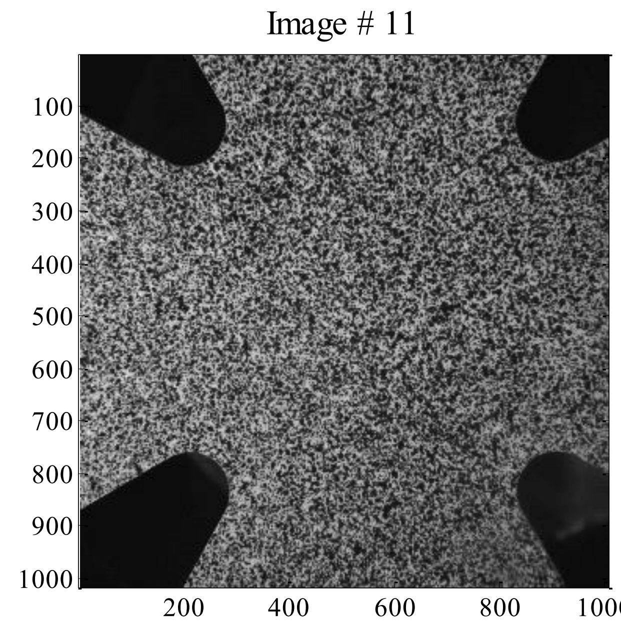
Relative displacement field?



Reference image

Displacement field u_y





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Displacement Fields Are Nice, But…

Can we get more?

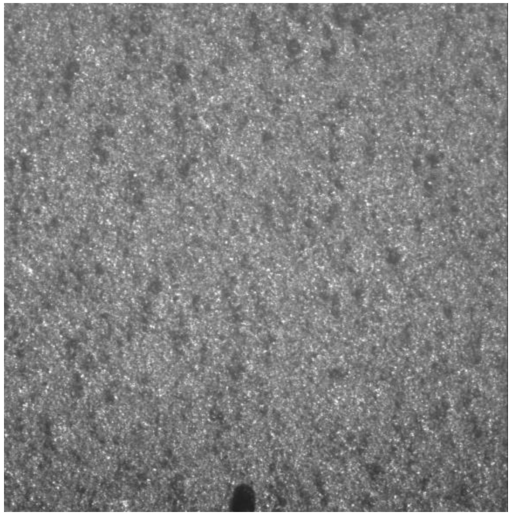


Image 1

Stress intensity factor, crack geometry

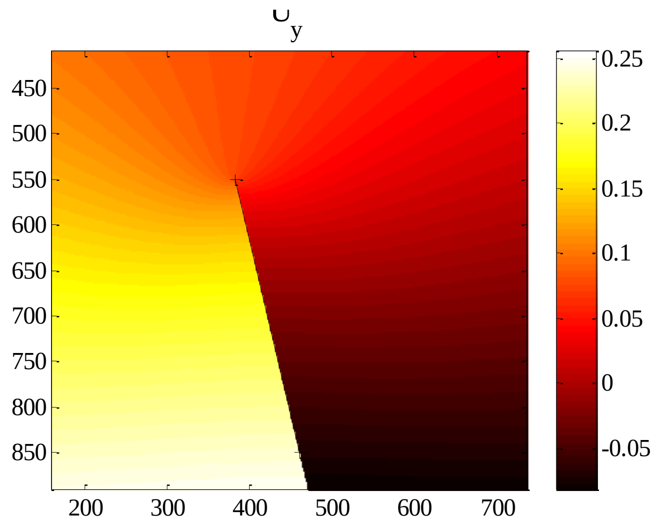
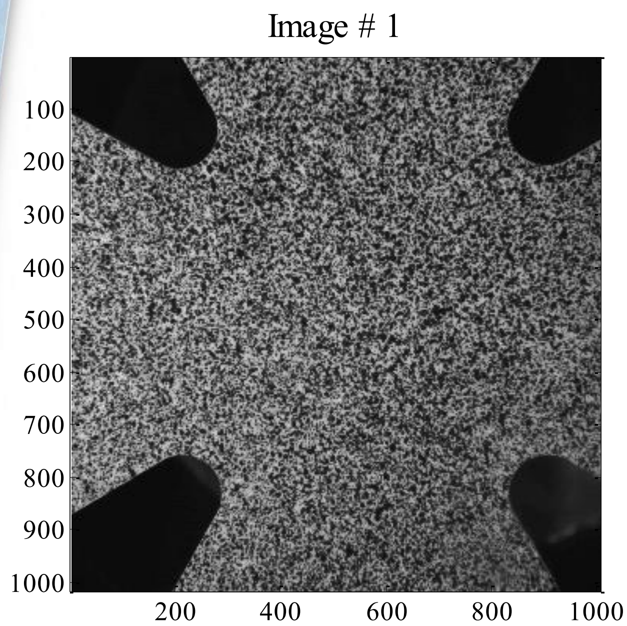


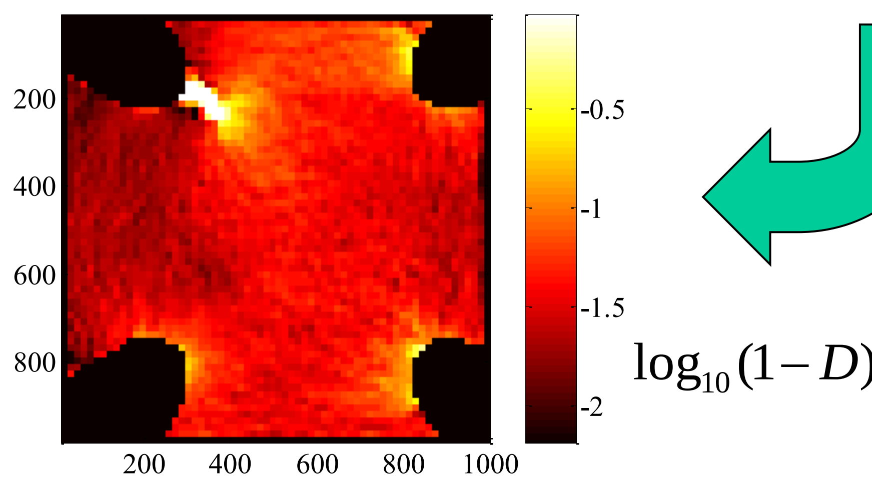
Image 2

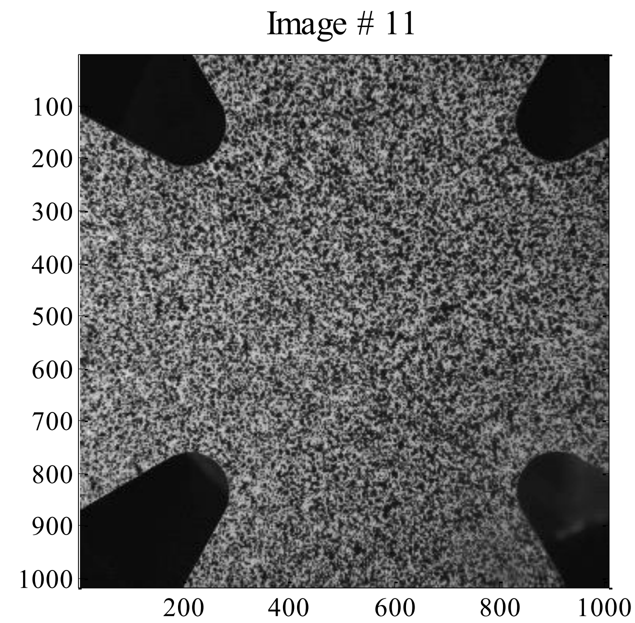
[SR *et al.*, 2006, *Int. J. Fract.* 140 pp. 141-157]



Reference image

Damage field





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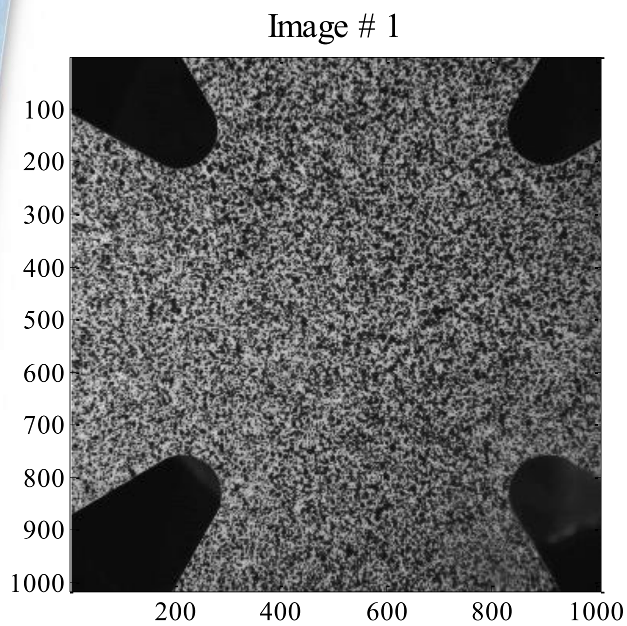
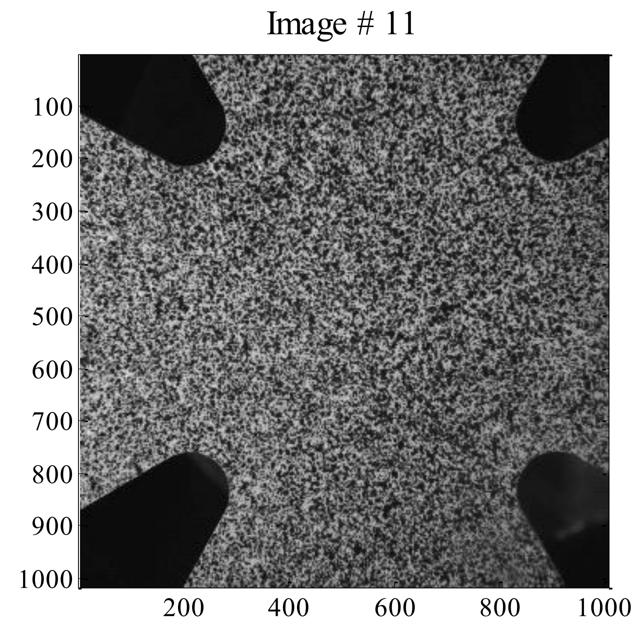


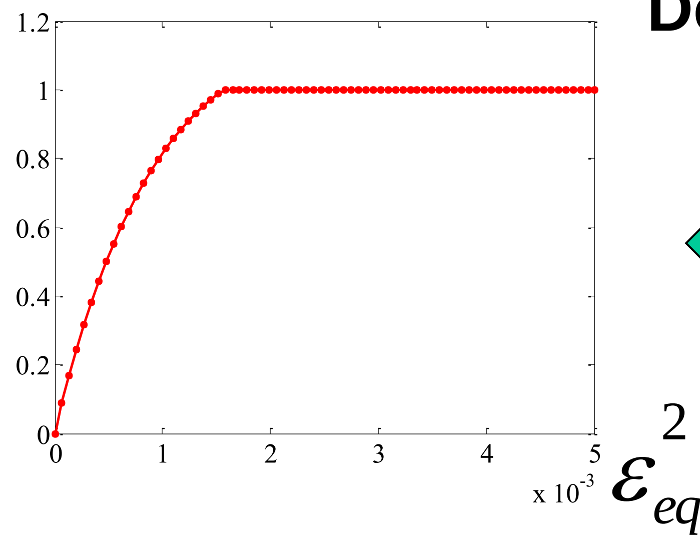
Reference image

Constitutive law



Deformed image

D



[SR *et al.*, 2008, *Exp. Mech.* 48 pp. 495-508]

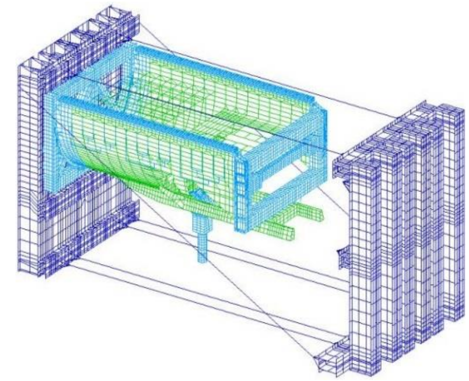
Question

What is the potential of this technique as compared to other means of measuring displacements/strains?

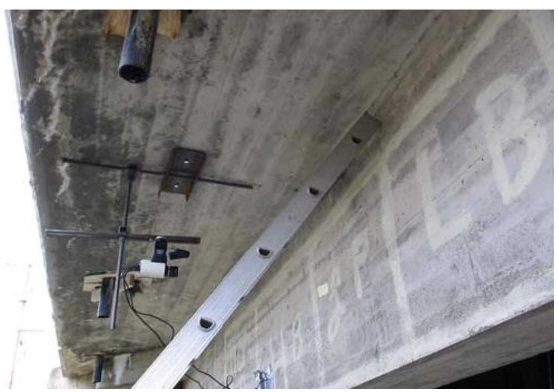
DIC Pros and Cons

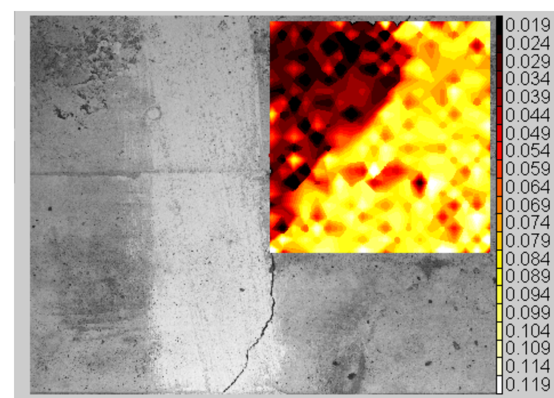
- Materials
- Scales
- Type of mechanical test
- Environment/temperature
- Type of behavior
- Uncertainty

Why Digital Image Correlation?



Digital image correlation provides natural link between experiments and simulations*,**





*[Chambon *et al.*, 2005, in Proc. *Photomécanique 2005*]
**[Küntz *et al.*, 2006, *Canad. J. Civil Eng.*, 33, pp. 1418-1425]

Measurement Challenges

- The measurement may *change the phenomenon, body, or substance* under study such that the quantity that is actually measured differs from the measurand
- The measurement may be affected by *noise and/or artifacts*

Outline

- Imaging devices
- Images
- Optical flow
- Ill-posedness and regularization
- Displacement basis
- Algorithms
- A priori error estimator
- Examples

IMAGING DEVICES

Question

What imaging technique do you know?

(e.g., scale, acquisition time, sensitivity, restrictions)

Optical Cameras

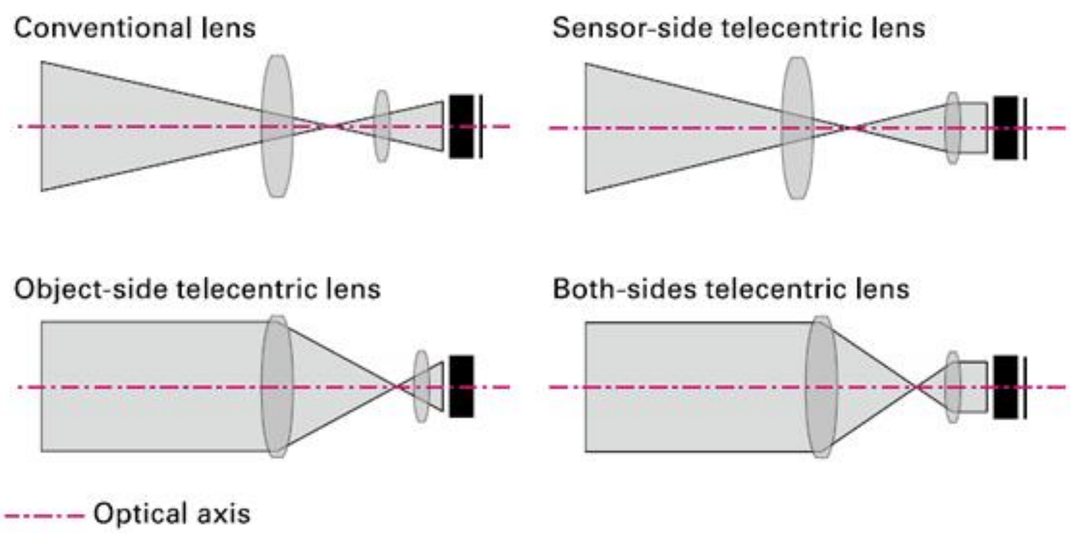
- Sensor technology CMOS/CCD
- Image definition
- Acquisition rate
- Color / BW
- Dynamic range
- SNR

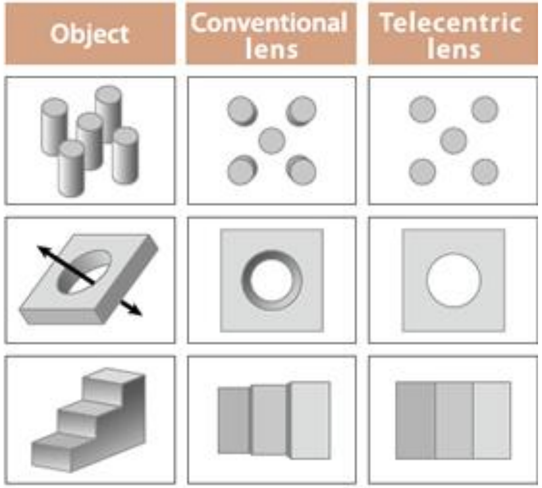
Optical Lenses

Long distance microscope

Optical Lenses

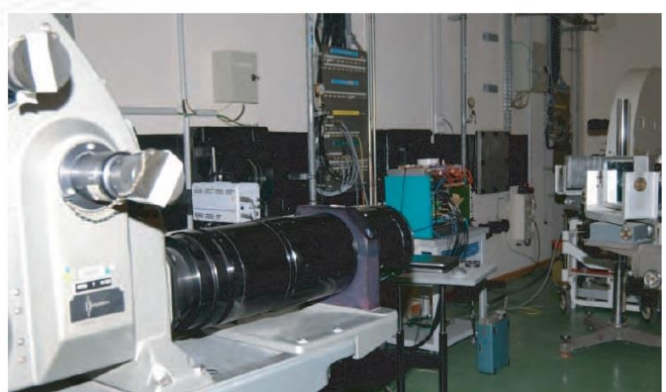
Telecentric lenses





Ultra-High Speed Cameras

Up to 10,000,000 frame/s



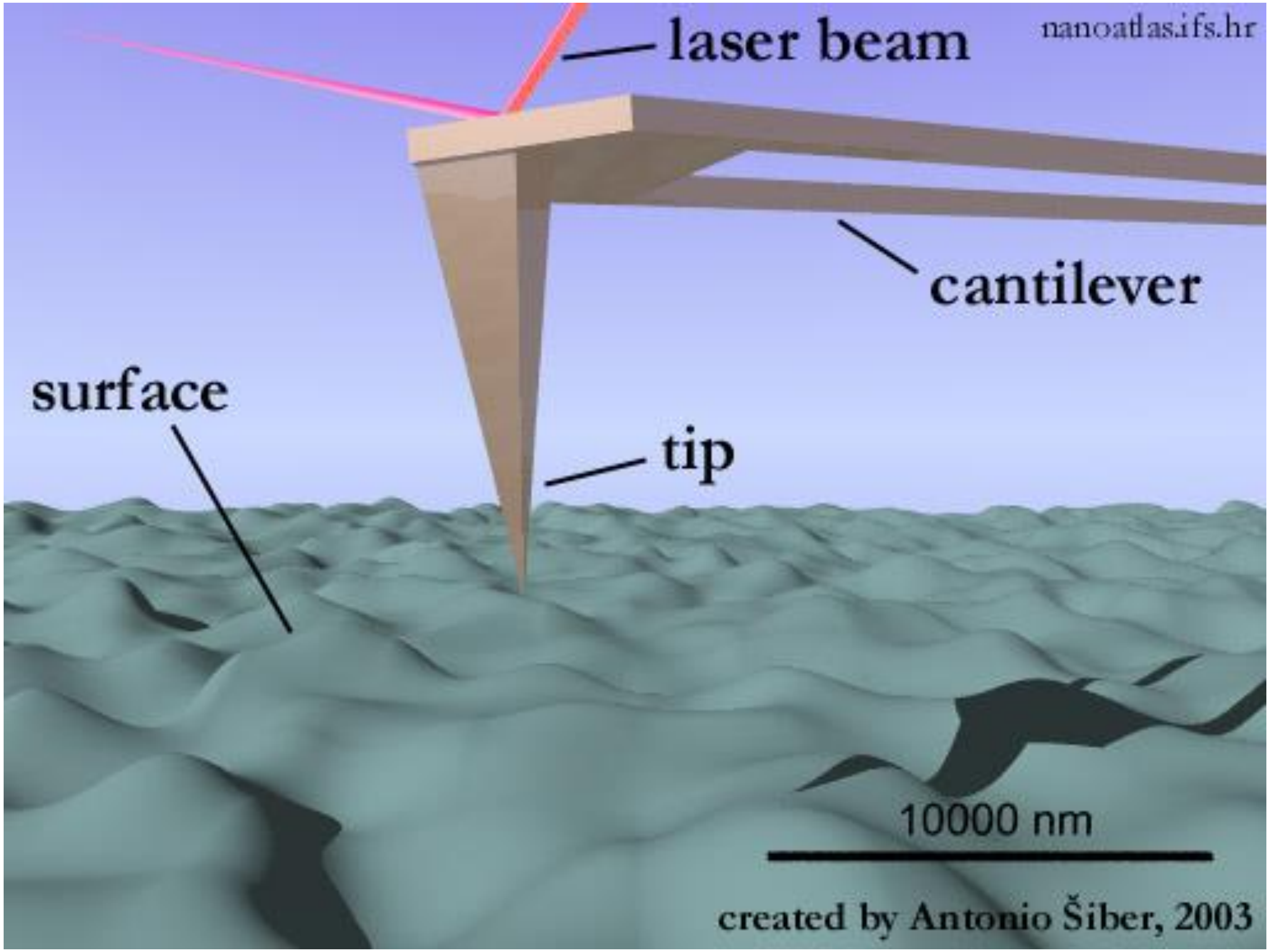
IR Cameras

Various spectral range

Scanning Electron Microscopes

- Different imaging modes SE/BSE
- Potentially 3D from BSE multi-detector
- + EBSD (crystallography)
- + EDS (chemical composition)

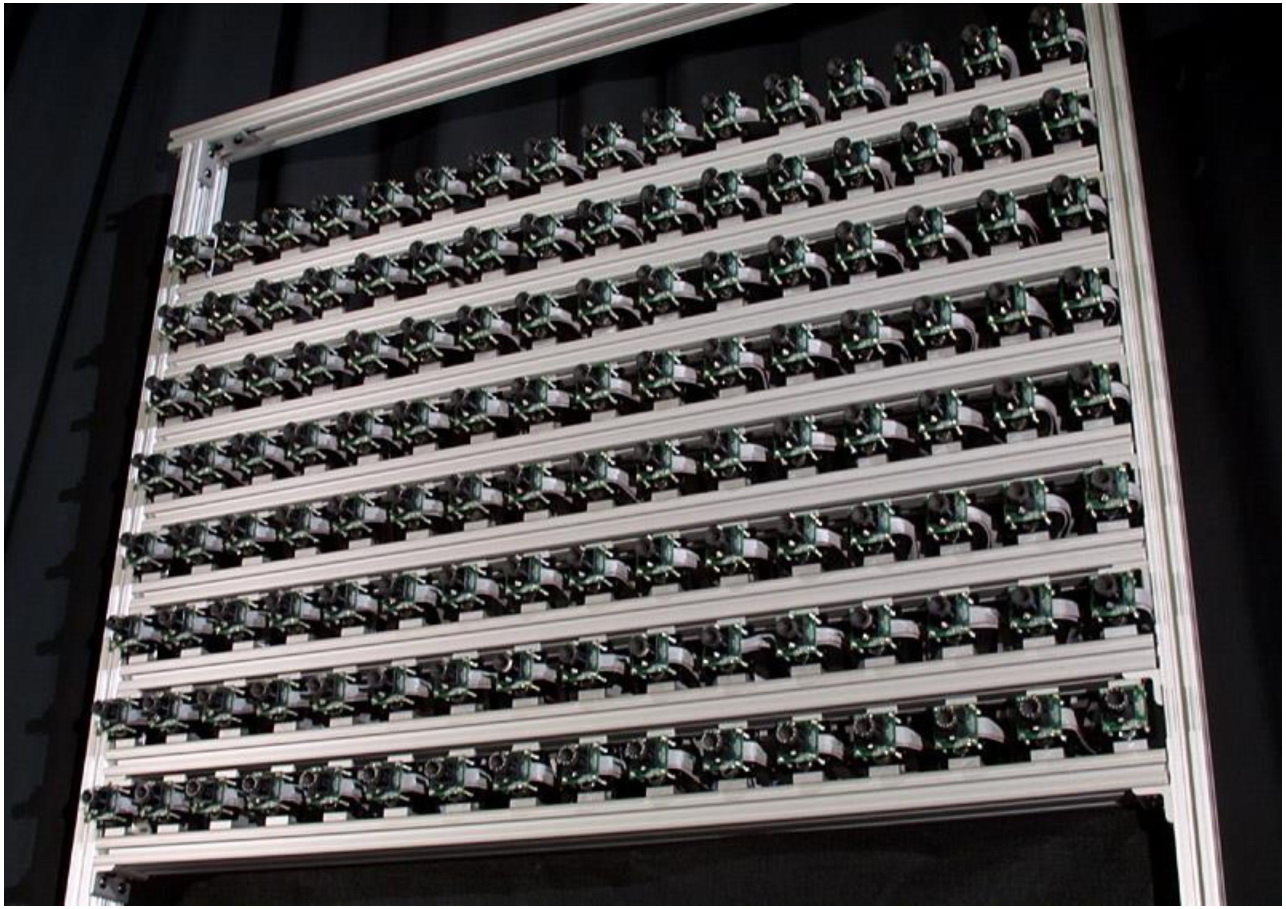
Atomic Force Microscopes



Multi Camera Systems

Stereovision

Multi Camera Systems



Stanford multi-camera array (128 cam.)

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Confocal Microscopes

3D images of surfaces

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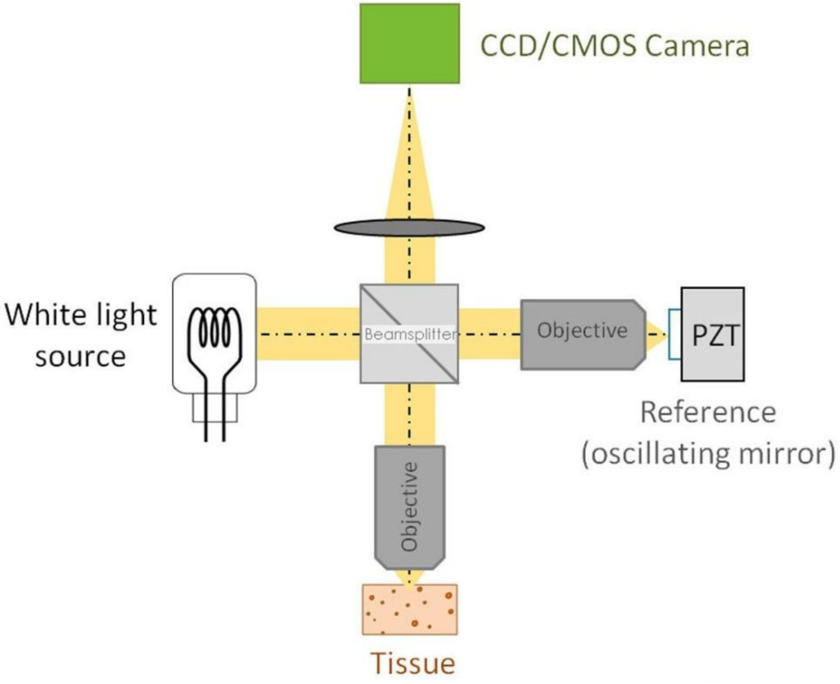
Optical Coherent Tomography

3D images of scatterers in semi-transparent media (biological tissues)



Tomography

ESRF

with X-rays…

ILL

… or neutrons

in Grenoble

X-Ray Tomography

… in the lab!

30

Magnetic Resonance Imaging

From nano... to Macro: The Same Technique!

Digital images obtained with:

- Optical camera
- High speed camera
- Camcorder
- IR camera
- SEM
- AFM
- Multiple cameras
- Satellite pictures

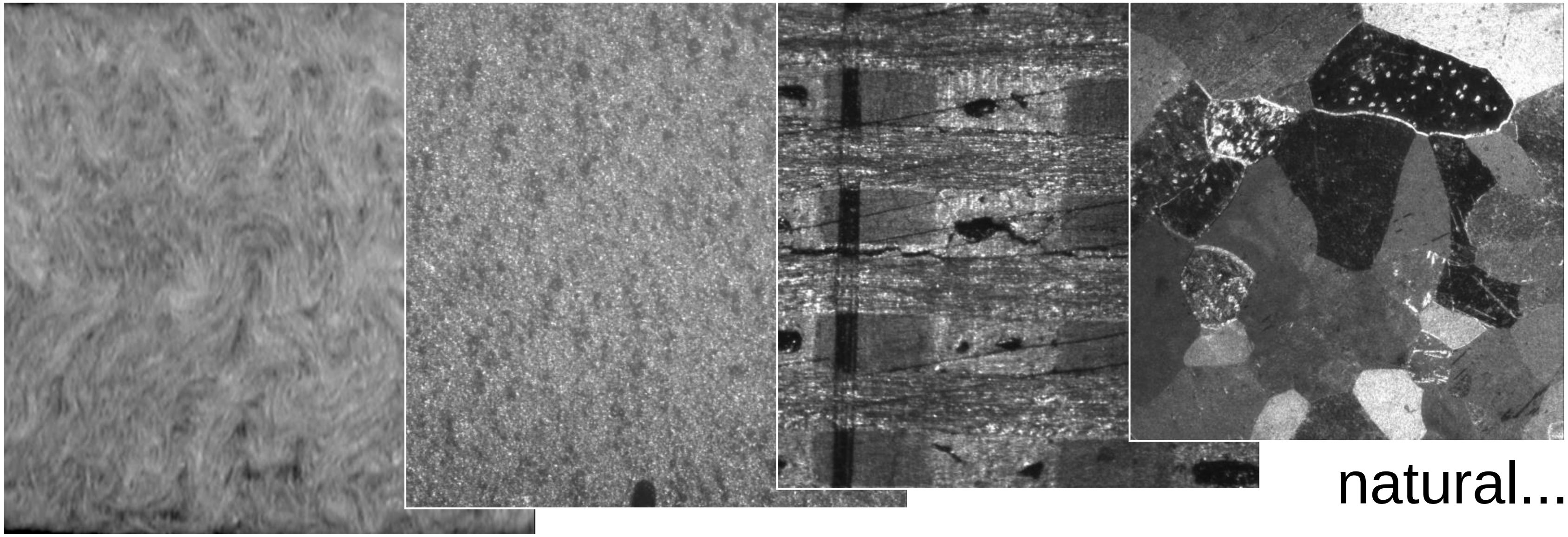
} Stereo-correlation 3D fields in 2D

- XRay-µTomography
- MRI
- OCT

} 3D fields in 3D

IMAGES

Characterization of ‘Texture’



natural...

or artifical...

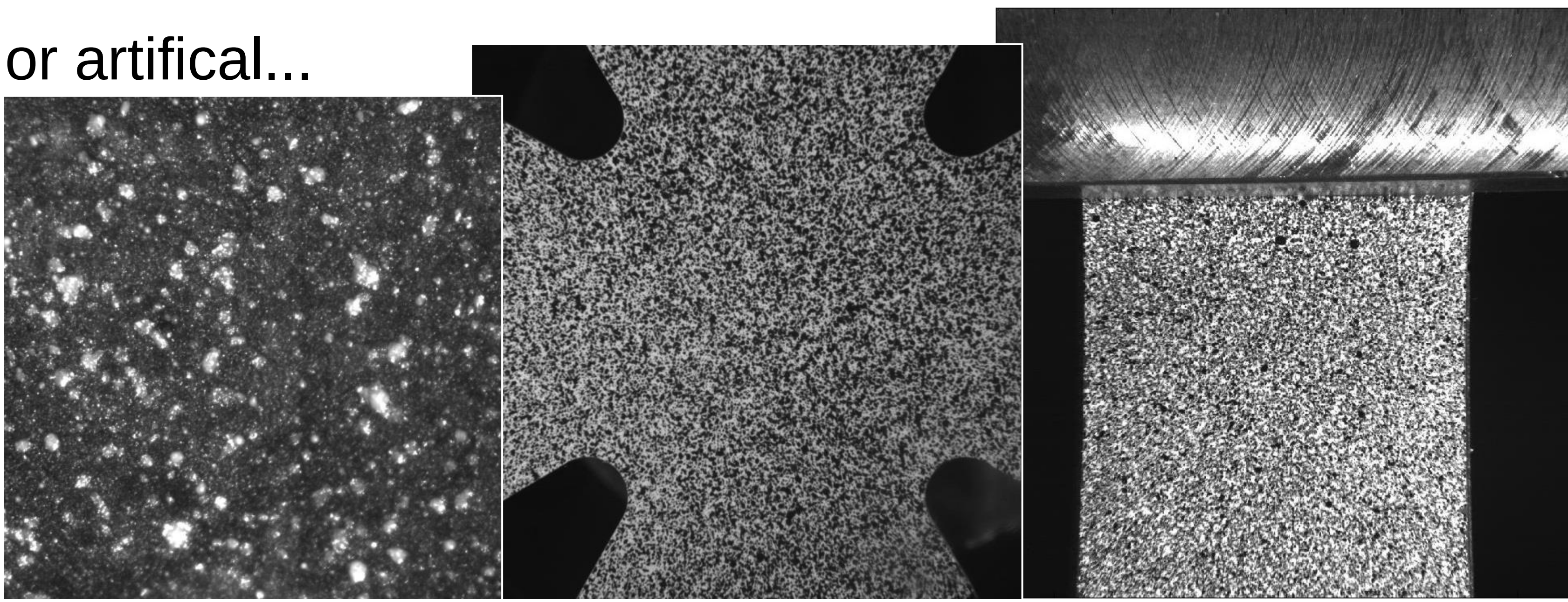


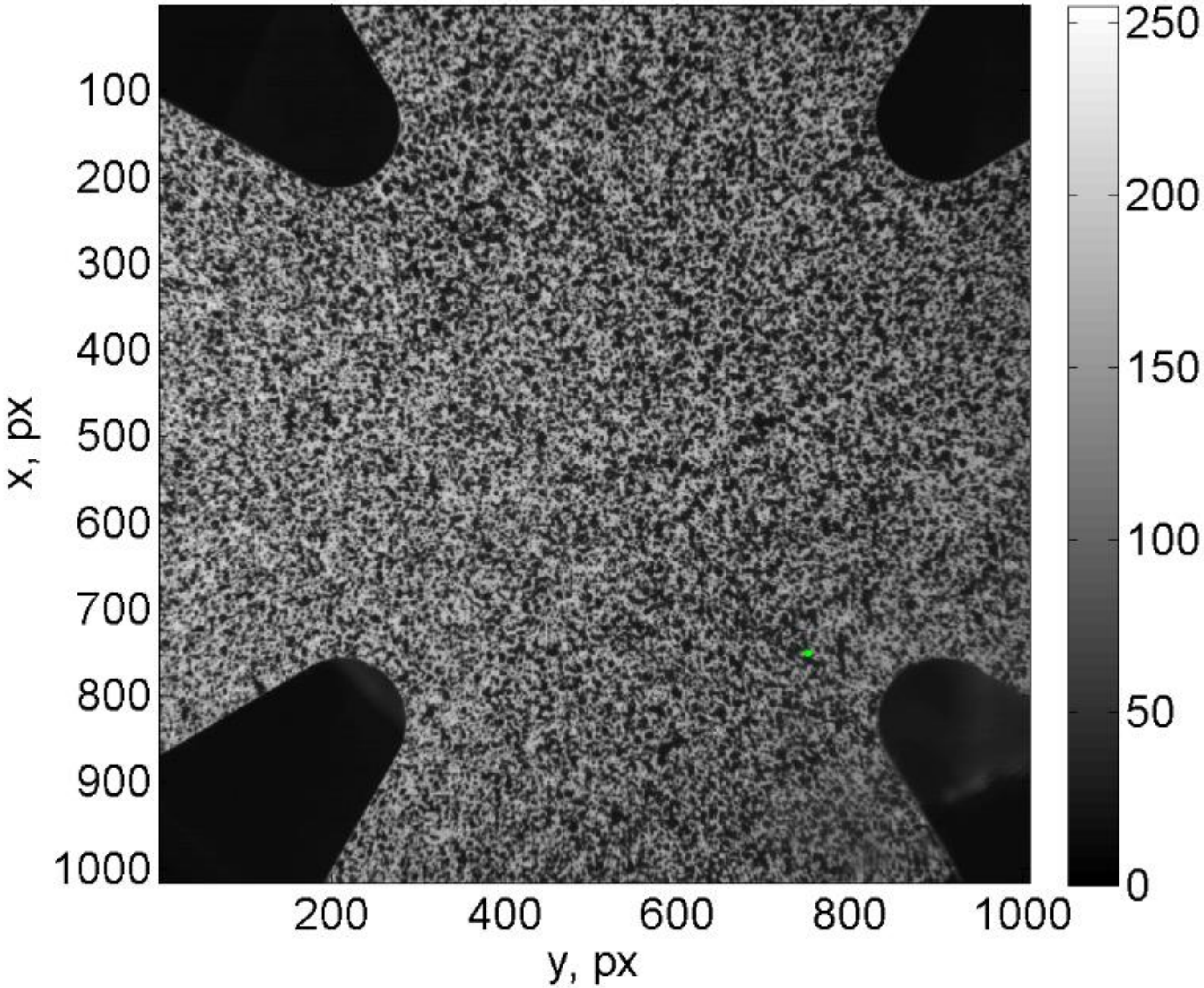
Image 'Texture'

Scalar *field of passive markers*

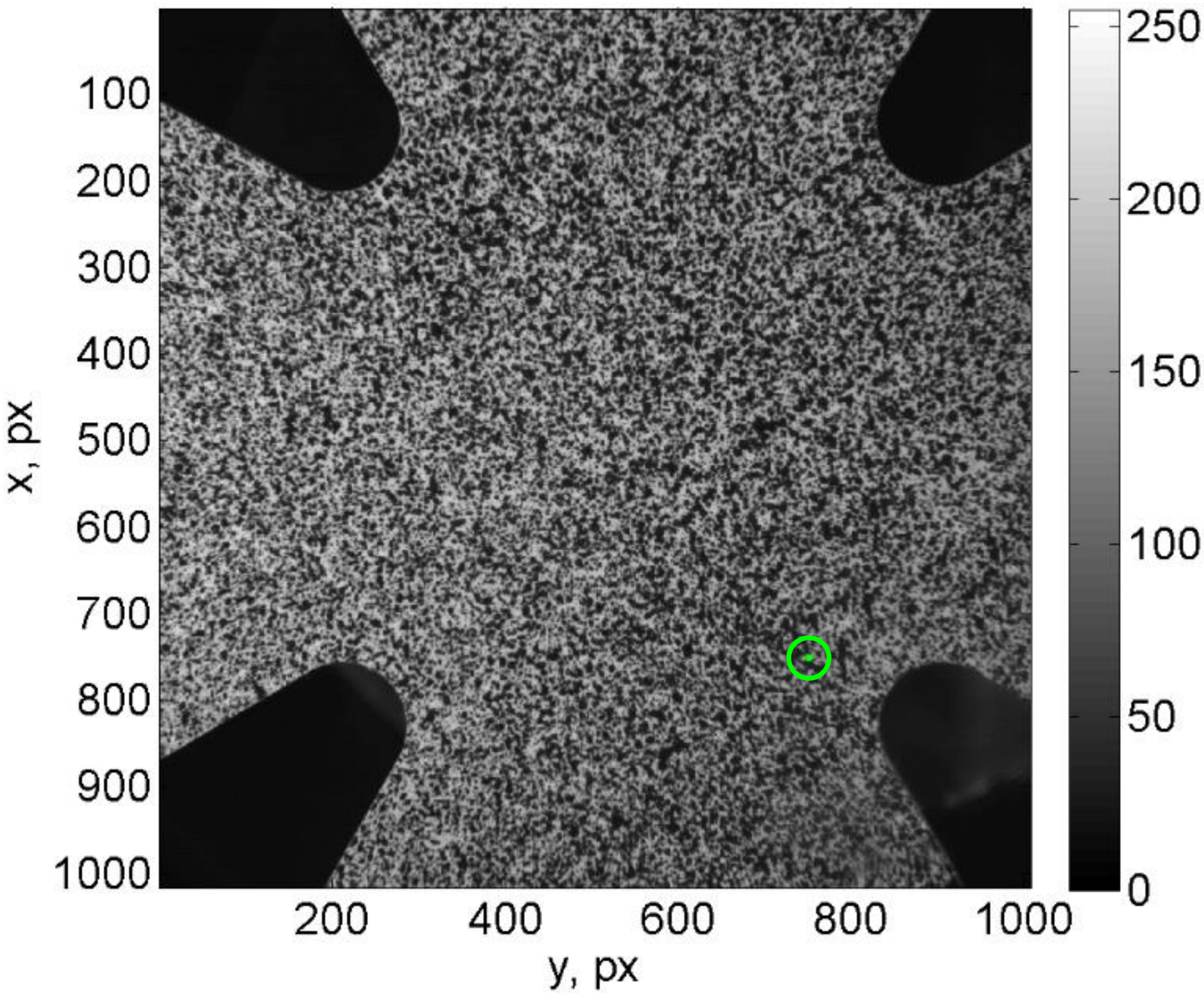
$$f(\mathbf{x})$$

- $\mathbf{x}$ is discrete: pixel (= pic<u>t</u>ure el<u>e</u>ment)
- f is discrete: gray levels

What is an Image?



What is an Image?



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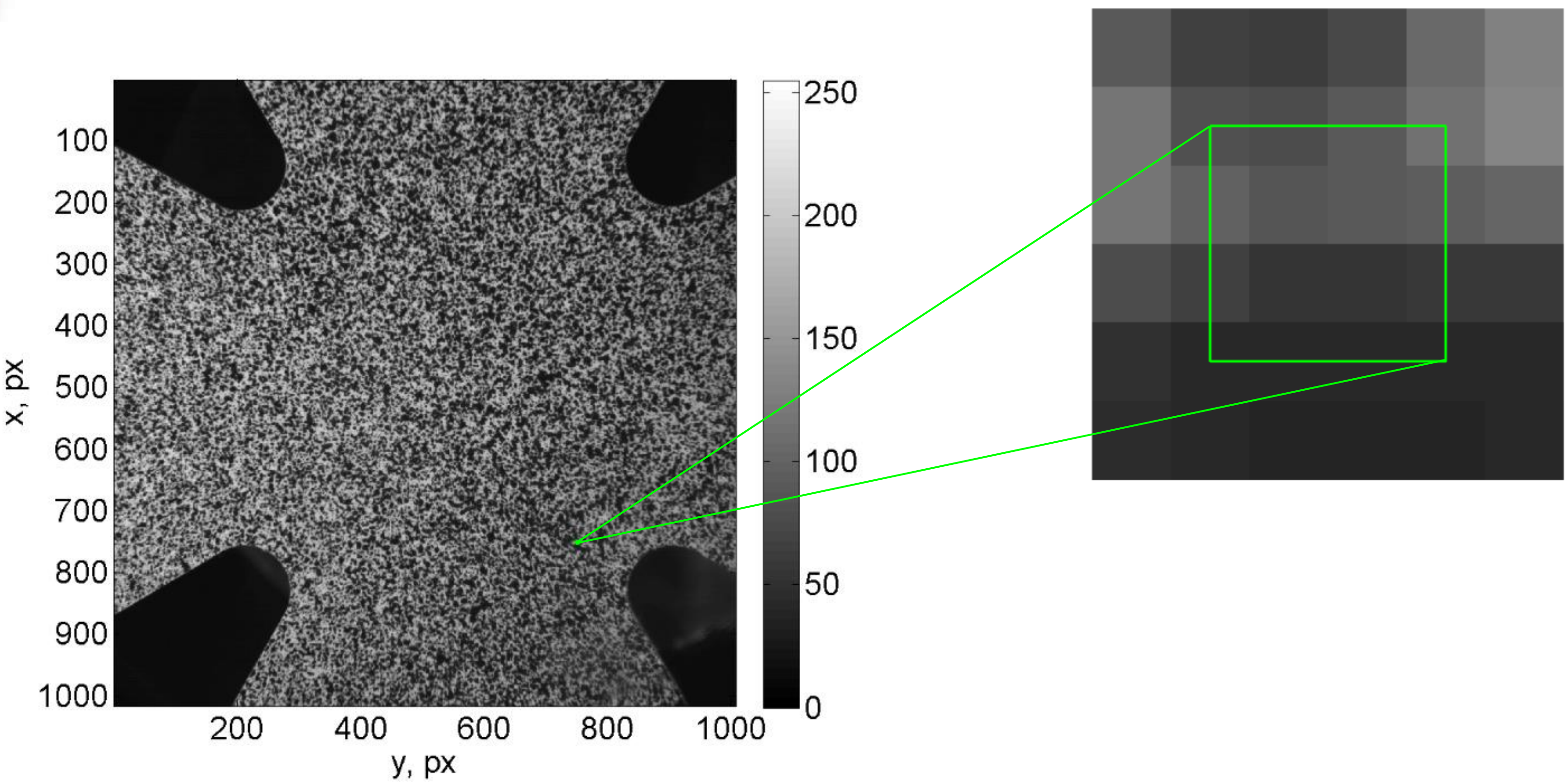

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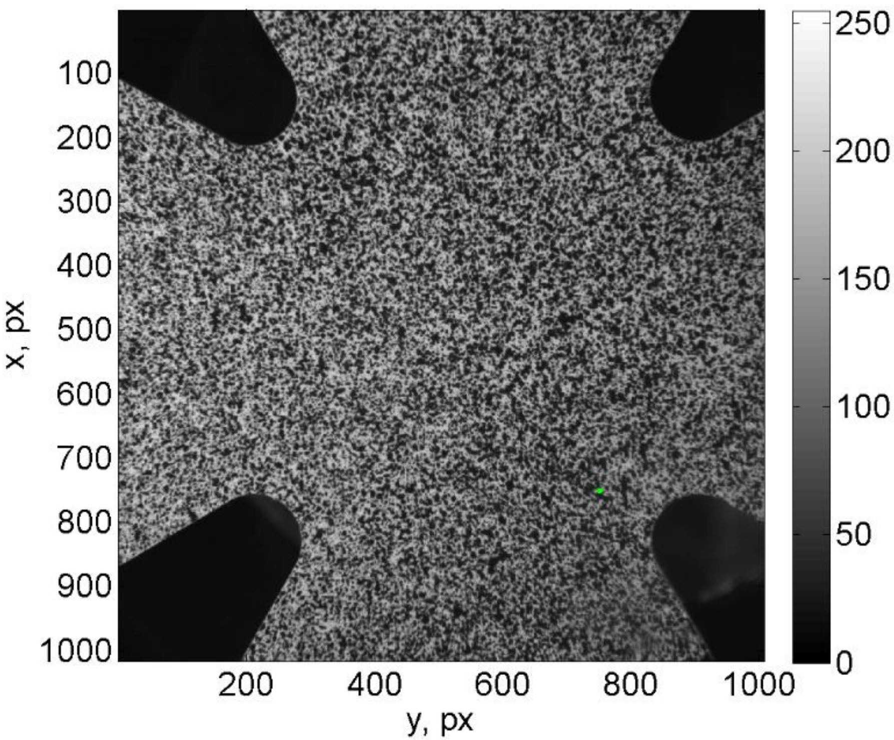
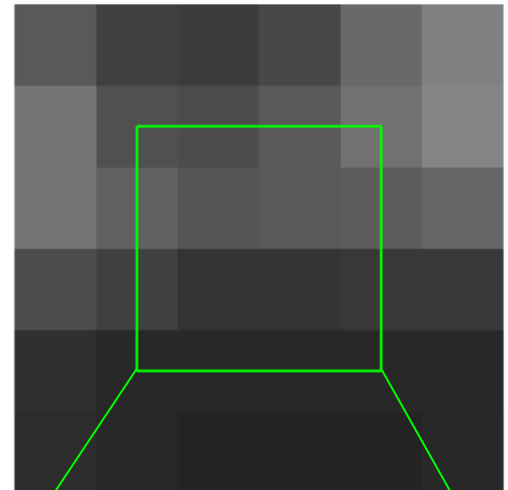


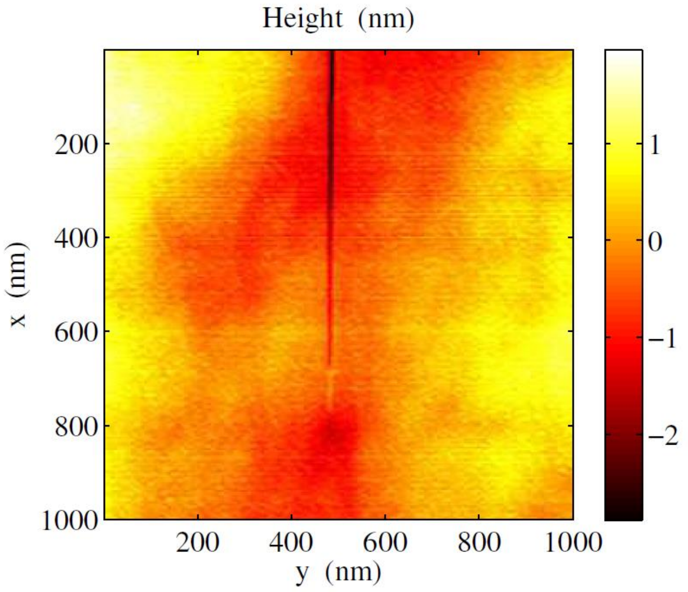


82	78	90	113
96	84	90	92
67	54	54	59
43	40	40	42

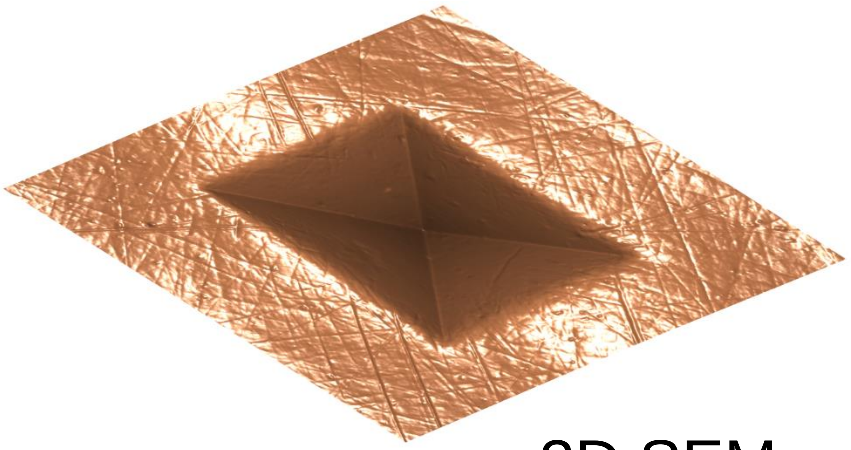
Usual format : uint8

Image ‘Texture’

Sometimes f has a different meaning e.g., related to temperature (IR-camera), or topography (AFM, confocal)



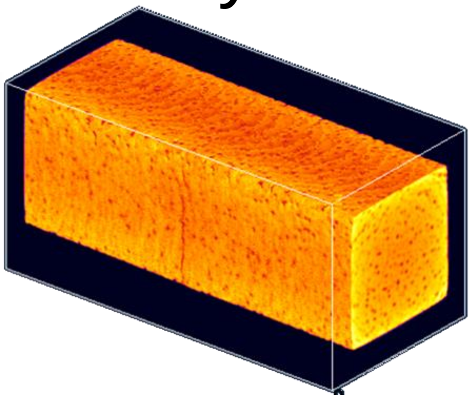
AFM

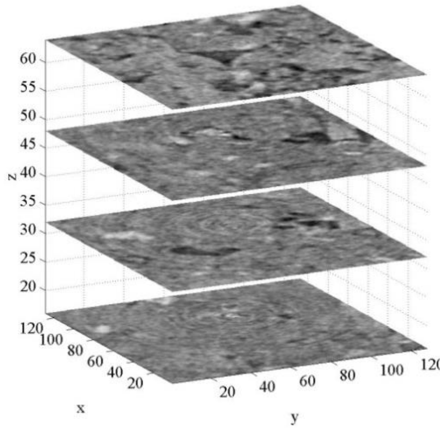


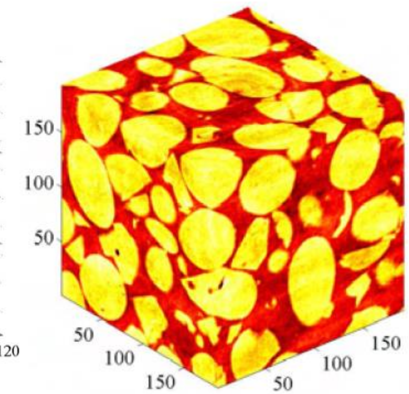
3D SEM

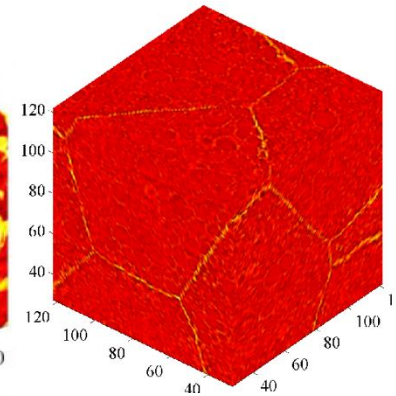
3D Images

- Synchrotron XCT

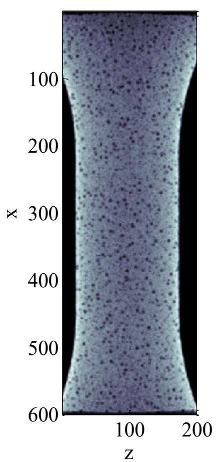


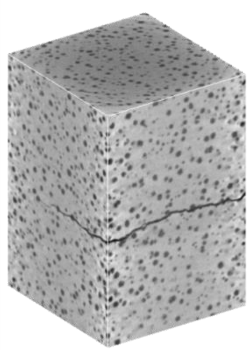


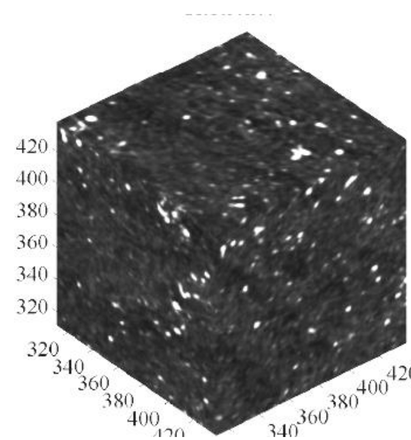


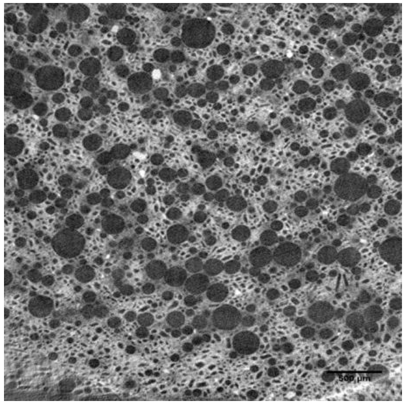


- Lab scale XCT



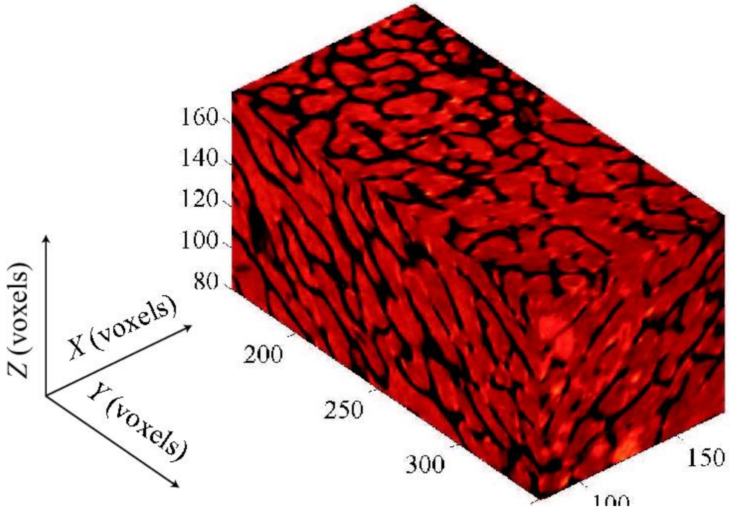




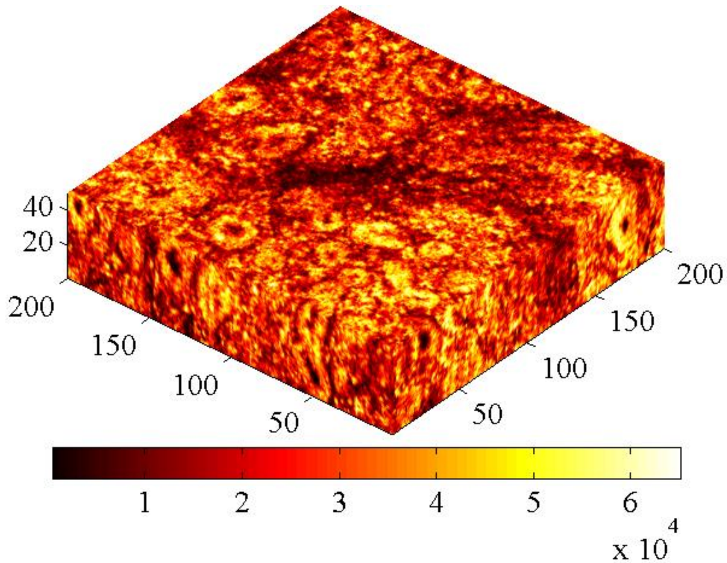


3D Images

- MRI



- OCT

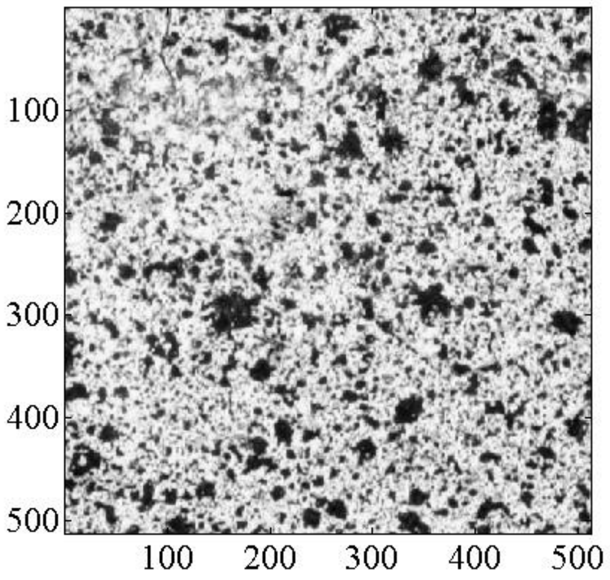


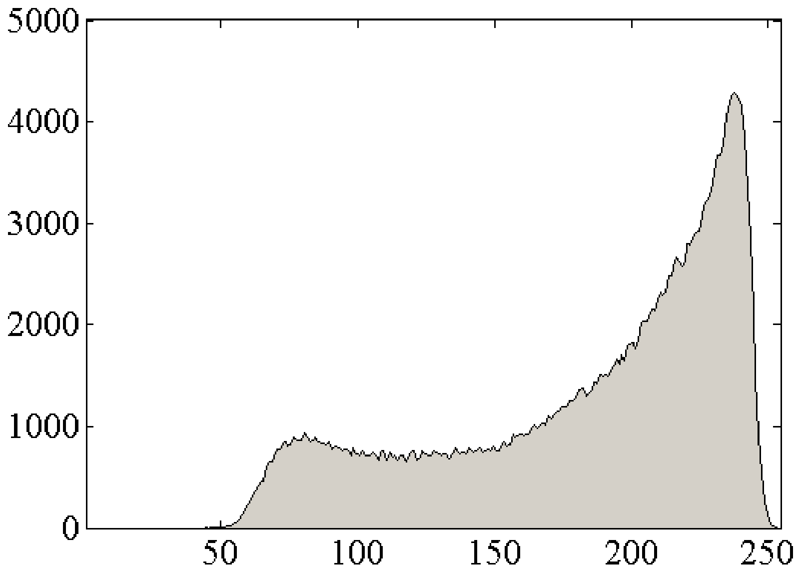
Question

How should the images be characterized?

Histogram

Gray levels



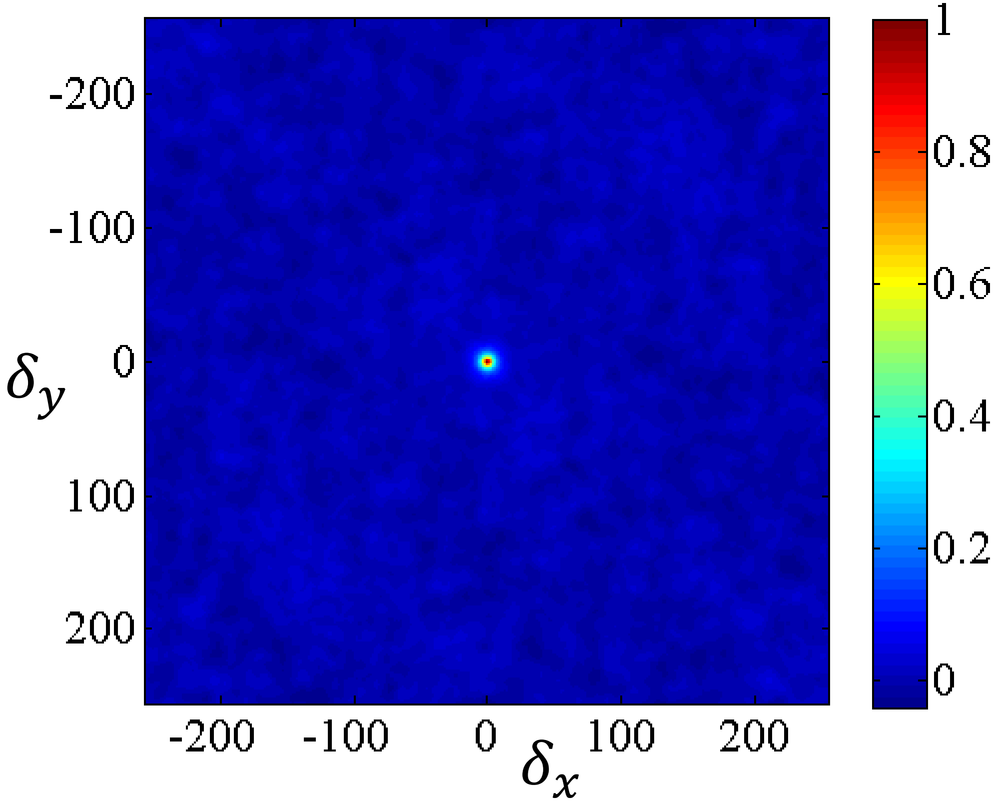


$$\langle f \rangle = \langle f(\mathbf{x}) \rangle_{\mathbf{x}}$$

$$\sigma^2 = \langle (f(\mathbf{x}) - \langle f \rangle)^2 \rangle_{\mathbf{x}}$$

Auto-Correlation

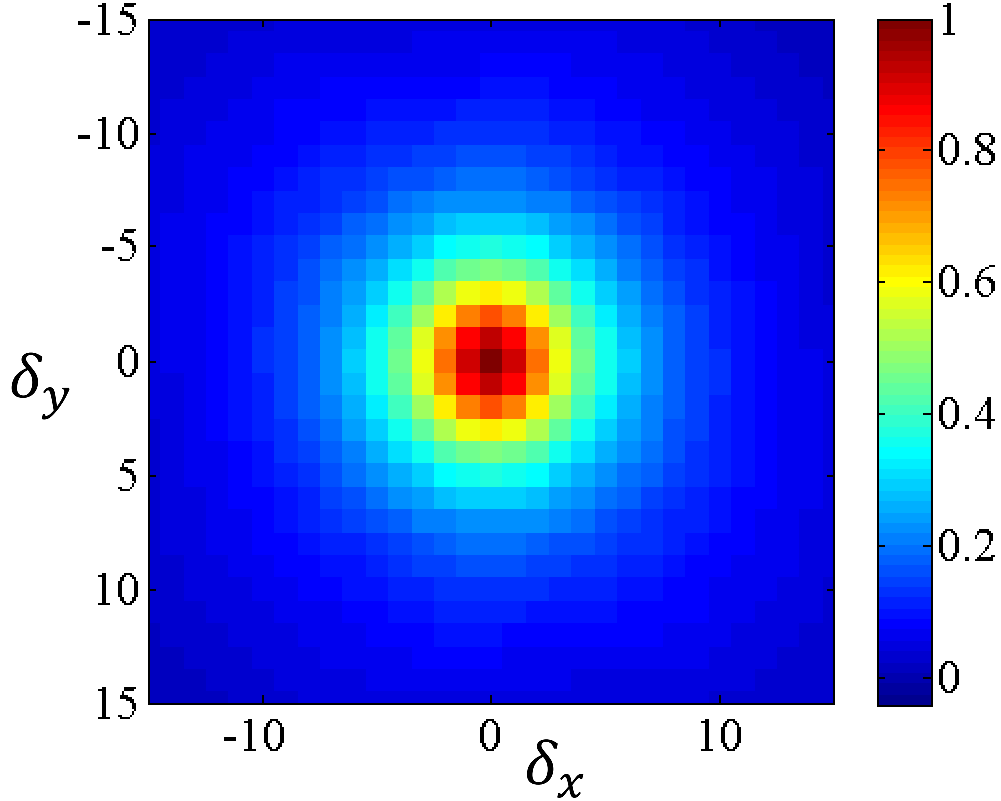
$$C(\boldsymbol{\delta}) = \frac{1}{\sigma^2}\langle (f(\mathbf{x}) - \langle f \rangle)(f(\mathbf{x}+\boldsymbol{\delta}) - \langle f \rangle)\rangle_{\mathbf{x}}$$



Auto-Correlation

$$C(\boldsymbol{\delta}) = \frac{1}{\sigma^2}\langle(f(\mathbf{x}) - \langle f\rangle)(f(\mathbf{x}+\boldsymbol{\delta}) - \langle f\rangle)\rangle_{\mathbf{x}}$$

Zoom



Auto-Correlation

- How does the gray levels vary with the distance?

$$\left\langle \left(f(\mathbf{x}+\boldsymbol{\delta}) - f(\mathbf{x}) \right)^2 \right\rangle$$

$$= \langle (f(\mathbf{x}+\boldsymbol{\delta}) - \langle f \rangle - f(\mathbf{x}) + \langle f \rangle)^2 \rangle$$

$$= \langle (f(\mathbf{x}+\boldsymbol{\delta}) - \langle f \rangle)^2 \rangle + \langle (f(\mathbf{x}) - \langle f \rangle)^2 \rangle$$

$$-2\langle (f(\mathbf{x}+\boldsymbol{\delta}) - \langle f \rangle)(f(\mathbf{x}) - \langle f \rangle) \rangle$$

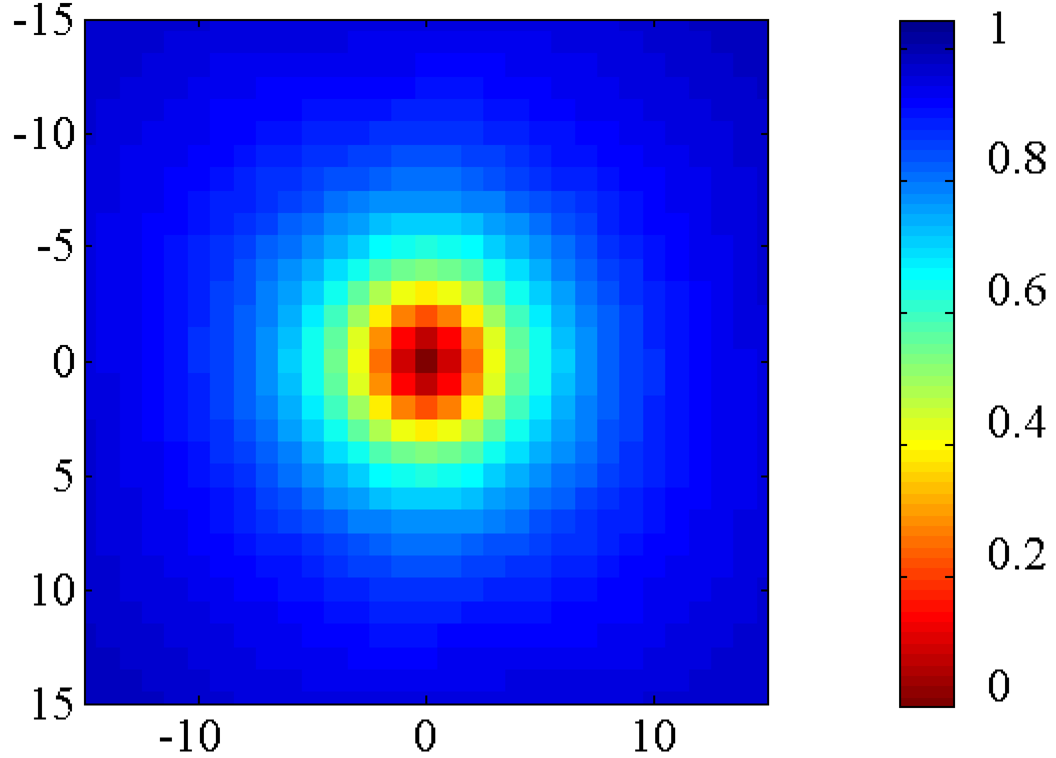
$$= \sigma^2 + \sigma^2 - 2\sigma^2 \mathrm{C}(\boldsymbol{\delta})$$

$$= 2\sigma^2 \left(1 - C(\boldsymbol{\delta}) \right)$$

Auto-Correlation

$$\frac{1}{2\sigma^2}\langle(f(\mathbf{x}+\boldsymbol{\delta})-f(\mathbf{x}))^2\rangle_{\mathbf{x}}$$

Zoom



Practical computation

- Fourier Transform

$$\hat{f}(k) \equiv \int_{-\infty}^{+\infty} f(x) \exp(ikx)\, dx$$

$$\hat{C}(k) = \iint f(x)\, f(x+\delta) \exp(ik\delta)\, dx d\delta$$

$$= \iint f(x)\, f(x+\delta) \exp(ik(\delta+x) - ikx)\, dx d\delta$$

$$= \left(\int f(x) \exp(-ikx)\, dx \right) \left(\int f(x') \exp(ikx')\, dx' \right)$$

$$= \hat{f}(-k)\ \hat{f}(k)$$

$$= \left| \hat{f}(k) \right|^2$$

Practical computation

- In practice

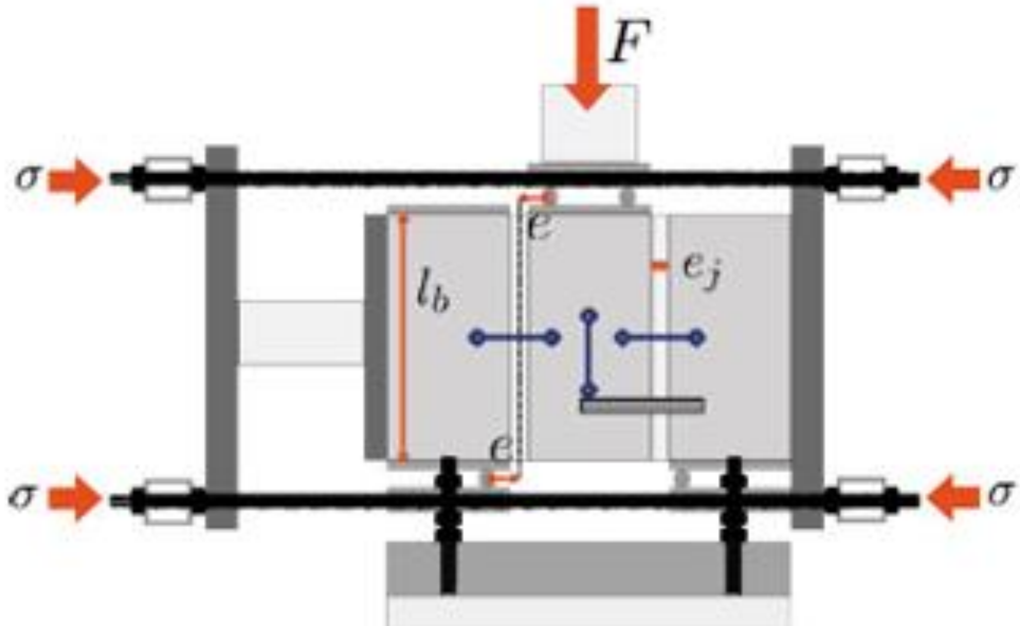
$$f(\mathbf{x}) \xrightarrow{\text{FFT}} \hat{f}(\mathbf{k}) \rightarrow \hat{C}(\mathbf{k}) = \left|\hat{f}(\mathbf{k})\right|^2 \xrightarrow{\text{FFT}^{-1}} C(\mathbf{x})$$

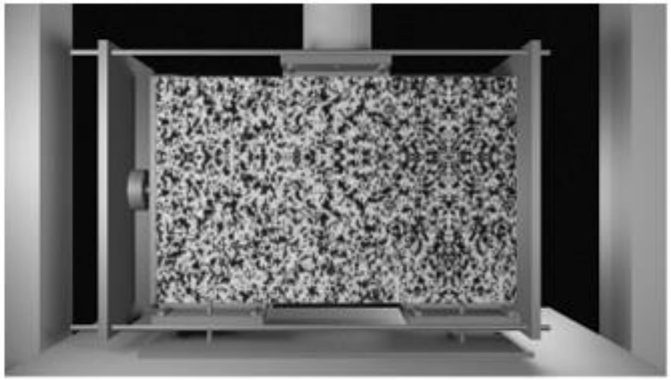
Auto-Correlation

- What is the gray level at non-integer pixel position?
- Appreciation of the regularity of the image legitimates a given subpixel interpolation scheme
- A pixel integrates information over some scale, and can be seen as a convolution providing smoothness

SOME EXAMPLES

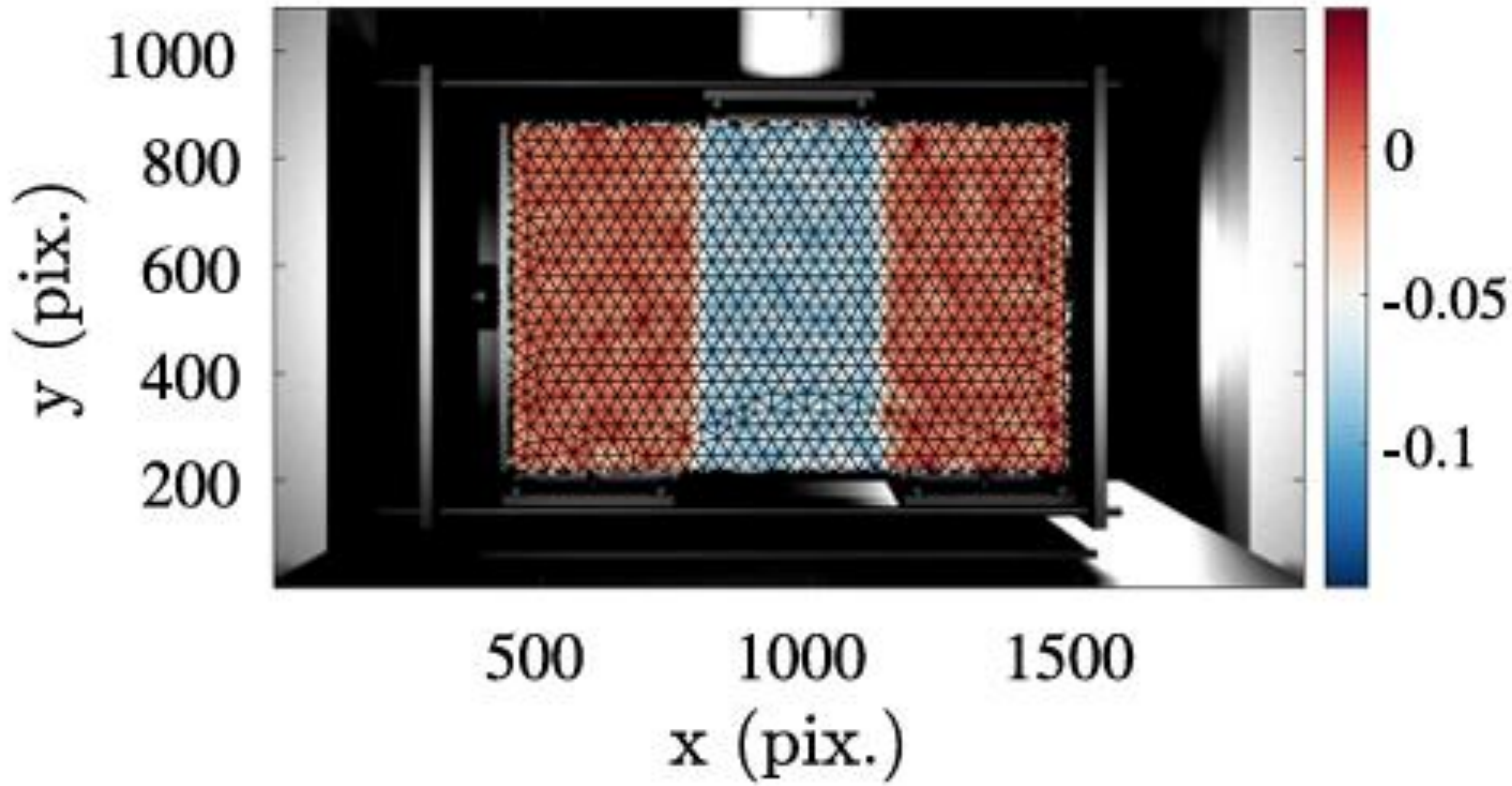
Triplet test on masonry





Collin et al,, *Materials and Structures*, 2025, 58, 203

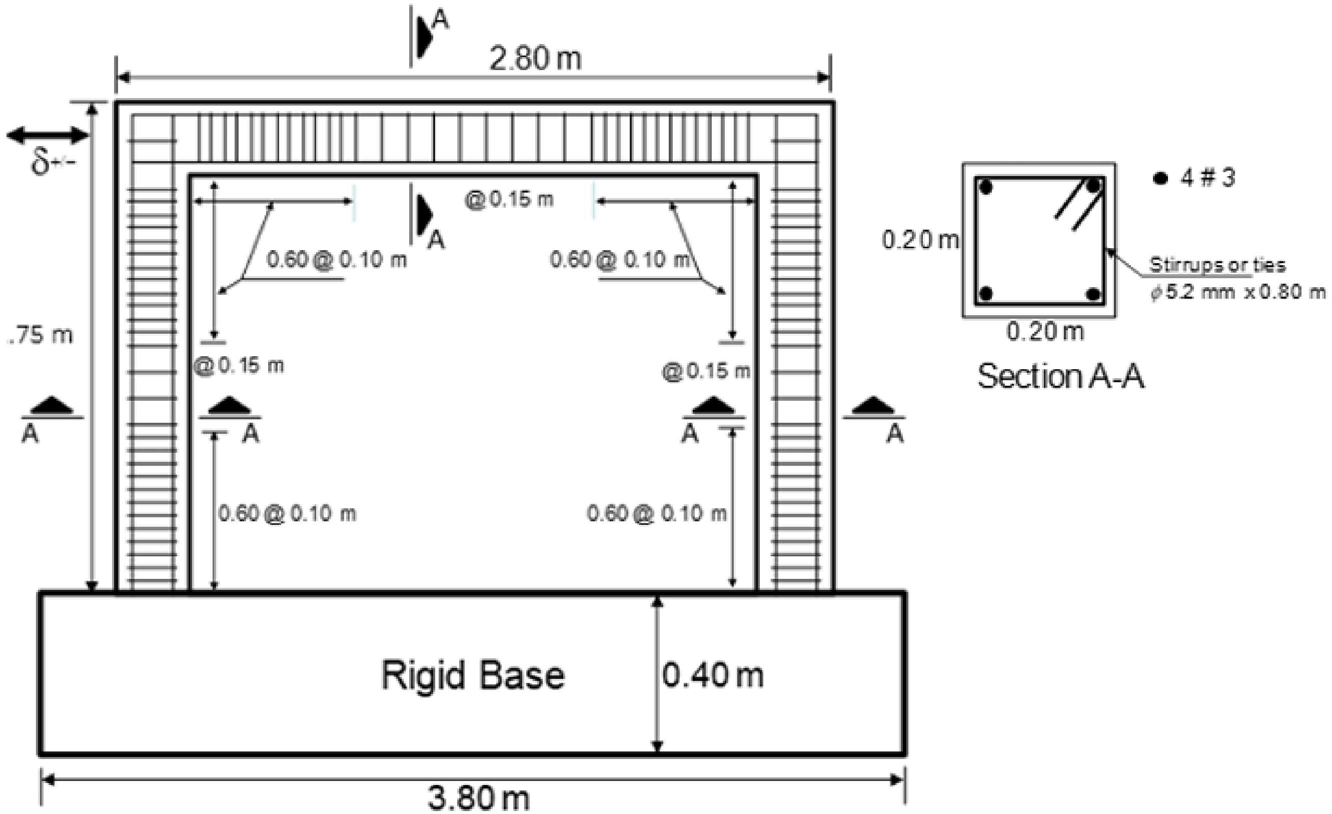
Triplet test on masonry



(a) Displacement field in the Y-direction.

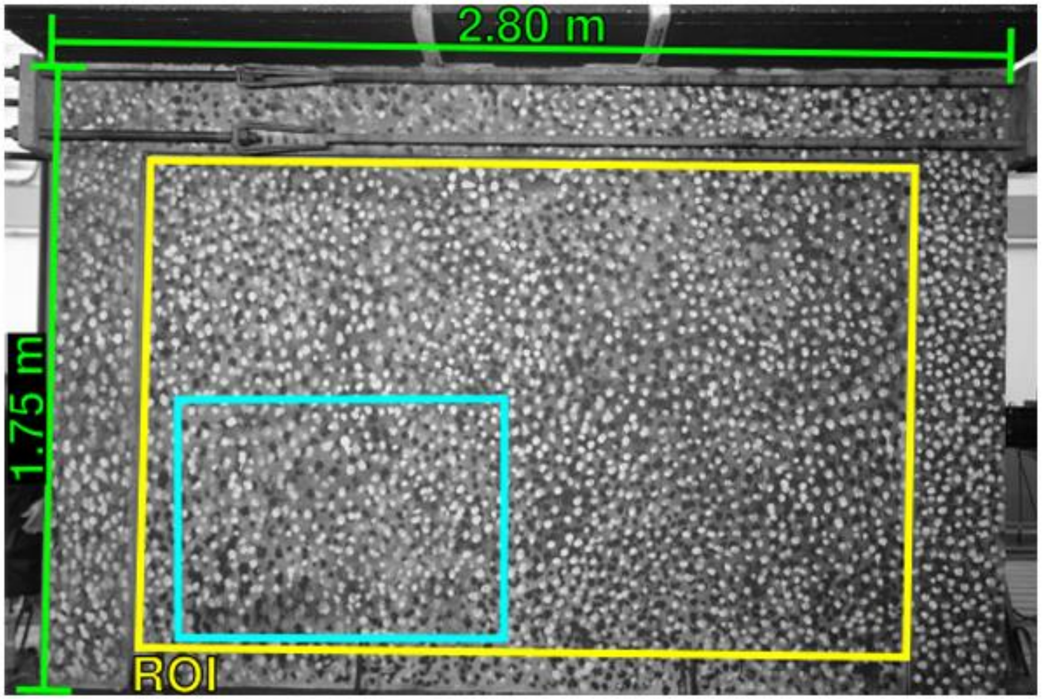
Collin et al,, *Materials and Structures*, 2025, 58, 203

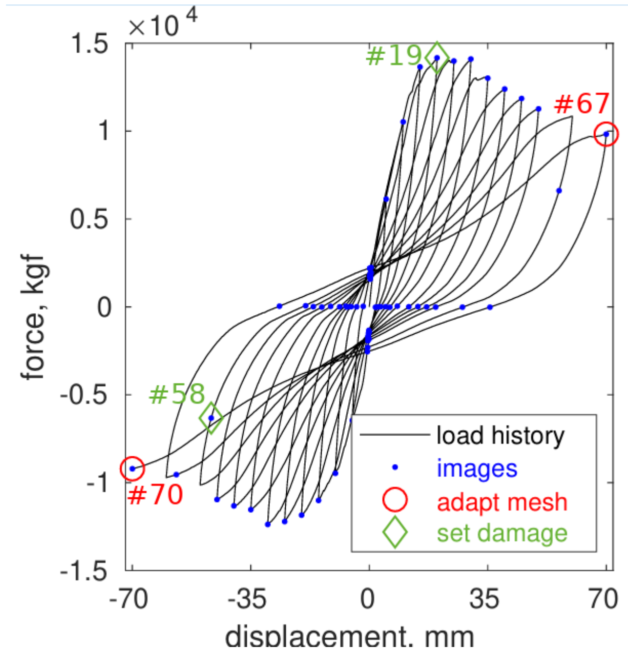
Masonry infilled frame



Sciutti et al, *Mathematics and Mechanics of Solids*, 2025, 30 (1), pp.93-115.

Masonry infilled frame

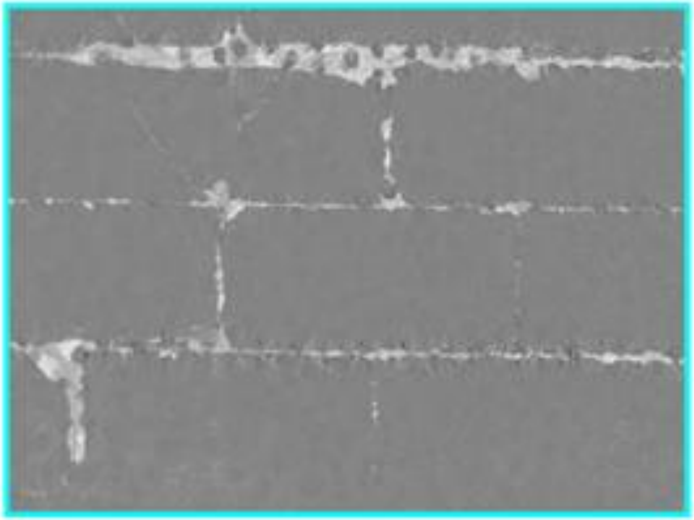




Sciutti et al, *Mathematics and Mechanics of Solids*, 2025, 30 (1), pp.93-115.

Masonry infilled frame

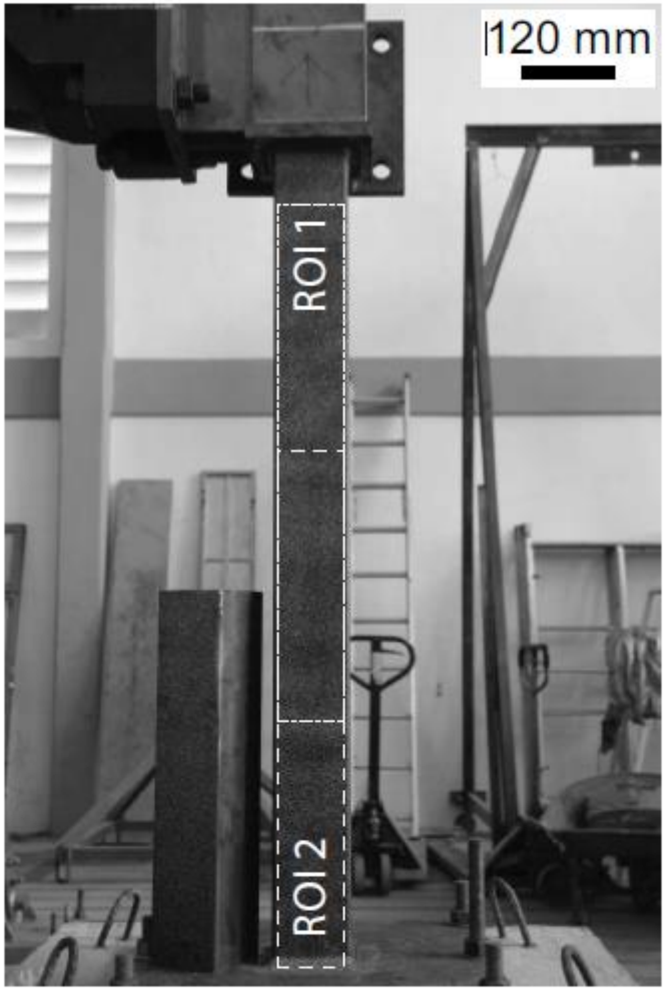
Discontinuities



Residual

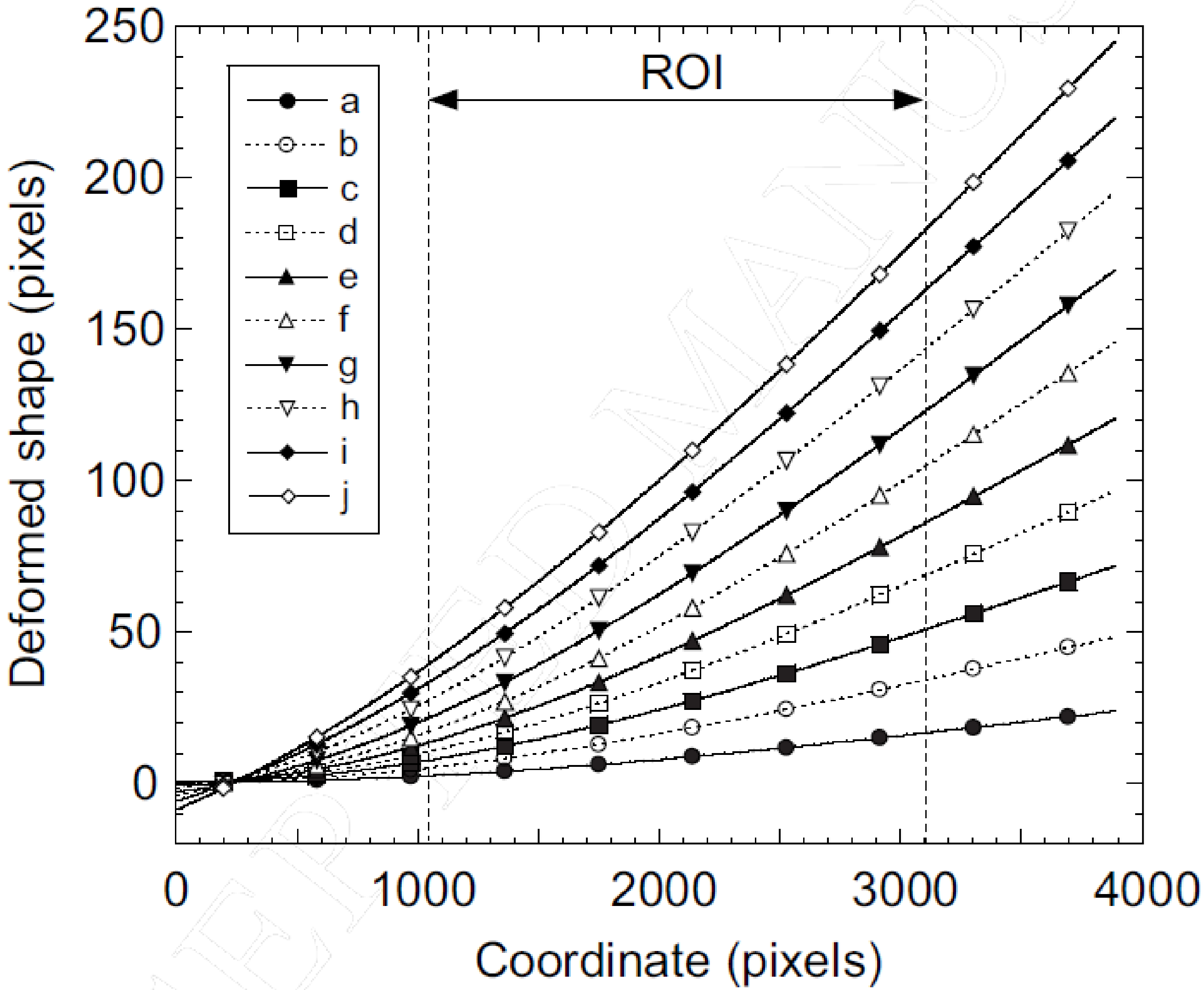
Sciutti et al, *Mathematics and Mechanics of Solids*, 2025, 30 (1), pp.93-115.

Steel beam



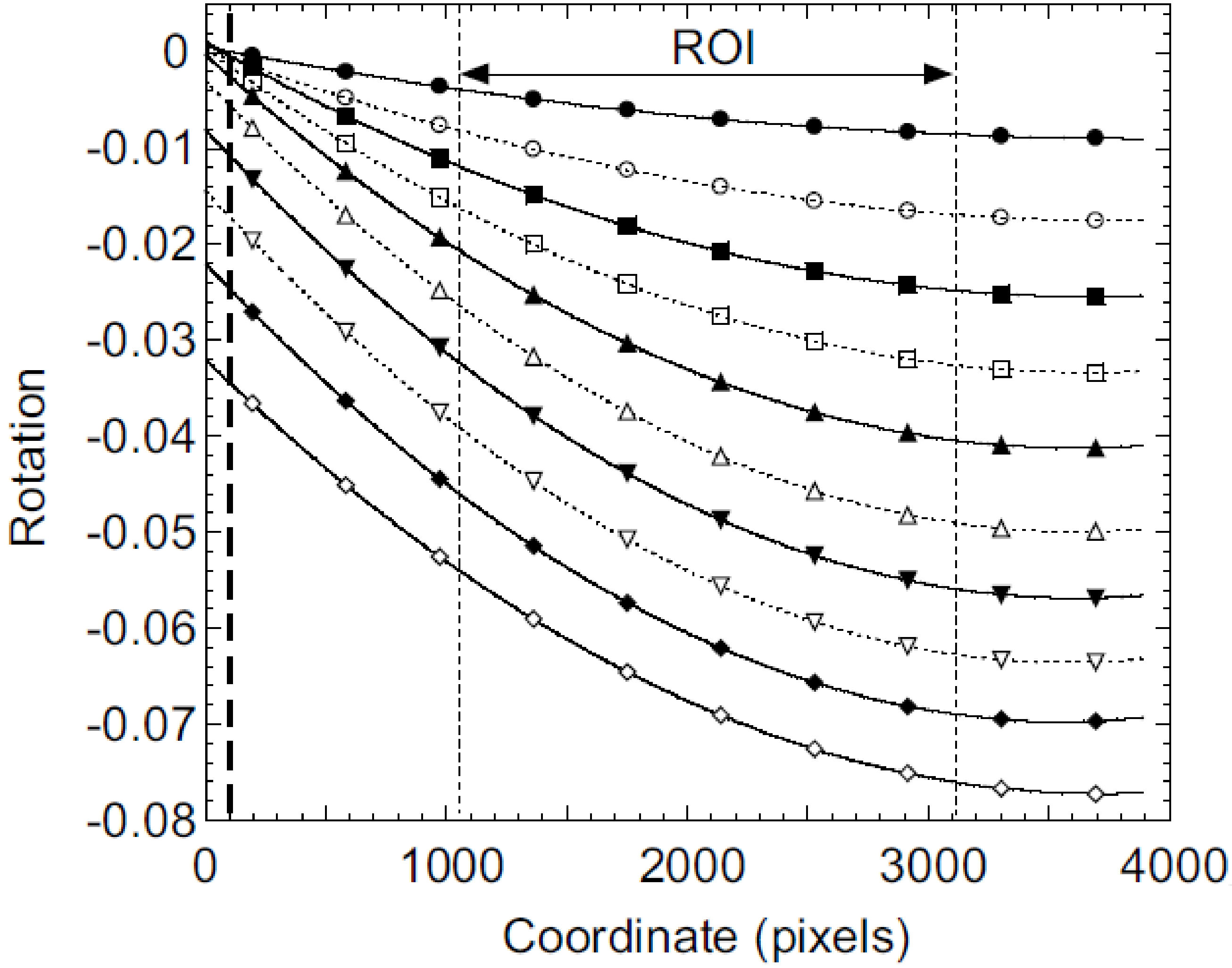
Hild et al., *European Journal of Mechanics A*, 2011, 30, 1-10

Steel beam



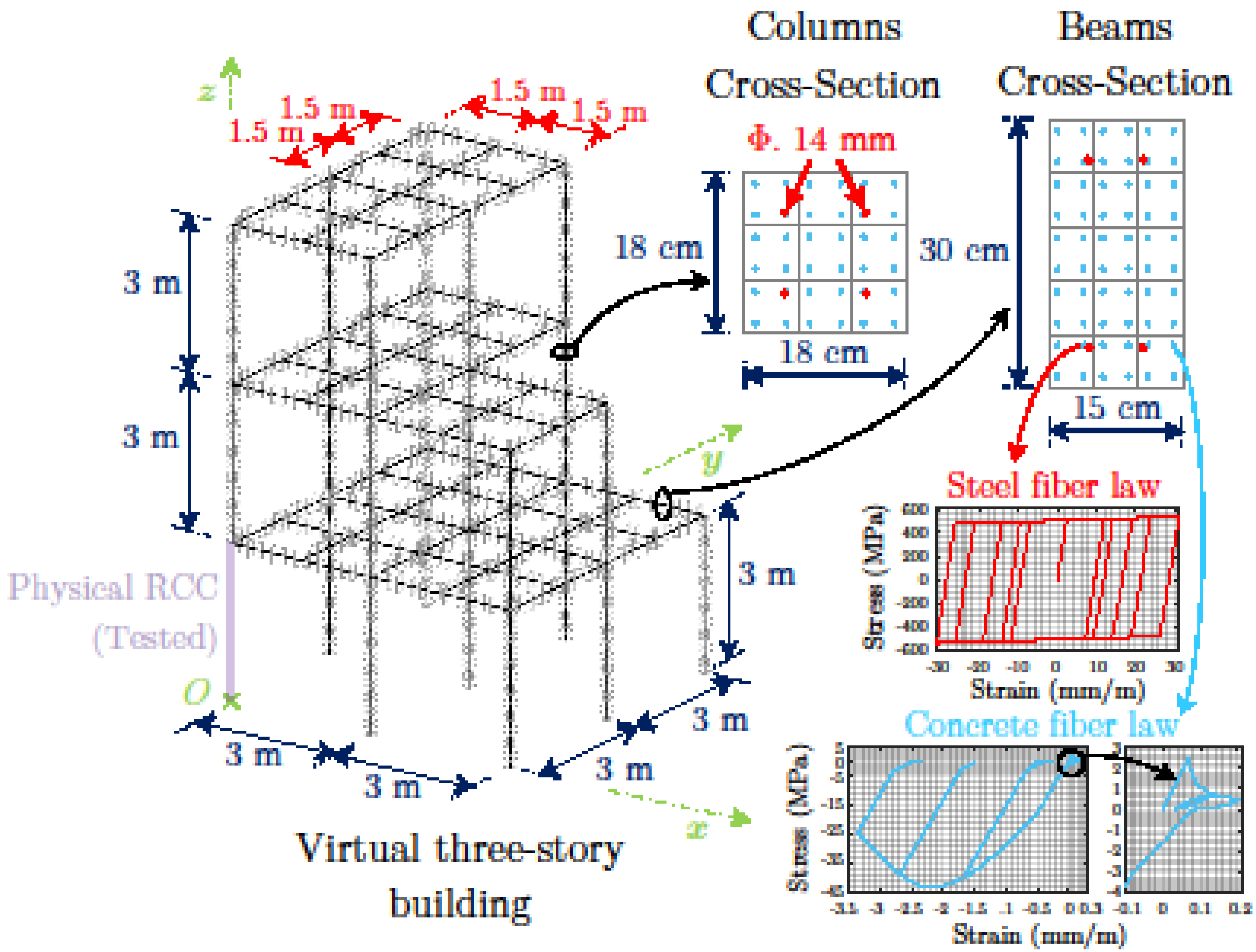
Hild et al., *European Journal of Mechanics A*, 2011, 30, 1-10

Steel beam



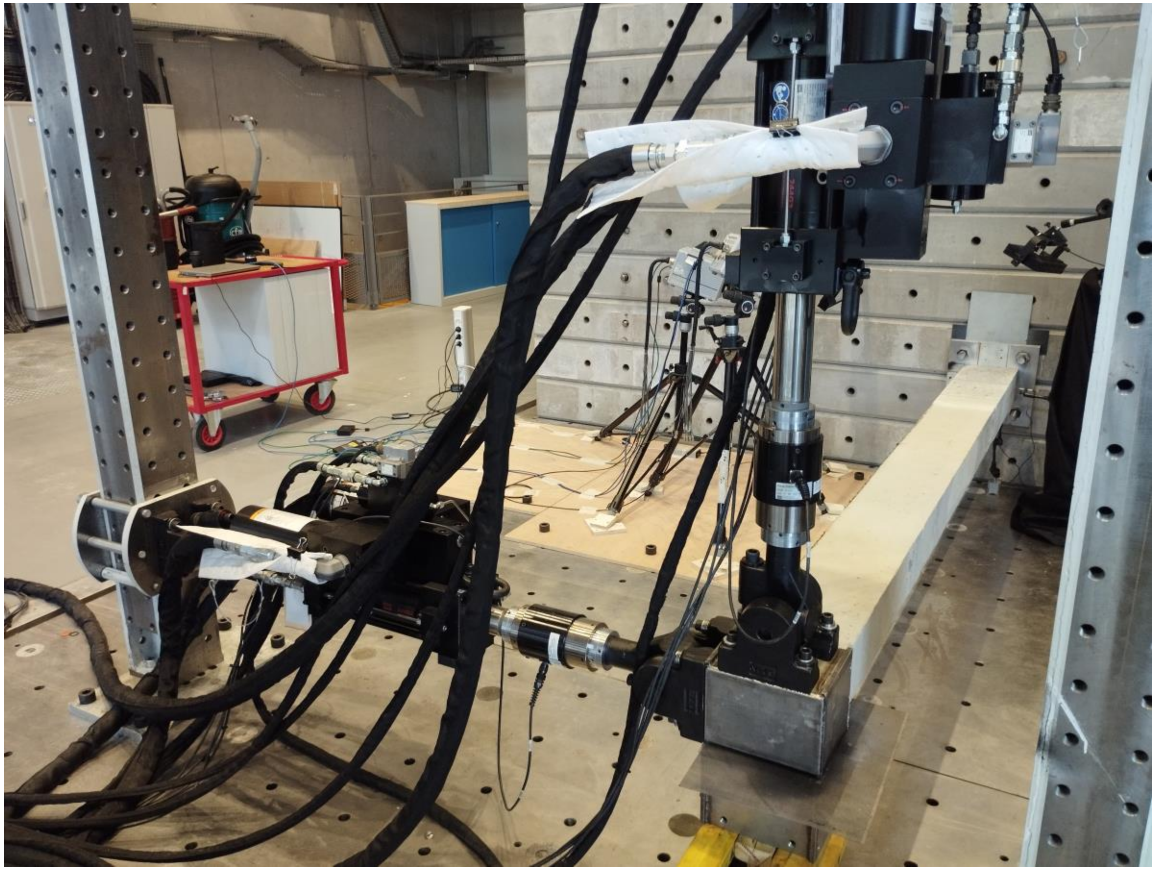
Hild et al., *European Journal of Mechanics A*, 2011, 30, 1-10

Hybrid testing



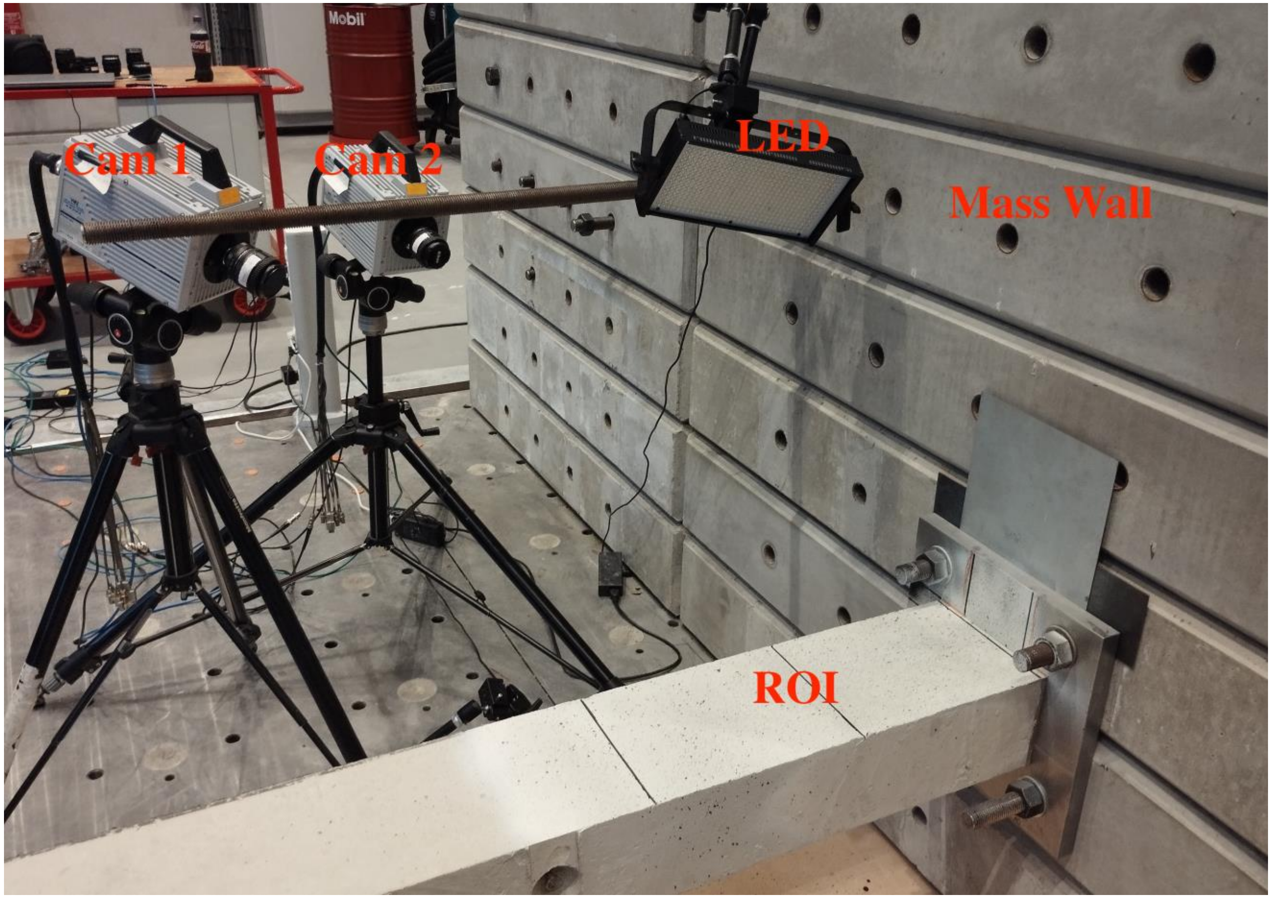
X. Chang et al, *Computer Methods in Applied Mechanics and Engineering*, 2025, 435, 117672

Hybrid testing



X. Chang et al, *Computer Methods in Applied Mechanics and Engineering*, 2025, 435, 117672

Hybrid testing

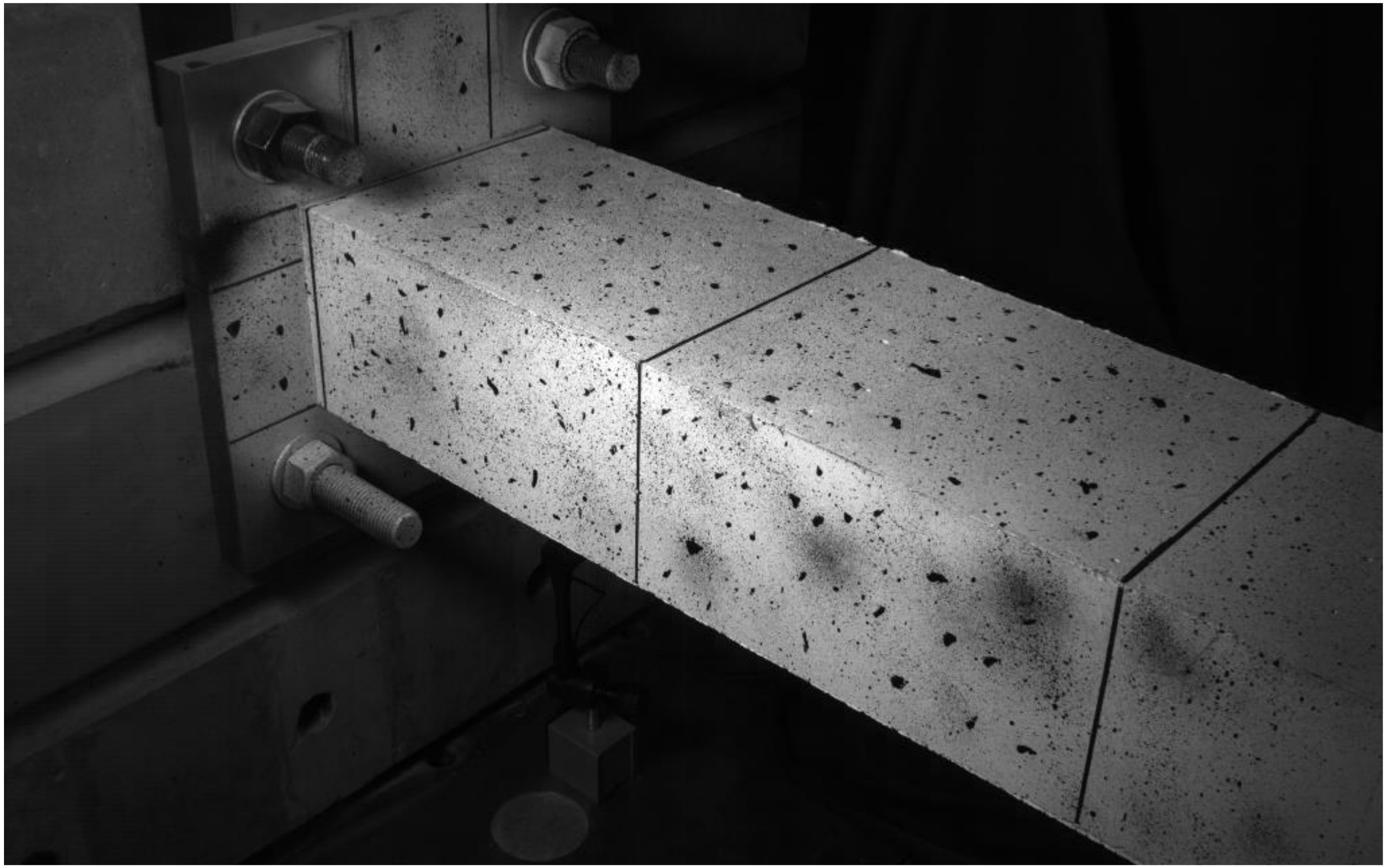


X. Chang et al, *Computer Methods in Applied Mechanics and Engineering*, 2025, 435, 117672

Hybrid testing

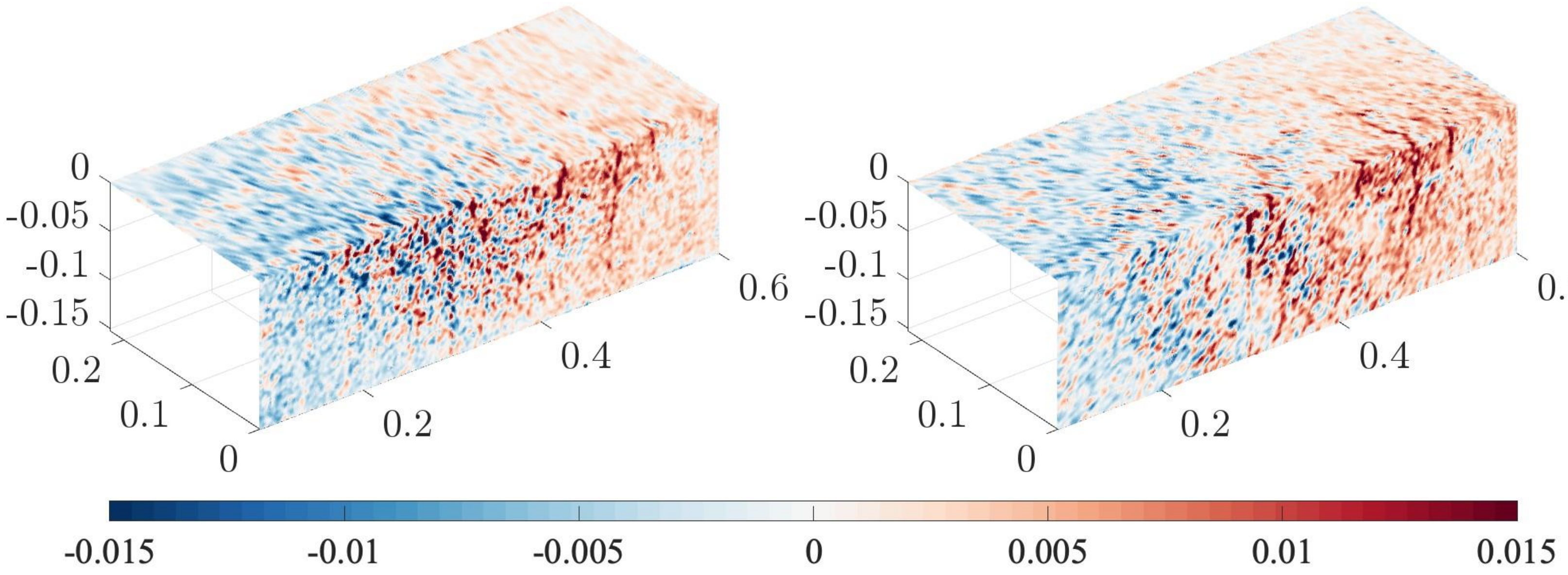
X. Chang et al, *Computer Methods in Applied Mechanics and Engineering*, 2025, 435, 117672

Hybrid testing



X. Chang et al, *Computer Methods in Applied Mechanics and Engineering*, 2025, 435, 117672

Hybrid testing



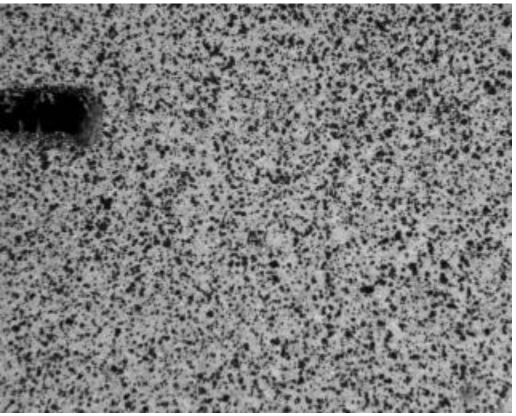
X. Chang et al, *Computer Methods in Applied Mechanics and Engineering*, 2025, 435, 117672

OPTICAL FLOW

Optical Flow

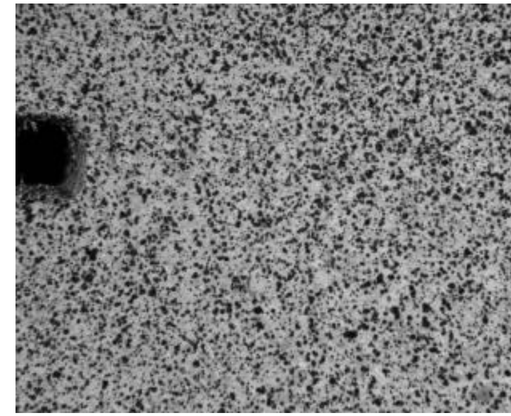
- It consists of expressing what is known (or assumed) on the temporal evolution of images
- It contains information on
  - motion!
  - observed specimen (e.g. drying, temperature changes)
  - environment (e.g. lighting)
  - imaging device (image formation model, distortions)

Gray Level Conservation



Most natural:

texture passively advected by motion



$$f(\mathbf{x}) = g(\mathbf{x} + \mathbf{u})$$

Fluid Mechanics: PIV

[Barker *et al.*, 1977; Grousson *et al.*, 1977; Dudderar and Simpkins, 1977]

Solid Mechanics: DIC

[Kanade *et al.*, 1981; Burt *et al.*, 1982; Sutton *et al.*, 1983]

Noise

- Accounting for noise

$$f(\mathbf{x}) = g(\mathbf{x} + \mathbf{u}) + \eta(\mathbf{x})$$

- Difficulty is that η is not known: Statistical tests have to be performed

Noise

- Noise can be measured (from image series of the same scene)

$$\eta(\mathbf{x})$$

- Its statistical characteristics (spatial/temporal correlation) can be studied
- Most often, a *white* *Gaussian* noise is relevant

Other Variants

- Change in lighting conditions for the scene, temperature, topography-valued image

$$f(\mathbf{x}) = (1 + a(\mathbf{x}))g(\mathbf{x} + \mathbf{u}) + b(\mathbf{x})$$

- Difficulty is that $a(\mathbf{x})$, $b(\mathbf{x})$ and $\mathbf{u}(\mathbf{x})$ are all unknown. More quantities are to be measured from the same amount of information

Optical Flow

- Note that the writing of the optical flow expresses an *a priori* knowledge on the evolution of images *without compromise nor approximation*
- In the following, strategies to approach the unknown fields are presented
- The optical flow allows for the evaluation of the quality of the estimated solution

Optical Flow

- It is important to distinguish between the satisfaction of the assumptions underlying the optical flow and the ability to capture the displacement field (and others if needed) with a specific choice of discretization and algorithm

ILL-POSEDNESS AND REGULARIZATION

Displacement Measurement

- Gray level images

$$f(\mathbf{x}) \qquad\qquad g(\mathbf{x})$$

- Passive advection

$$f(\mathbf{x}) = g(\mathbf{x} + \mathbf{u}(\mathbf{x}))$$

- Identify $\mathbf{u}(\mathbf{x})$

Cost function

- DIC is based on the minimization of

$$\mathrm{T}[\mathbf{v}] = \sum_{\Omega} \left(f(\mathbf{x}) - g\left(\mathbf{x} + \mathbf{v}(\mathbf{x})\right) \right)^2$$

$$\mathbf{u}_{sol} = \operatorname{argmin} \mathrm{T}[\,\mathbf{v}]$$

(Summation over all pixels belonging to Ω)

Notations

- Let us introduce the corrected deformed image $\tilde{g}(\mathbf{x})$

$$\tilde{g}_{\mathbf{v}}(\mathbf{x}) = g(\mathbf{x} + \mathbf{v}(\mathbf{x}))$$

- Gray level residuals

$$\begin{aligned} \rho(\mathbf{x}) &= f(\mathbf{x}) - g(\mathbf{x} + \mathbf{v}(\mathbf{x})) \\ &= f(\mathbf{x}) - \tilde{g}_{\mathbf{v}}(\mathbf{x}) \end{aligned}$$

- DIC is based on minimizing

$$\mathrm{T}[\mathbf{v}] = \|\rho\|^2$$

Choice of Norms

$$T[\mathbf{v}] = \|\rho\|^2$$

- Not all norms are equivalent
- The ‘best’ should be designed to limit the effect of noise
- For a uniform white Gaussian noise, the ‘best’ norm is simply the Euclidian norm

$$\|\phi(\mathbf{x})\|^2 = \sum_{pixel\ i} \phi^2(\mathbf{x}_i)$$

Part 2 of the lecture will be devoted to this specific point

Ill-Posedness

- Based on the sole gray level conservation, the displacement field is generally far from being unique if no additional constraints are enforced
- Mapping each pixel $g(\mathbf{x})$ with another pixel $f(\mathbf{y})$ having the same gray level gives a perfect match

$$\mathrm{T}[\mathbf{v}] = 0$$

Solution

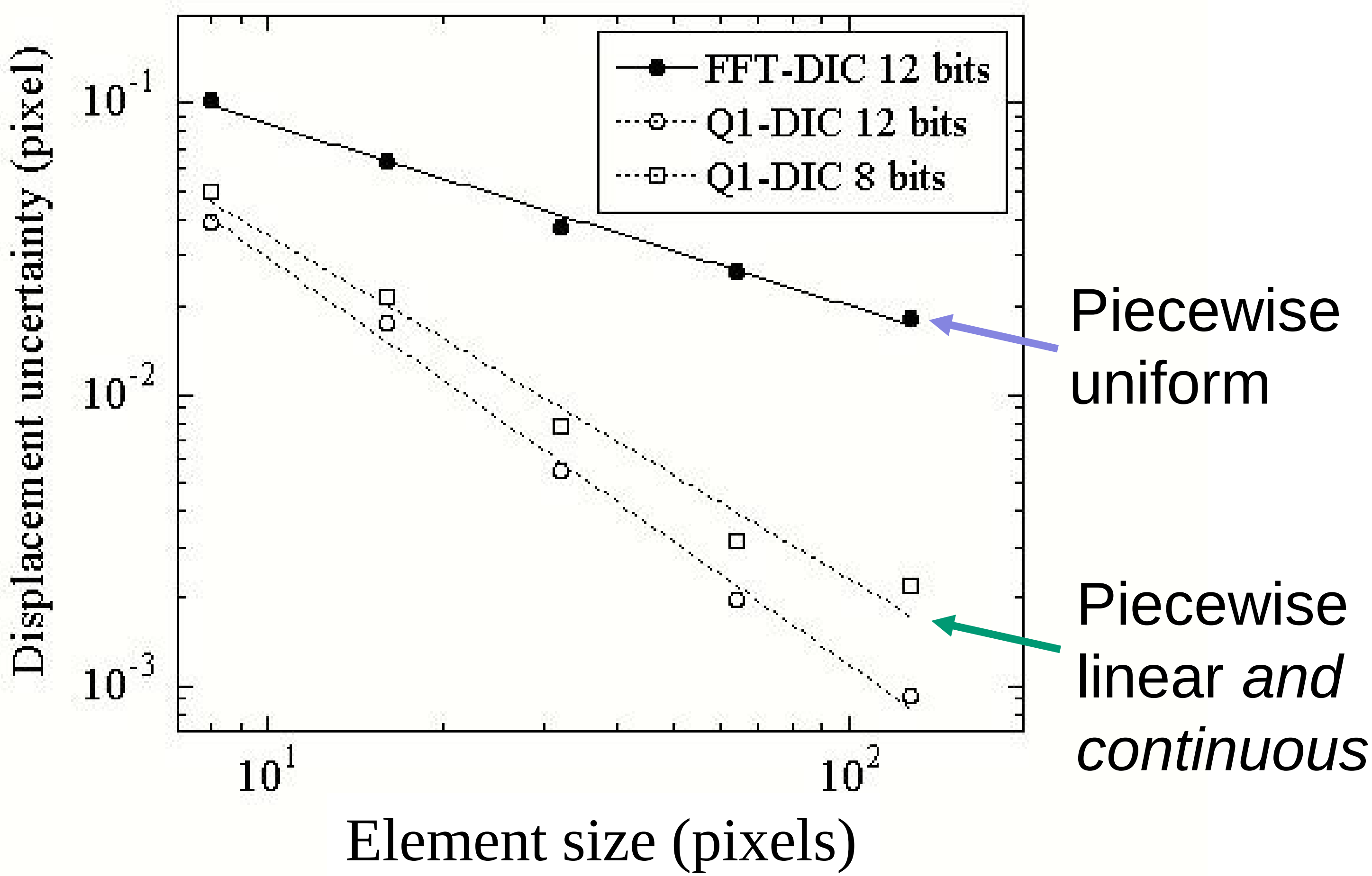
- Displacement fields such that $\mathrm{T}[\mathbf{v}] = 0$ generally are not satisfactory
- Missing information is the regularity of the displacement field (such as its continuity, or its limited variability over a given window)
- The general presence of noise, and interpixel displacement forbid reaching $\mathrm{T}[\mathbf{v}] = 0$ hence the choice for a variational formulation

‘Hard’ Regularization

The choice of given vector space, $\mathbf{F}$, for the displacement field

$$\mathbf{v} = \underset{\mathbf{v}\in\mathbf{F}}{\operatorname{argmin}}\ \mathrm{T}[\,\mathbf{v}]$$

Benefit of C_0 Regularization



DISPLACEMENT BASIS

Simplest Choice

- The Region of Interest (ROI) is partitioned into zones (ZOI) that are analyzed independently (local approach)

$$\mathrm{T}[\mathbf{v}] = \sum_{ZOI_i} \mathrm{T}_i[\mathbf{v}]$$

$$\mathrm{T}_i[\mathbf{v}] = \sum_{\underline{x} \in ZOI_i} \left(\tilde{g}_{\mathbf{v}}(\mathbf{x}) - f(\mathbf{x})\right)^2$$

$$\mathbf{v}_i = \underset{\mathbf{v}\ \text{uniform in ZOI}}{\operatorname{argmin}} \mathrm{T}_i[\mathbf{v}]$$

Piecewise Uniform Local DIC

- Elementary problem

$$\mathrm{T}_i[\,\mathbf{v}] = \sum_{\underline{\mathbf{x}} \in ZOI_i} \left(g(\mathbf{x} + \mathbf{v}(\mathbf{x})) - f(\mathbf{x})\right)^2$$

- Let us assume mere translation, and no noise

$$g(\mathbf{x} + \mathbf{u}) = f(\mathbf{x})$$

Piecewise Uniform Local DIC

The function reduces to

$$\mathrm{T}_i[\mathbf{v}] = \sum_{\underline{x} \in ZOI_i} \left(f(\mathbf{x}+\mathbf{v}-\mathbf{u}) - f(\mathbf{x})\right)^2$$

or

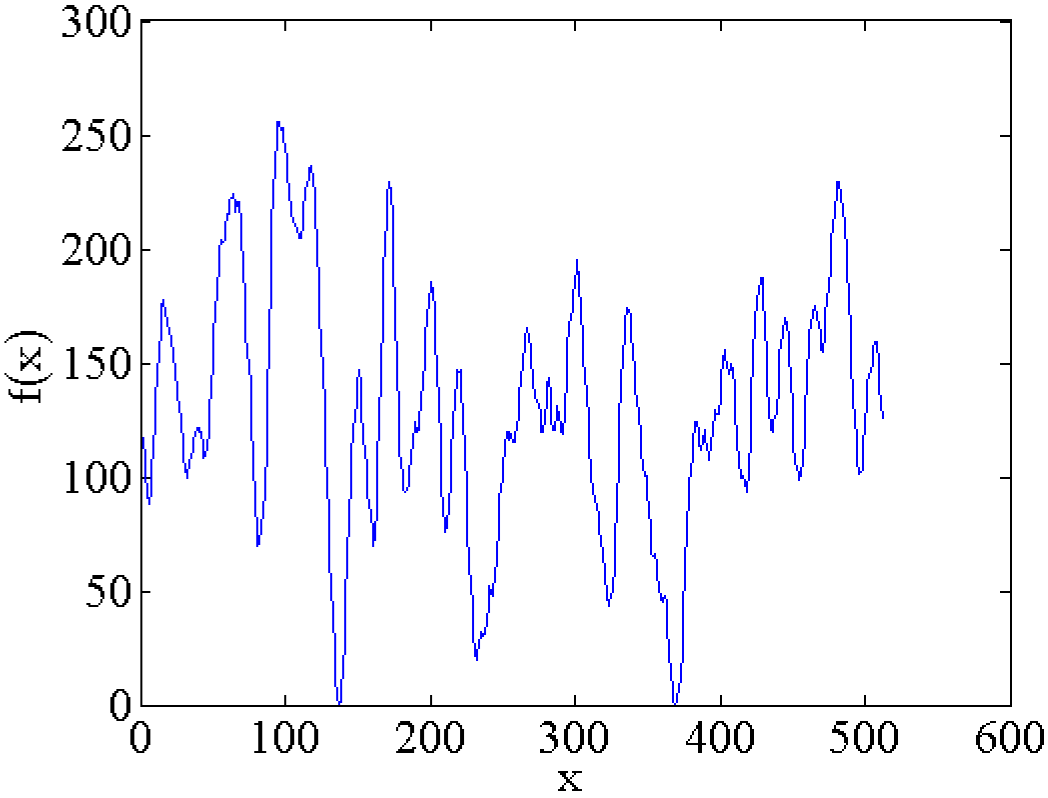
$$\mathrm{T}_i[\mathbf{v}] \propto 1 - \mathrm{C}_i(\mathbf{v}-\mathbf{u})$$

where

$$1 - \mathrm{C}_i(\boldsymbol{\delta}) = \frac{1}{2\sigma^2} \sum_{\underline{\mathbf{x}} \in ZOI_i} \left(f(\mathbf{x}+\boldsymbol{\delta}) - f(\mathbf{x})\right)^2$$

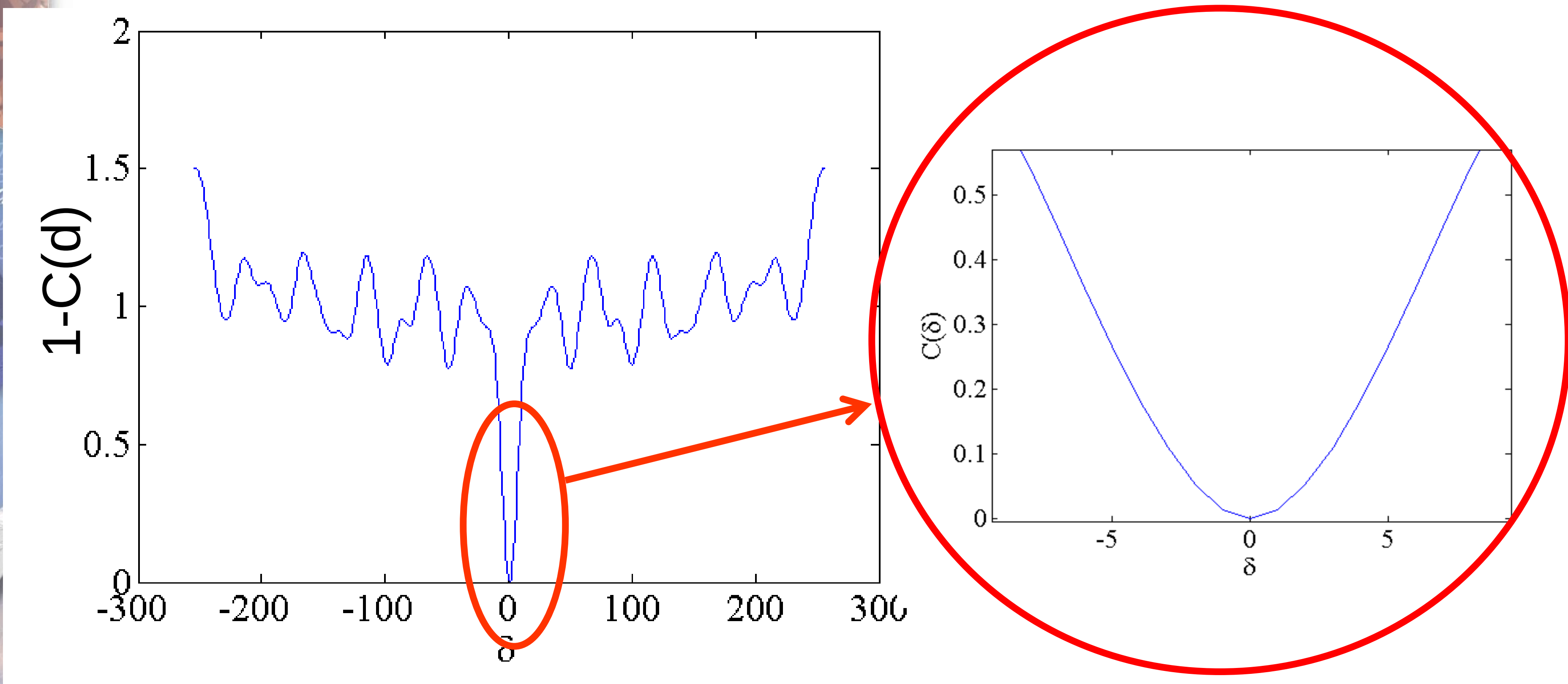
Example in 1D

Random signal



Example in 1D

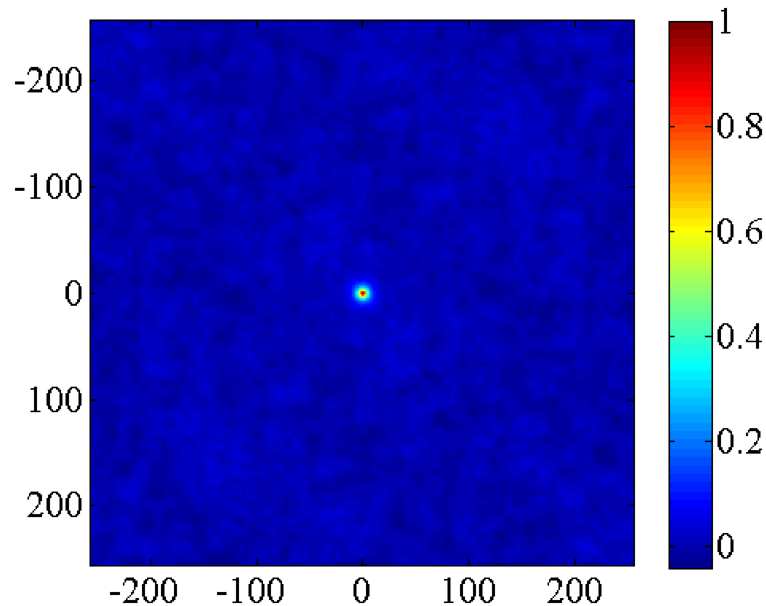
If signal *f* is smooth enough, then absolute minimum of 1-C is a nice parabolic well



DIC or The Art of Playing Golf

Functional $\mathrm{T}[\mathbf{v}]$ is $1 - C(\boldsymbol{\delta})$

$C(\underline{\boldsymbol{\delta}})$

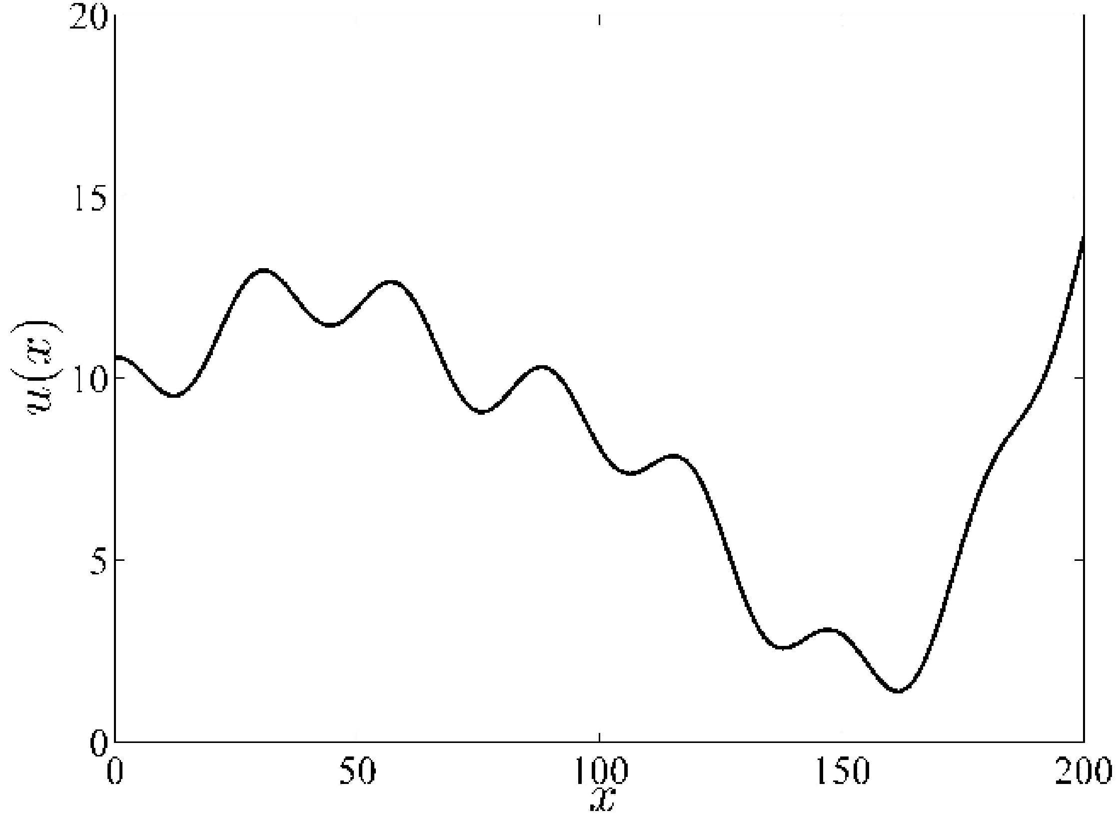


$$C(\boldsymbol{\delta}) = \frac{1}{\sigma^2}\langle (f(\mathbf{x}) - \langle f \rangle)(f(\mathbf{x}+\boldsymbol{\delta}) - \langle f \rangle)\rangle_{\mathbf{x}}$$

Uniform Displacement

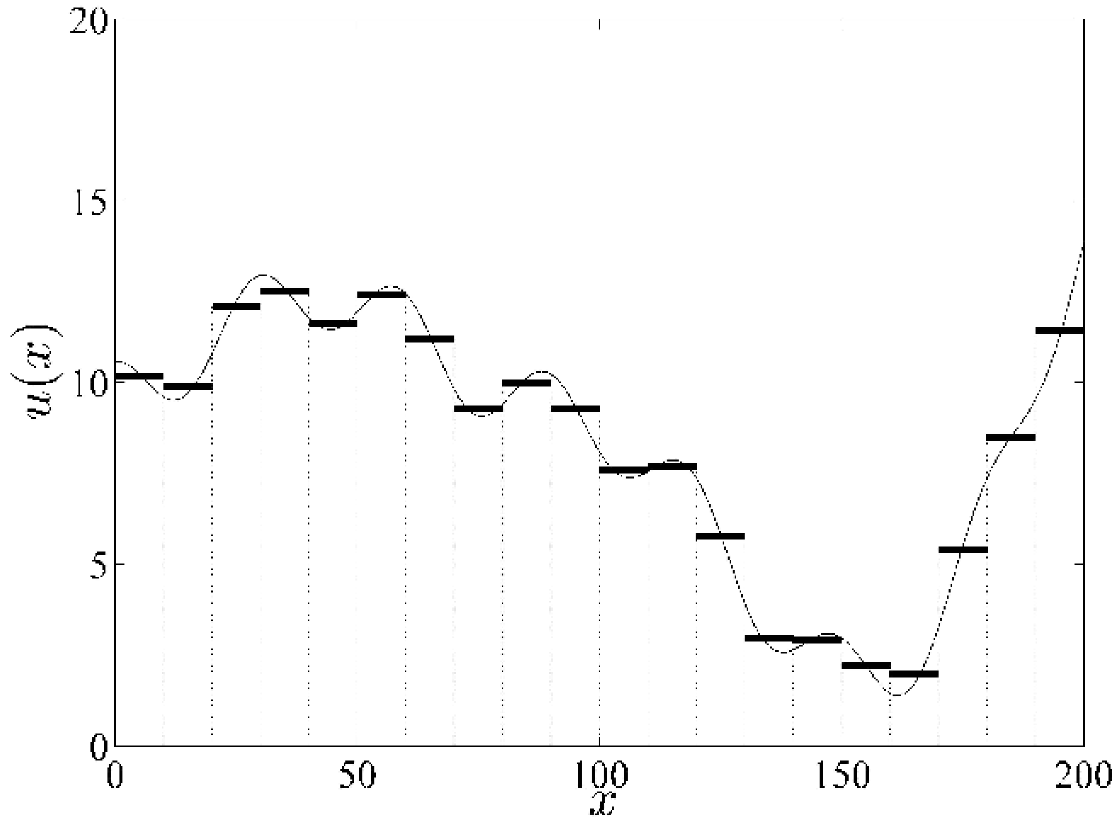
- In the very specific case of uniform displacement, the cross-correlation function is an effective way of computing $\mathrm{T}_i[\mathbf{v}]$ for all $\mathbf{v}$, and hence the absolute minimum can be found
- When the displacement is not uniform, a different approach has to be followed (next section)

Displacement Discretization



Reference u

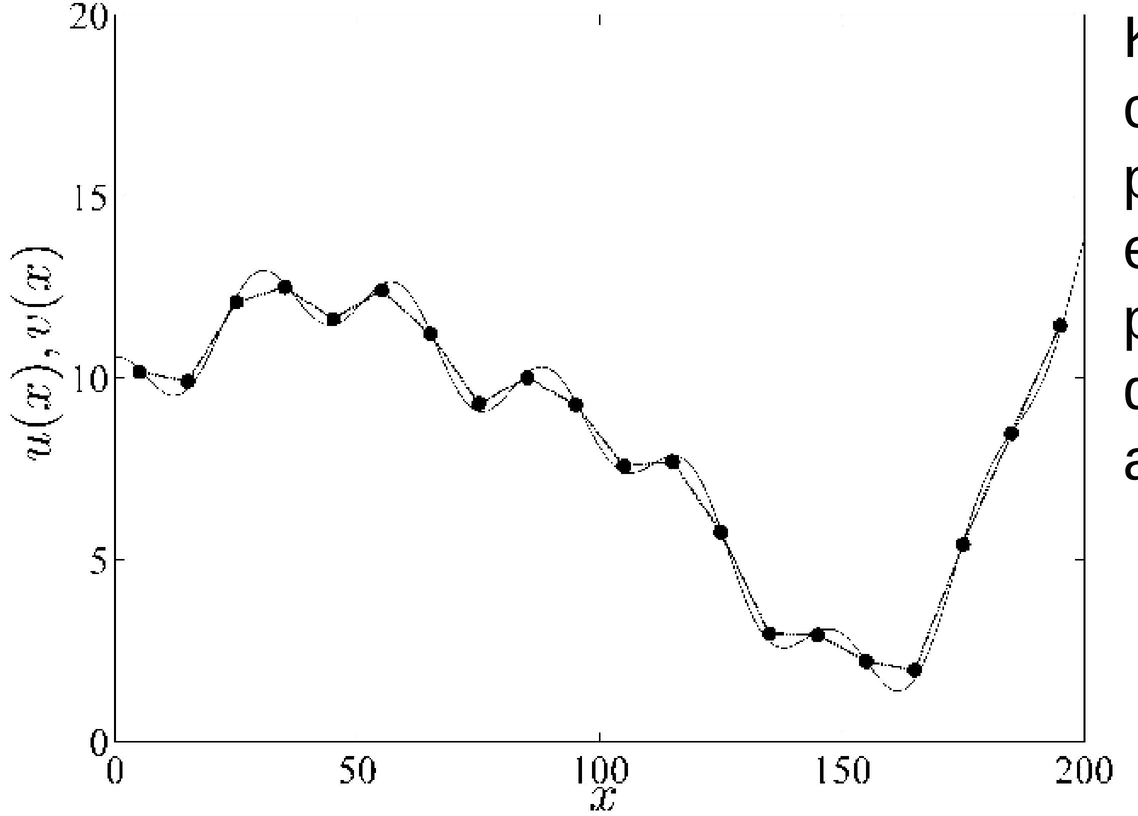
Discretization (Local)



Piecewise constant

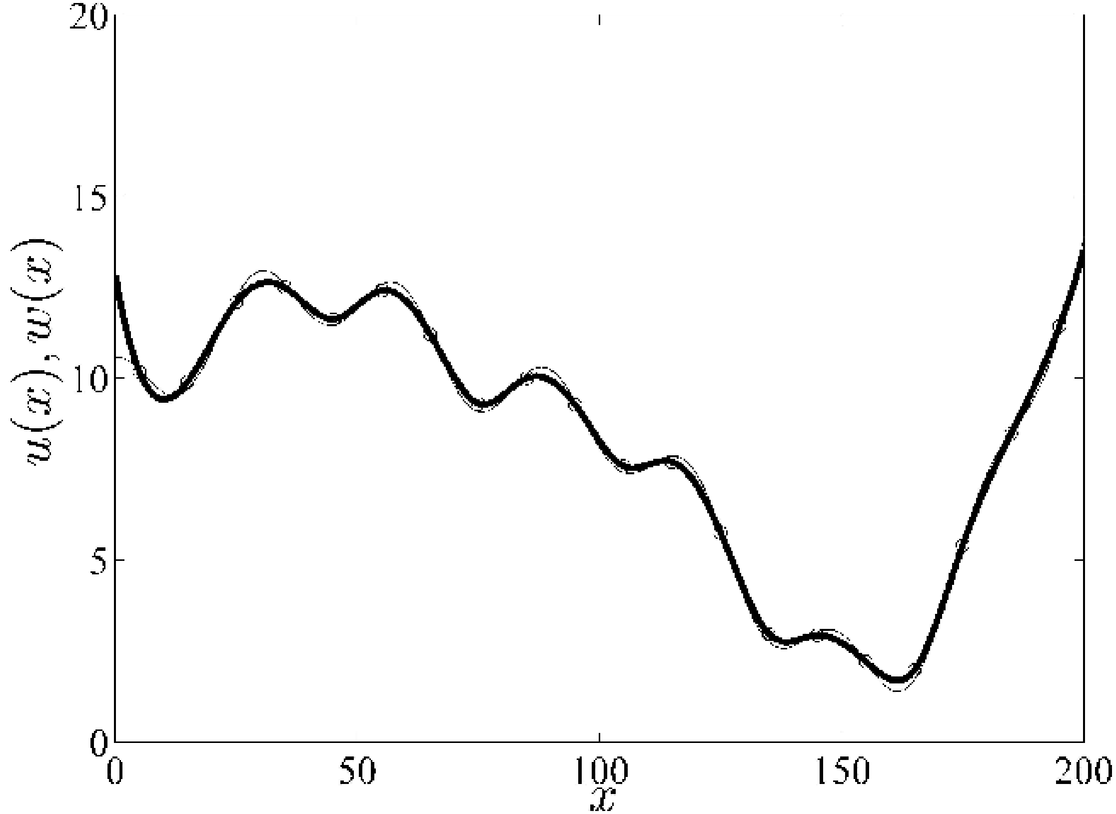
PIV (or DIC)

Discretization (Local)



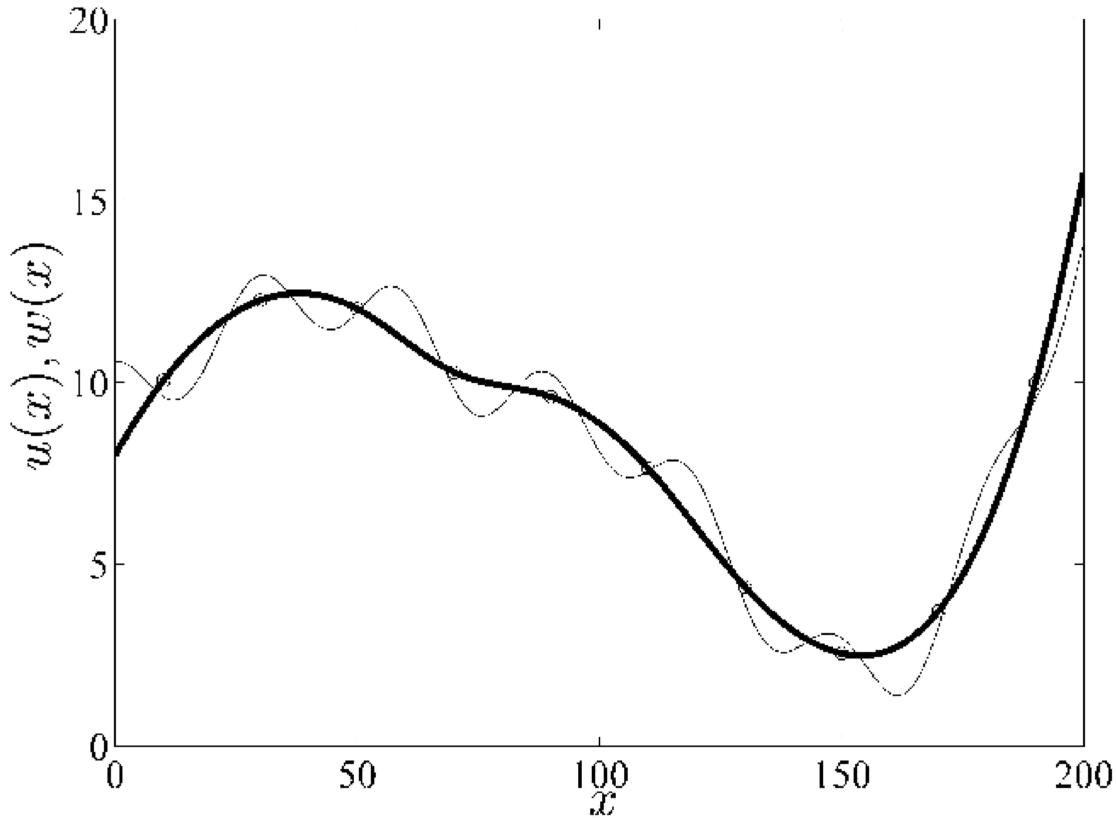
Keeping central position of each interval provides different approximation

Discretization (Local)



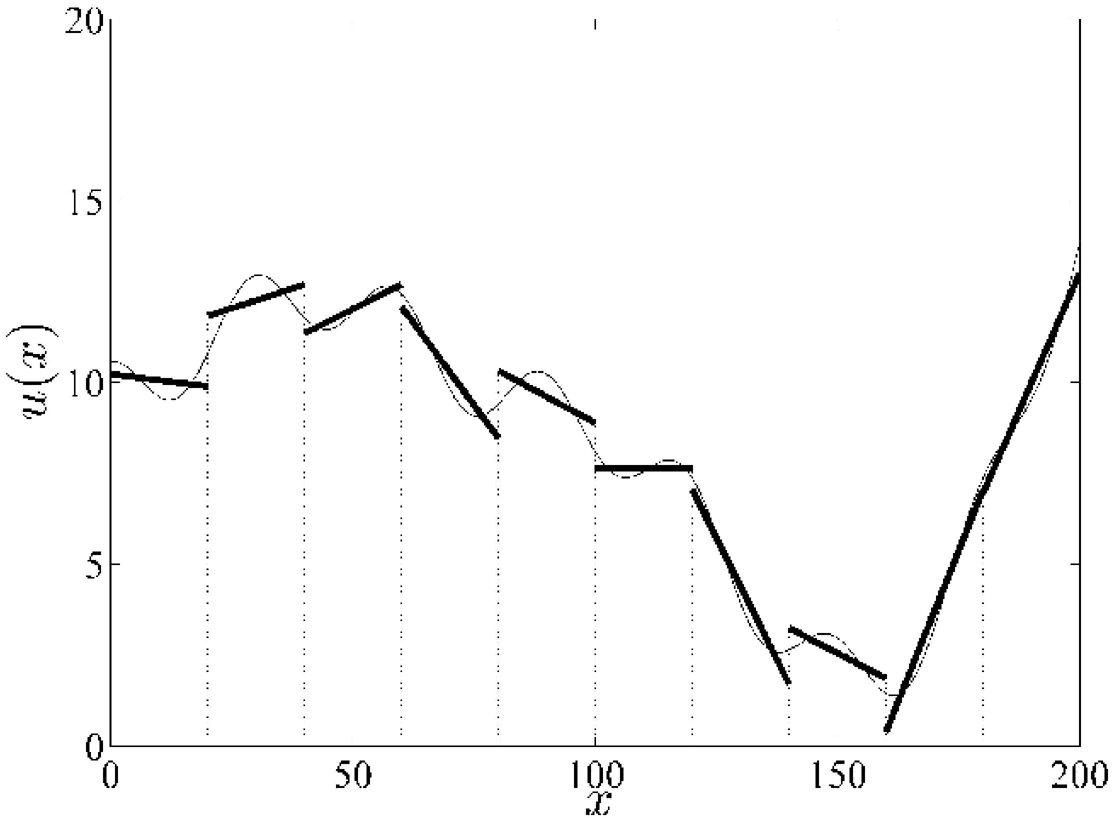
Spline interpolation of those discrete estimates gives very close description

Discretization (Local)



Note that same procedure with larger intervals results in much poorer approximation

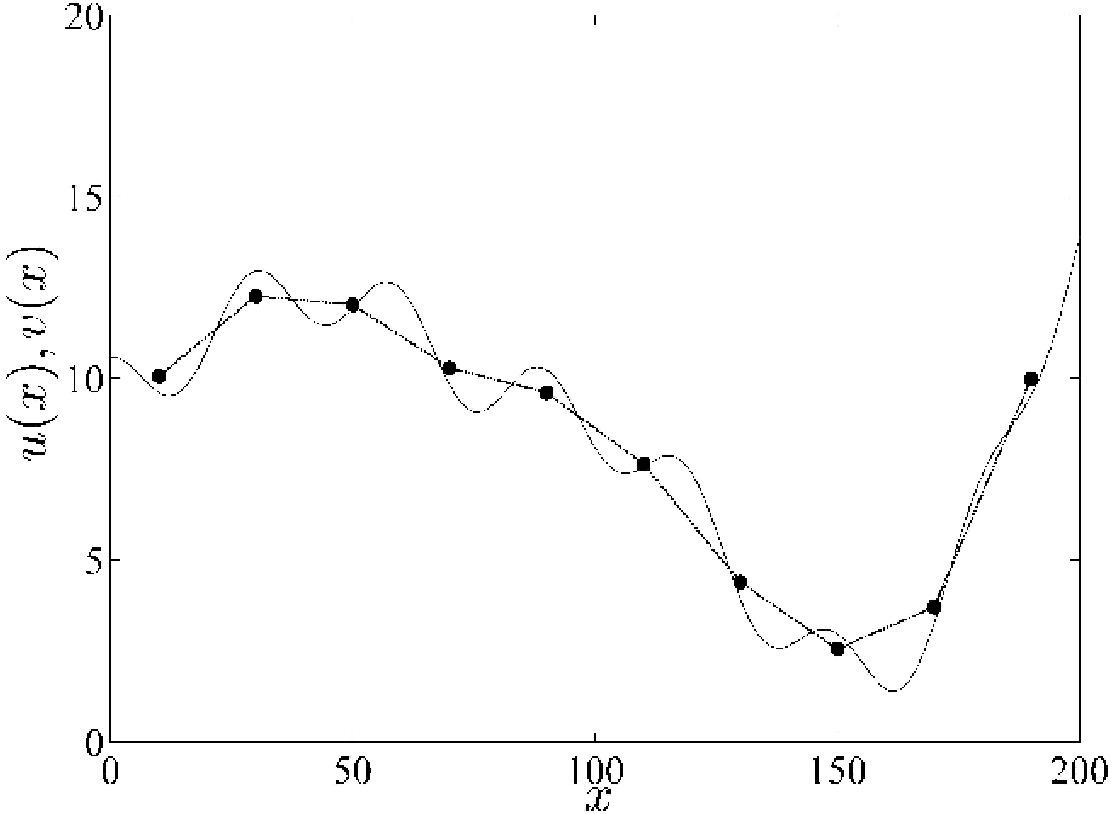
Discretization (Local)



Piecewise linear description is initially a good (although discontinuous) description

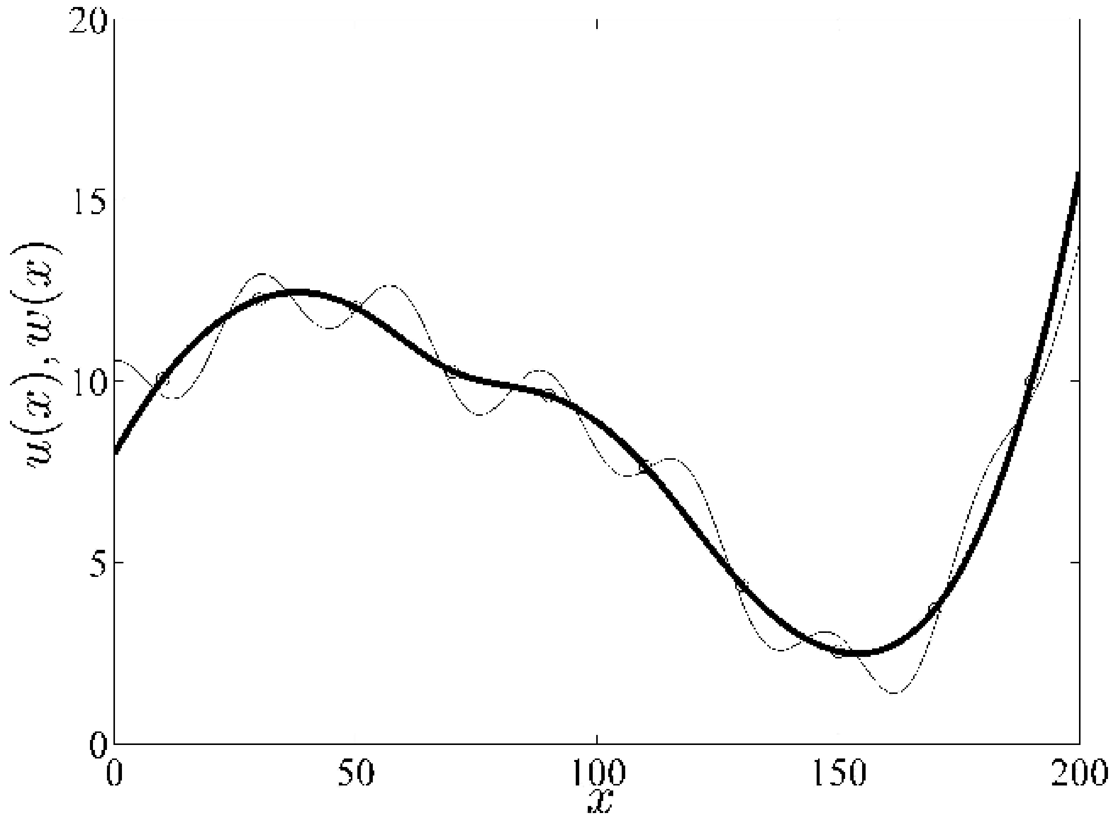
Here the same dof # is used

Discretization (Local)



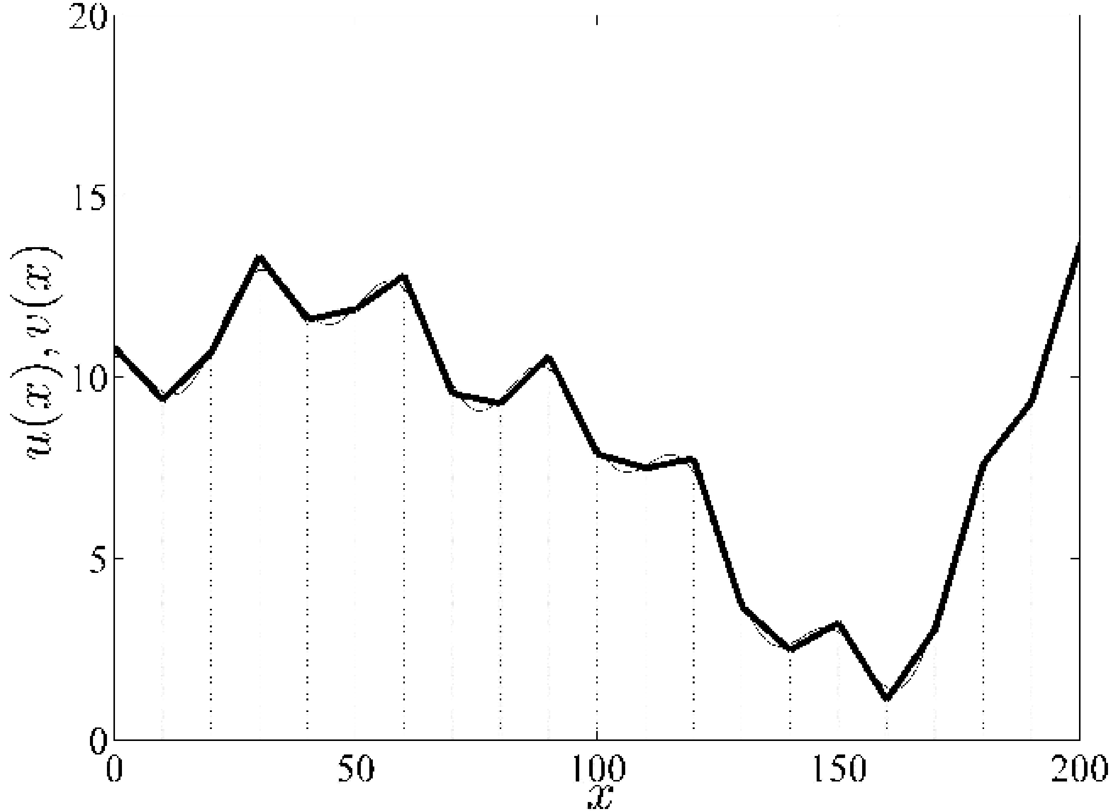
Central position of each interval is kept

Discretization (Local)



Spline interpolation misses high frequency components

Discretization (Global)



Finite element description of displacement field

Here the same dof # is used

BEWARE!

Although post-processing is often used in commercial codes, so as to provide nice-looking results, this additional step is not performed in connection with image registration and hence is disregarded from the present discussion

Regularization

The displacement field in $\mathbf{F}$ is written

$$\mathbf{v}(\mathbf{x}) = \sum_i v_i \mathbf{\Psi}_i(\mathbf{x})$$

for all methods, be they local or global

Regularization

Measuring the displacement field consists of minimizing

$$\mathrm{T}[\{\mathbf{v}\}] = \sum_{\mathbf{x}} \left(\tilde{g}_{\{v\}}(\mathbf{x}) - f(\mathbf{x}) \right)^2$$

$$\tilde{g}_{\{v\}}(\mathbf{x}) = g\left(\mathbf{x} + v_i \boldsymbol{\Psi}_i(\mathbf{x}) \right)$$

with respect to the discrete set of kinematic parameters v_i gathered in $\{\mathbf{v}\}$

Choice of Kinematic Basis

Haute couture

Ready to wear

- The selection of a suited regularization is an opportunity to inject prior knowledge on the analyzed test
  This may lead to hard or soft strategies that are tailored to a particular test (and image set)
- Otherwise, the strategy is to rather select a general basis that fits robustly most situations

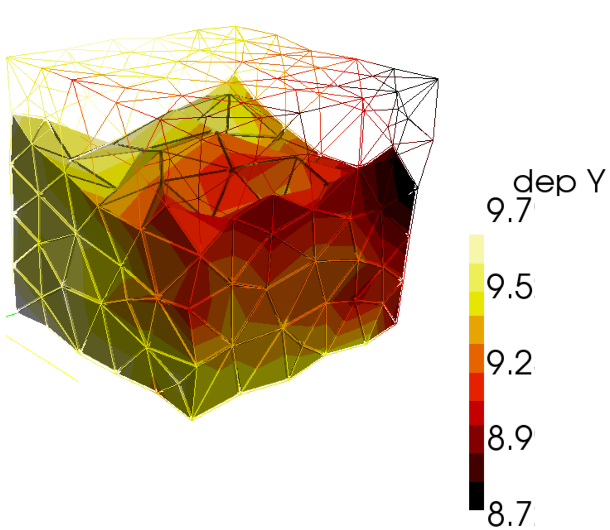
Choice of Kinematic Basis

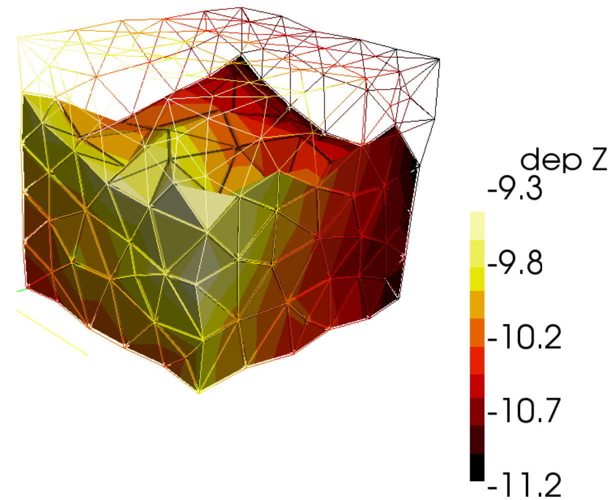
local
- Piece-wise constant displacements
- Piece-wise P1 (or higher order) field

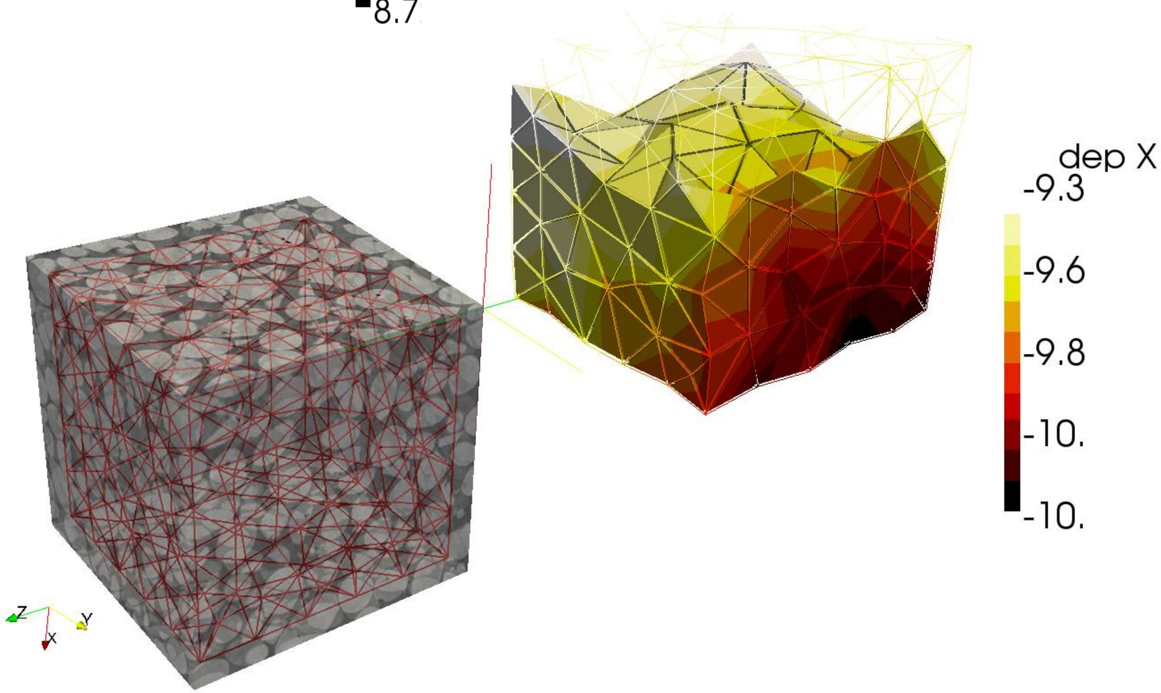
- Finite element kinematics (structured or unstructured mesh)
- Spline, Fourier, Wavelets
- Beam kinematics (Euler Bernoulli)
- Elastic analytical solution (crack, contact, simple mechanical tests)
- Numerically generated basis (for identification)
- Other (e.g., opt. distortions, AFM artifacts)

global

T4-DVC



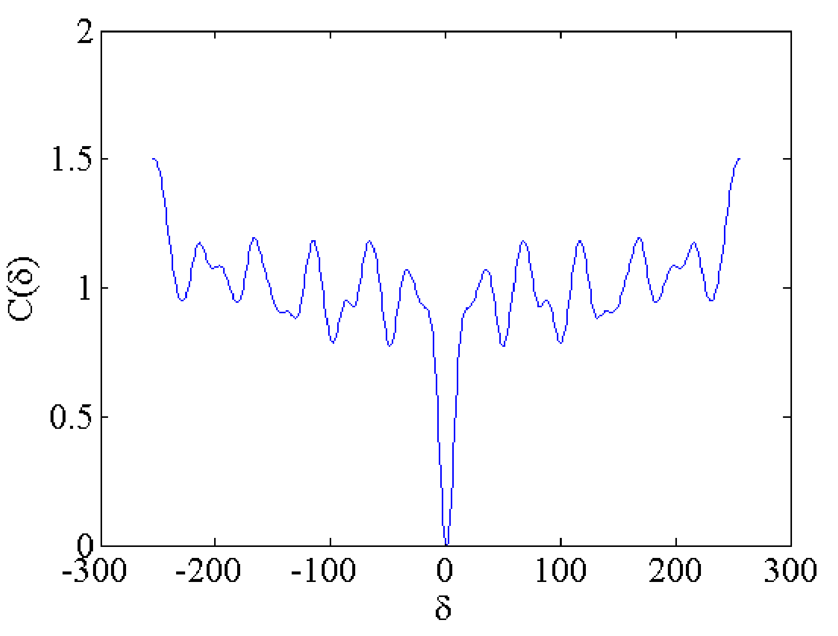




ALGORITHMS

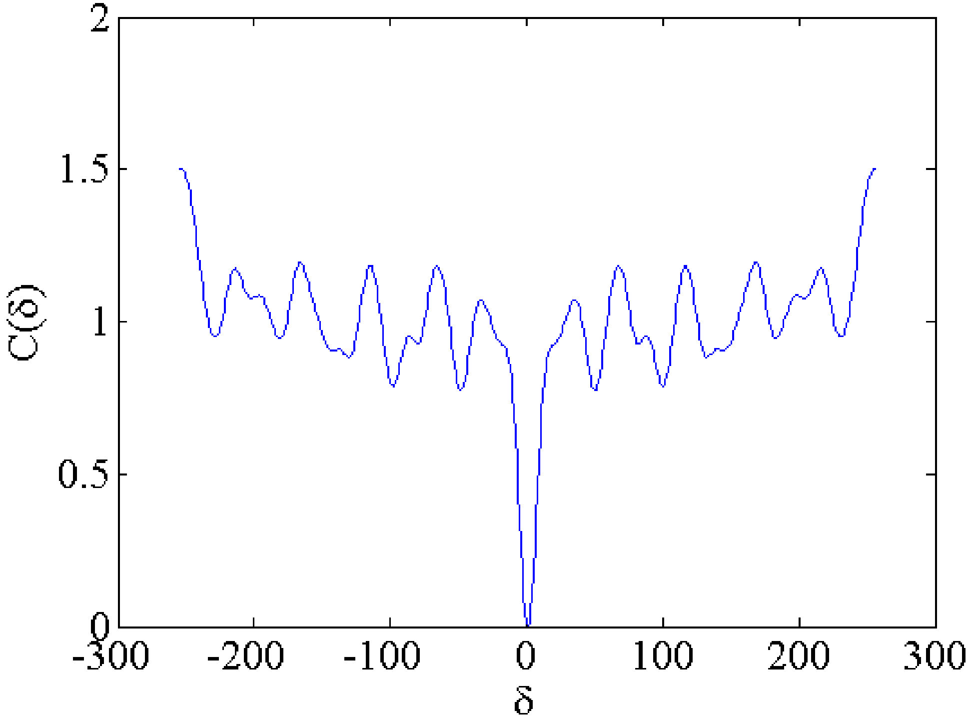
Approach

Qualitatively, the problem consists of finding the minimum of a ‘potential’ constructed from the auto-correlation function of the reference image



Approach

Hence, it will be simple to reach the absolute minimum *iff* the initial guess is close to the actual solution



Relaxation

Let us assume that the displacement $\{\alpha^{(n)}\}$ is close to the solution

$$\mathrm{T}(\{\mathbf{v}+\delta\mathbf{v}\}) = \sum_{\mathbf{x}} \left(\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}+\delta v_i \boldsymbol{\Psi}_i(\mathbf{x})) - f(\mathbf{x})\right)^2$$

$$\approx \sum_{\mathbf{x}} \left(\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) + \delta v_i \boldsymbol{\Psi}_i(\mathbf{x}) \cdot \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) - f(\mathbf{x})\right)^2$$

The latter is a mere quadratic form easy to minimize

Relaxation

$$\frac{\mathrm{dT}(\{\mathbf{v}\} + \{\delta\mathbf{v}\})}{\mathrm{d}\{\delta\mathbf{v}\}} = \{\mathbf{0}\}$$

leads to

$$\sum_{\underline{\mathbf{x}}} \left(\mathbf{\Psi}_i(\mathbf{x}) \cdot \nabla \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right) \left(\mathbf{\Psi}_j(\mathbf{x}) \cdot \nabla \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right) \delta v_j$$

$$= \sum_{\underline{\mathbf{x}}} \left(\mathbf{\Psi}_i(\mathbf{x}) \cdot \nabla \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right) \left(f(\mathbf{x}) - \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)$$

Relaxation

$$\mathrm{M}_{ij}\delta v_j = b_i$$

with

$$\mathrm{M}_{ij} = \sum_{\mathbf{x}} \left(\boldsymbol{\Psi}_i(\mathbf{x}) \cdot \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)\left(\boldsymbol{\Psi}_j(\mathbf{x}) \cdot \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)$$

$$b_i = \sum_{\mathbf{x}} \left(\boldsymbol{\Psi}_i(\mathbf{x}) \cdot \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)\left(f(\mathbf{x}) - \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)$$

Relaxation

Finally, the displacement is updated

$$v_i^{(n+1)} = v_i^{(n)} + \delta v_i$$

as well as the corrected image

$$\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) = g\left(\mathbf{x} + v_i^{(n+1)}\mathbf{\Psi}_i(\mathbf{x})\right)$$

as needed for computing [**M**] and {***b***}

Question

What are the properties of [**M**]?

$$\mathrm{M}_{ij} = \sum_{\mathbf{x}} \left(\boldsymbol{\Psi}_i(\mathbf{x}) \cdot \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)\left(\boldsymbol{\Psi}_j(\mathbf{x}) \cdot \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)$$

$$\mathrm{M}_{ij} = \sum_{\mathbf{x}} \left(\boldsymbol{\Psi}_i(\mathbf{x}) \otimes \boldsymbol{\Psi}_j(\mathbf{x})\right) : \left(\boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) \otimes \boldsymbol{\nabla}\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)$$

Remark

- It is worth emphasizing that the linearization of the objective functional is only performed to evaluate the correction $\{\delta\mathbf{v}\}$ but the complete (non-linear) functional is reverted to (without compromise) at each iteration
- Correlation residuals

$$\rho(\mathbf{x}) = \begin{cases} f(\mathbf{x}) - \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) \\ f(\mathbf{x}) - g(\mathbf{x} + v_i^{(n)}\mathbf{\Psi}_i(\mathbf{x})) \end{cases}$$

Residuals

- After registration the difference, or *residual*,

$$\rho(\mathbf{x}) = \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) - f(\mathbf{x})$$

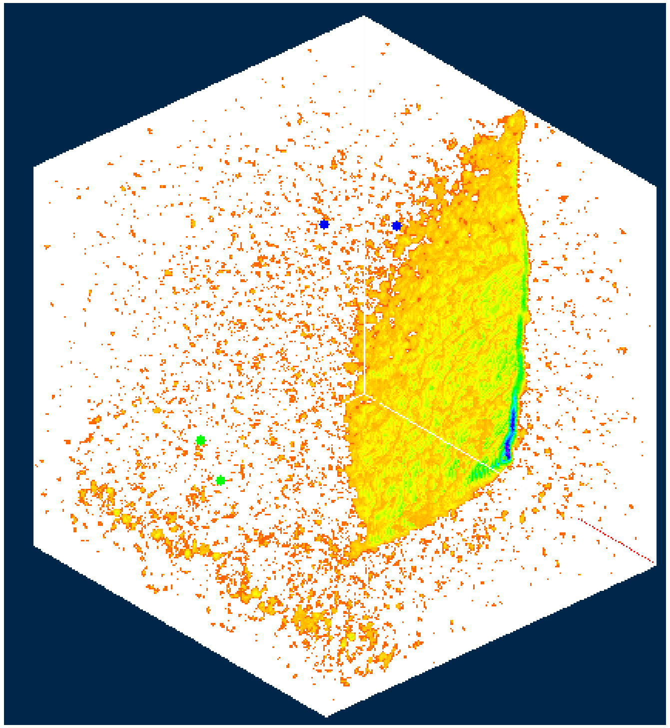
allows the user to estimate how good/bad the DIC procedure performed

- It may indicate if it is useful to revisit starting assumptions (e.g., optical flow, kinematic basis)

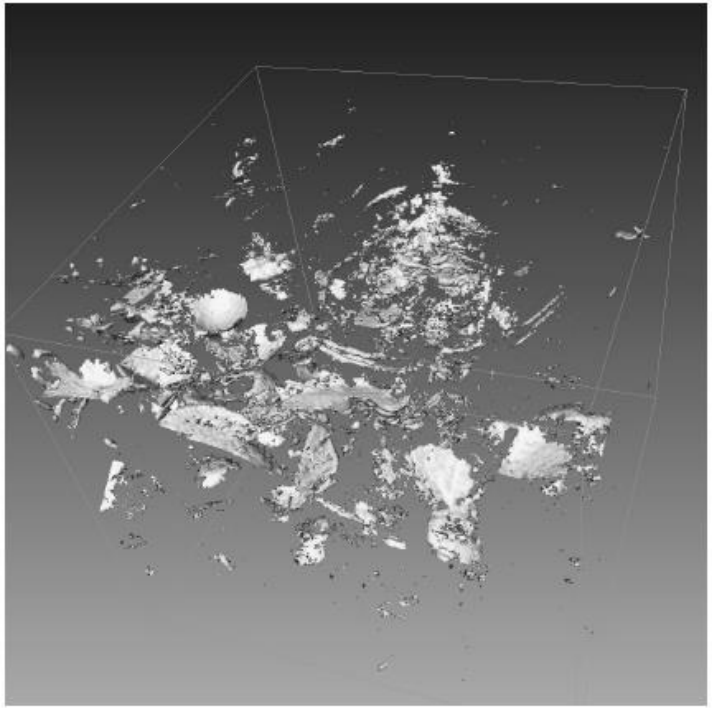
DVC

Used for residuals: crack or cracks

Al alloy



Bi-material with debonding at interfaces



Variants

- Because $\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) \xrightarrow[n\to\infty]{} f(\mathbf{x})$ one may save computation time on $[\mathbf{M}]$ and $\{\boldsymbol{b}\}$

$$\mathrm{M}_{ij} = \sum_{\mathbf{x}} \left(\boldsymbol{\Psi}_i(\mathbf{x}) \cdot \boldsymbol{\nabla} f(\mathbf{x})\right)\left(\boldsymbol{\Psi}_j(\mathbf{x}) \cdot \boldsymbol{\nabla} f(\mathbf{x})\right)$$

$$b_i = \sum_{\mathbf{x}} \left(\boldsymbol{\Psi}_i(\mathbf{x}) \cdot \boldsymbol{\nabla} f(\mathbf{x})\right)\left(f(\mathbf{x}) - \tilde{g}_{\{\mathbf{v}\}}(\mathbf{x})\right)$$

Note that $[\mathbf{M}]$ no longer depends on iteration number

Important Feature

- Because the displacement is not integer valued, a sub-pixel interpolation scheme is to be chosen

$$\tilde{g}_{\{\mathbf{v}\}}(\mathbf{x}) = g(\mathbf{x} + v_i^{(n)} \boldsymbol{\Psi}_i(\mathbf{x}))$$

- Spline or Fourier interpolations reveal superior (although costly), and allow one to reach errors well below 10^{-2} pixel

Initialization

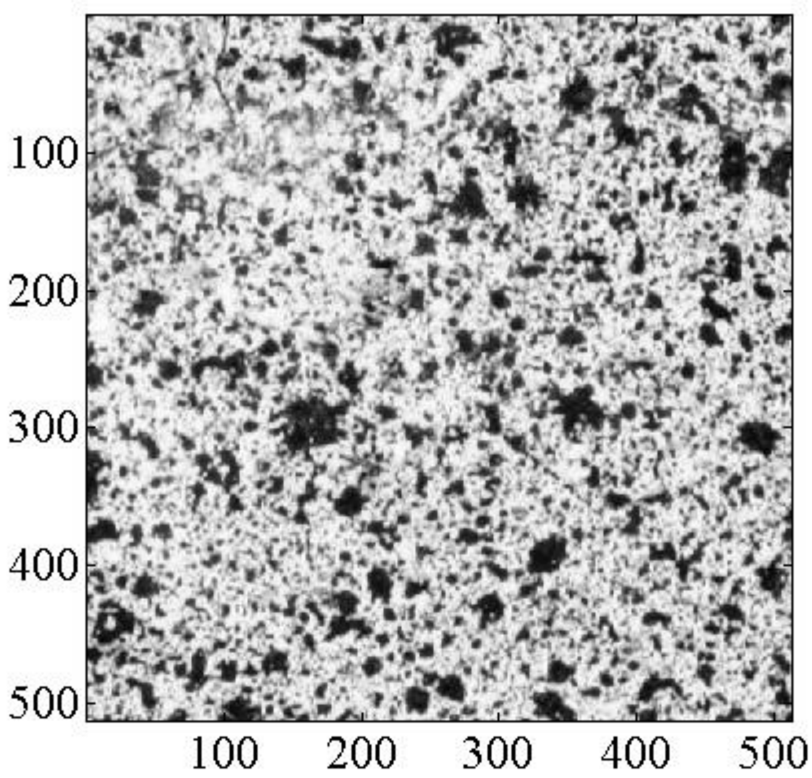
- The previous algorithm works fine when approaching the final solution. However, it assumes that the initial displacement is sufficiently close to the solution since relying on first order Taylor expansion

- The basin of attraction of the displacement is limited by the peak width of the auto-correlation function. It is an intrinsic feature of the reference texture

Initialization

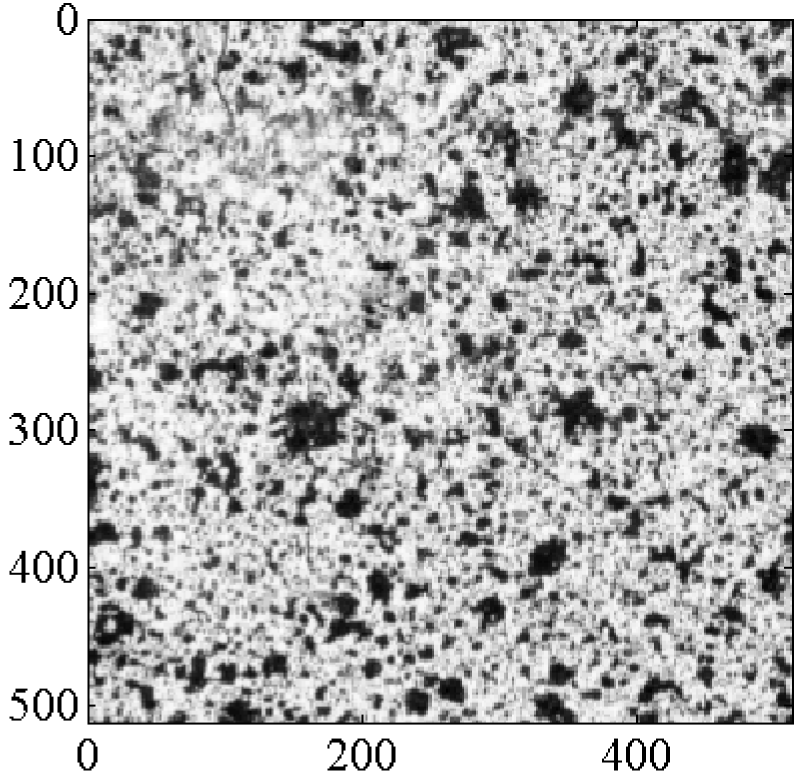
- A way to widen auto-correlation function is to filter out high frequencies, or to smooth images (both reference and deformed images)
- A smoothed image may be sampled at a coarser scale (reducing the image size and hence the computation cost)
- This procedure can be performed repeatedly (pyramidal structure)

Example (scale 0)



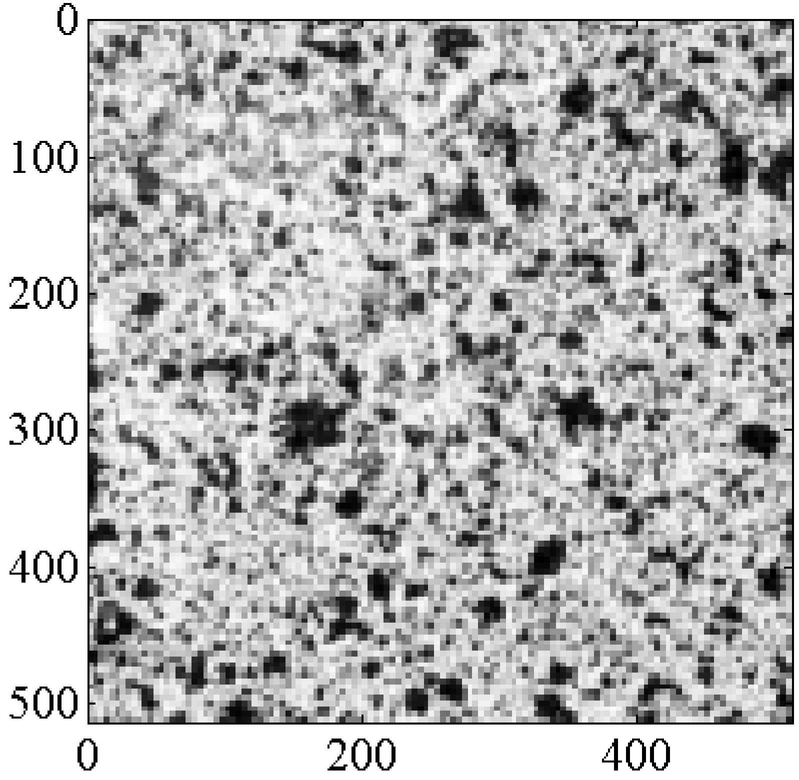
512×512

Example (scale 1)



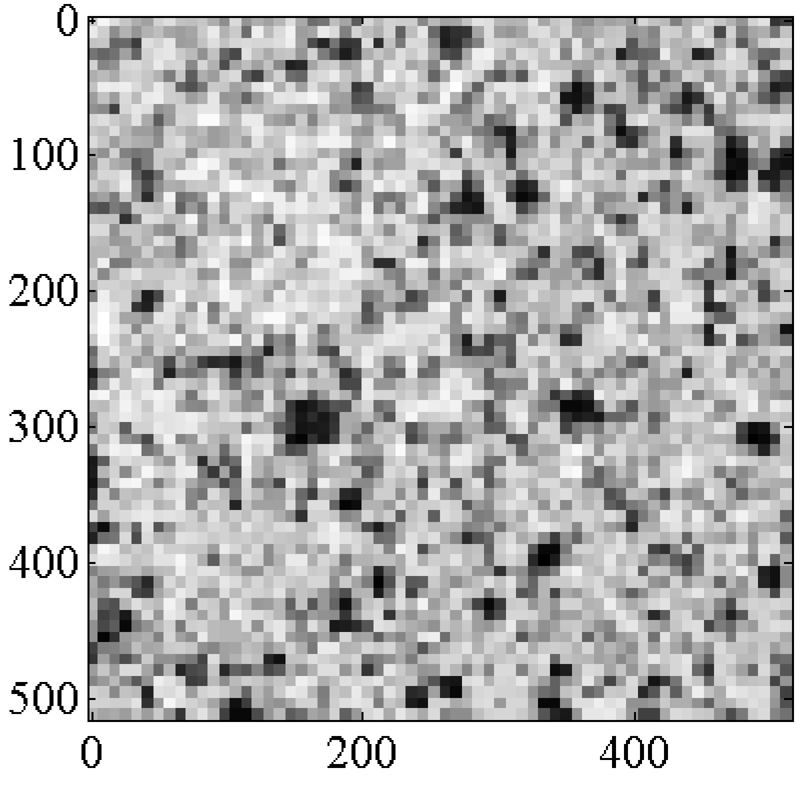
256×256

Example (scale 2)



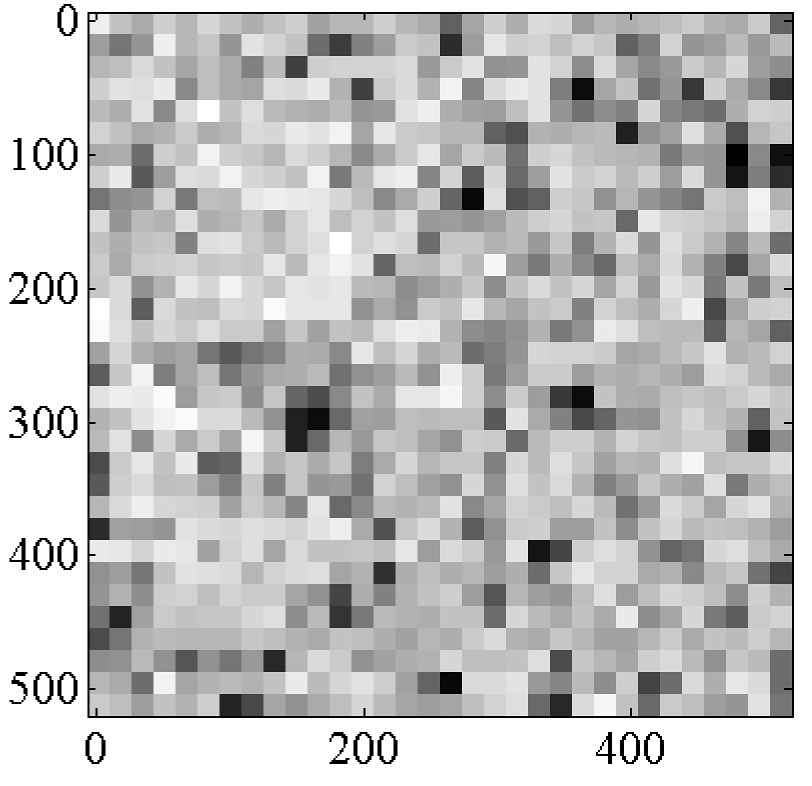
128×128

Example (scale 3)



64×64

Example (scale 4)



32×32

Multiscale Algorithm

- The usual first step is to evaluate the mean displacement through a simple cross-correlation procedure (cheap step)
- Then DIC is performed on the coarsened image, where the coarsening scale has been adjusted above the maximum expected displacement with respect to the mean
- The measured displacement at convergence is used as the initialization of the finer definition

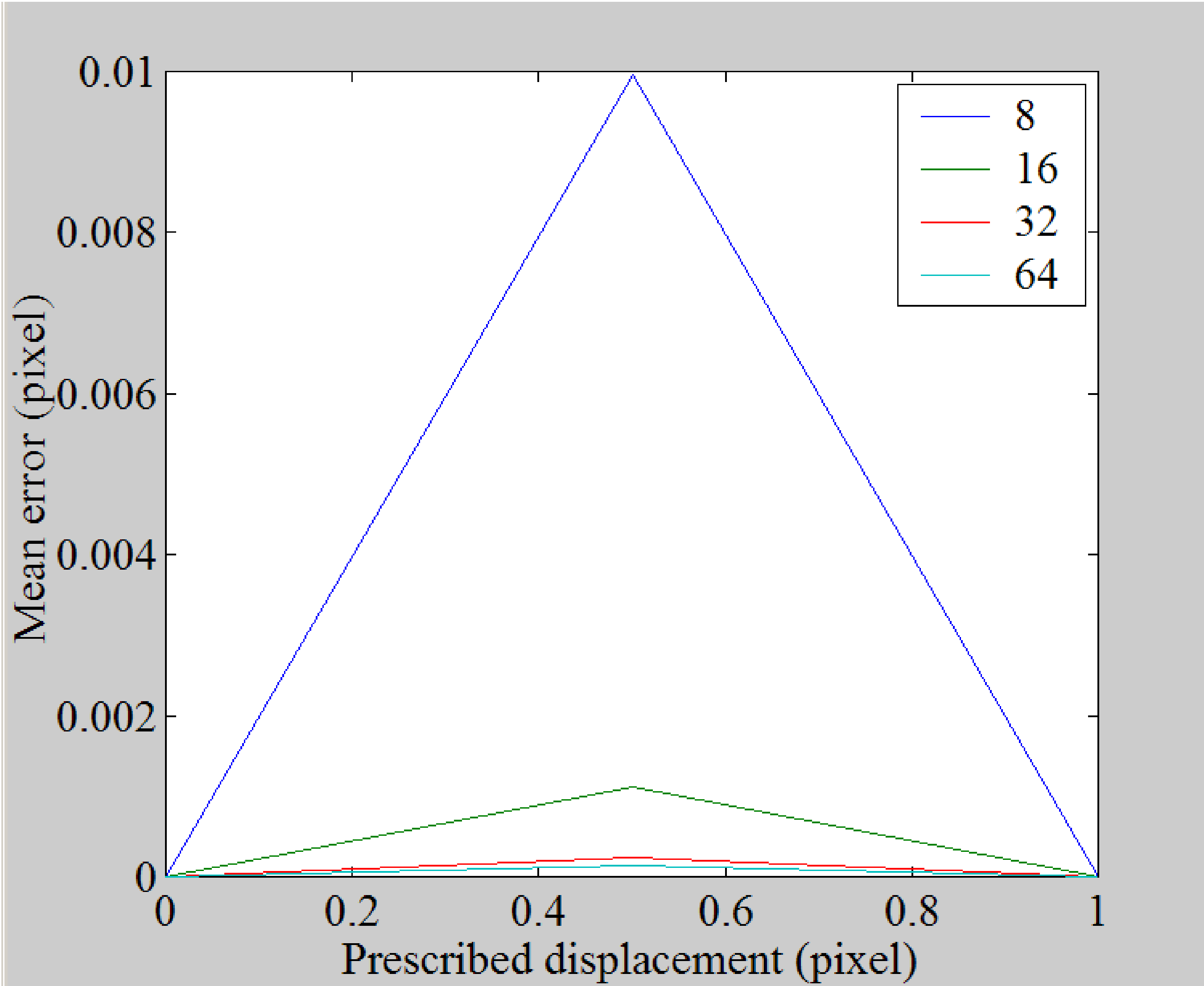
A PRIORI ERROR ESTIMATOR

A Priori Error Estimator

- Generate a fake image with subpixel displacement
- Evaluate mean displacement (bias) and standard uncertainty
- This can be repeated for different bases to select the appropriate versatility / noise immunity
- The same exercise can be performed based on experimental images (involving displacement)

Mean Error vs. Prescribed Displacement

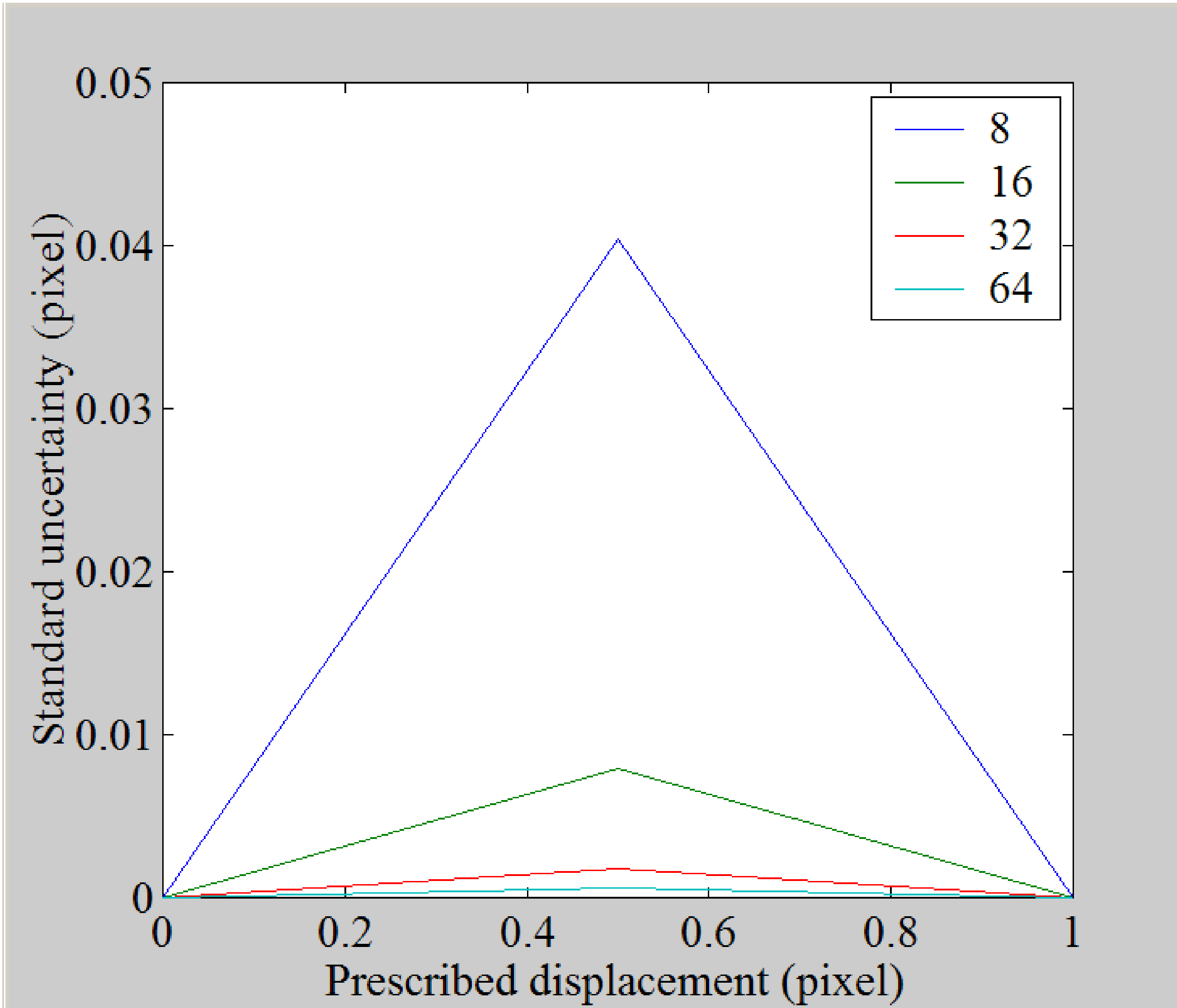
composite0.bmp



Q4 FE basis

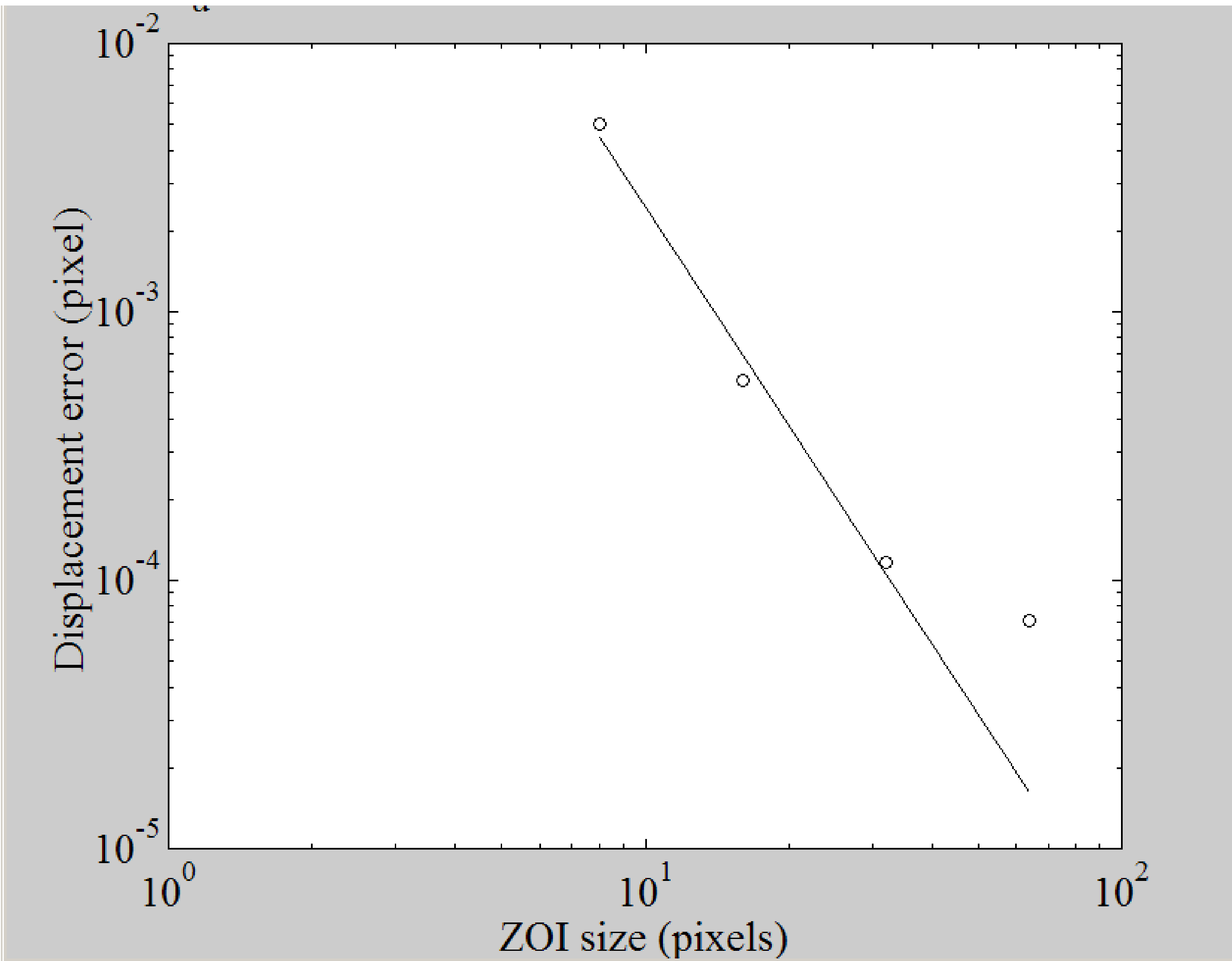
Standard Uncertainty vs. Prescribed Displacement

composite0.bmp



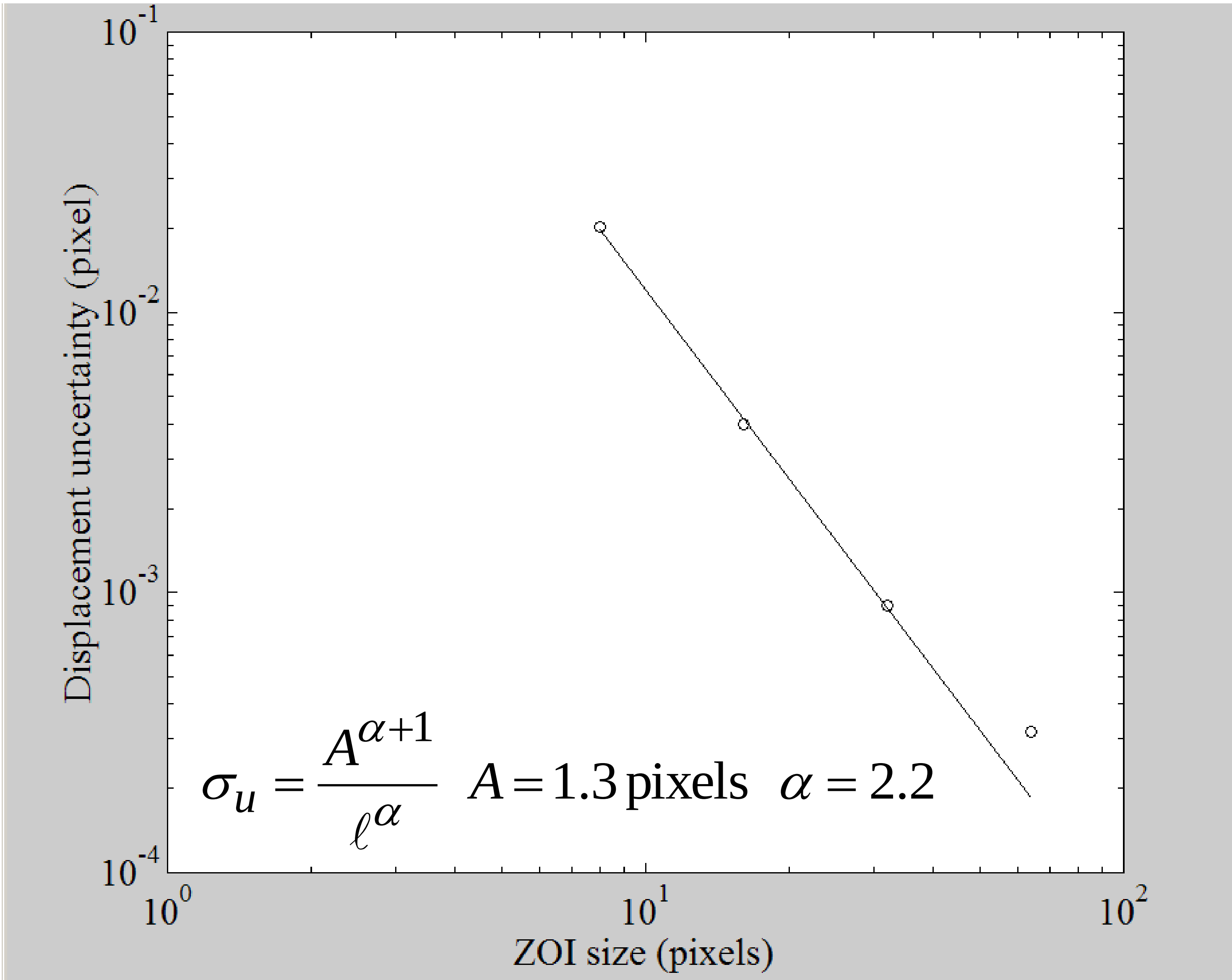
Displacement Error vs. Element Size

composite0.bmp



Displacement Uncertainty vs. Element Size

composite0.bmp



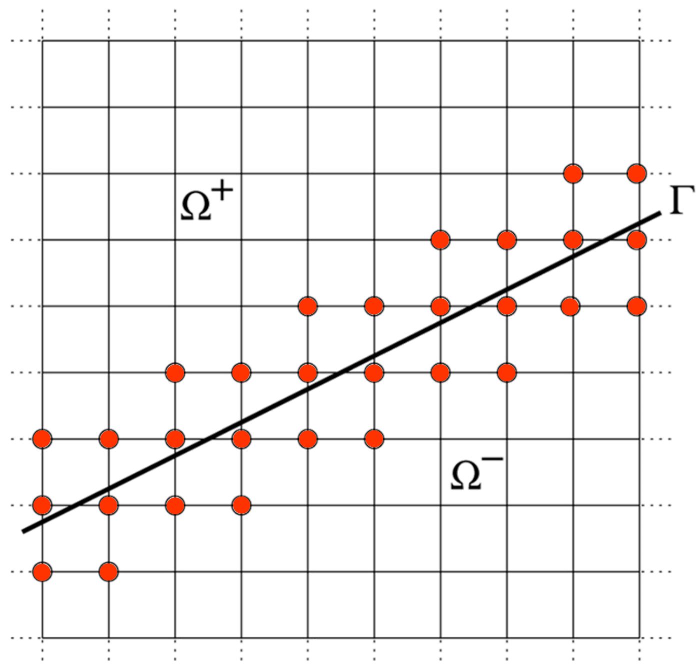
EXAMPLES

EXTENDED DIC

*[Réthoré *et al.*, 2007, *C. R. Mécanique*, 335, pp. 136-137]
*[Réthoré *et al.*, 2007, *CMAME*, 196, pp. 5016-5030]

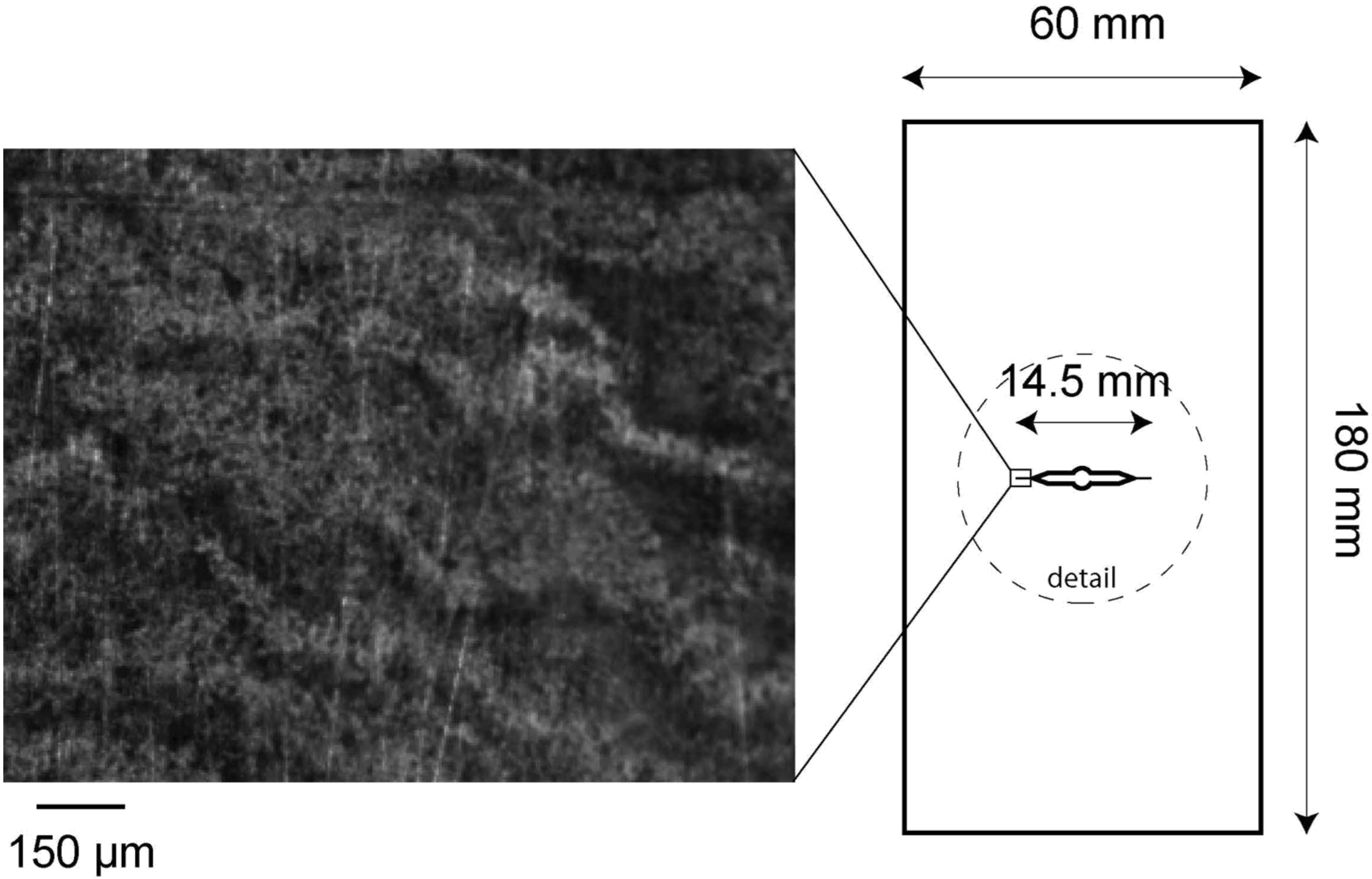
Enriched Kinematics à la X-FEM

$$\underline{u}(\underline{x}) = \underbrace{\sum_{n \in N} \sum_{i} v_{ni}^{e} N_n(\underline{x}) \underline{e}_i}_{\text{standard (e.g., Q4)}}$$



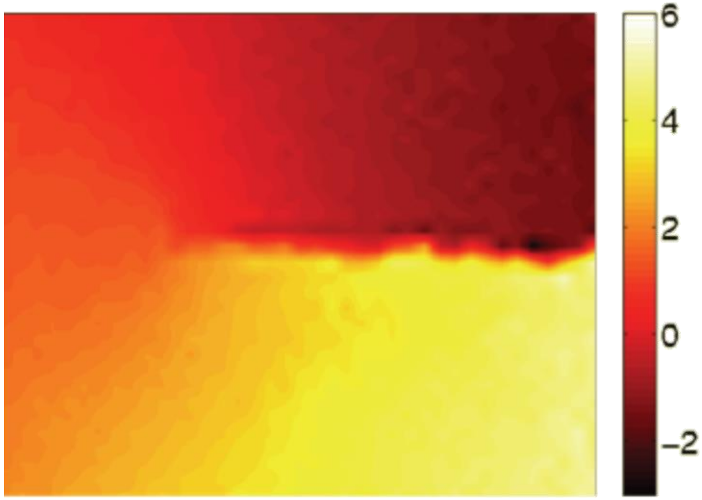
*[Belytschko and Black, 1999, *IJNME* 45 601–620; Moës *et al.*, 1999, *IJNME* 46 133–150]

Steel Fatigue Crack



Long distance microscope: 1 pixel ↔ 2.08 µm

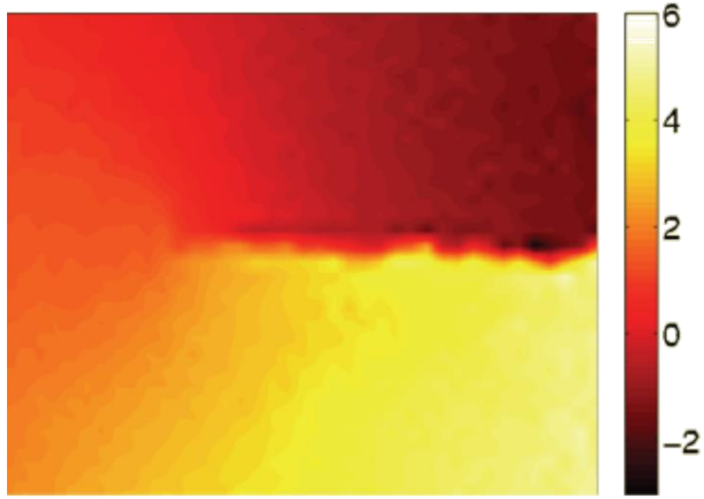
Steel Fatigue Crack

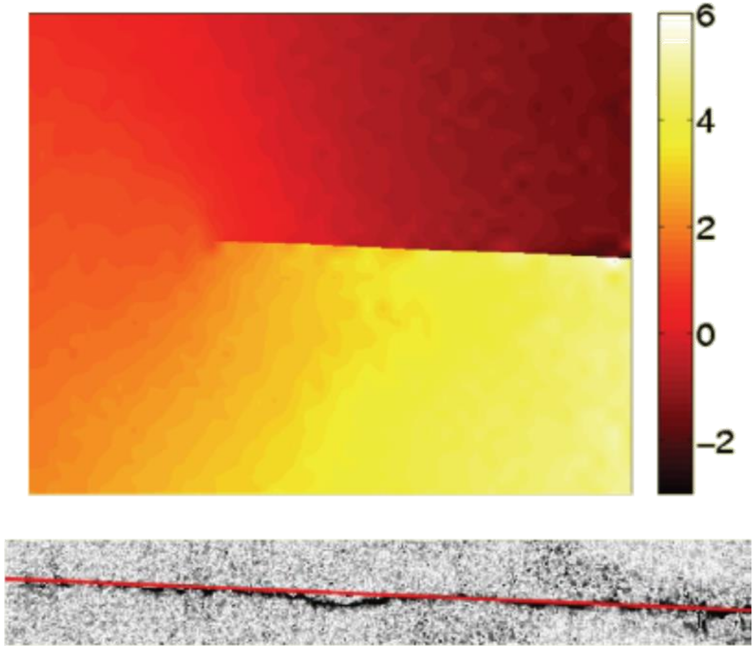


Finite element basis

1 pixel = 2.08 µm

Steel Fatigue Crack





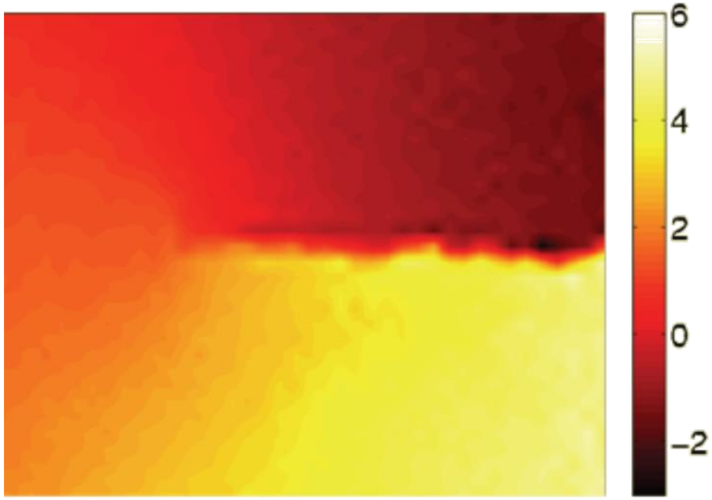
Finite element basis

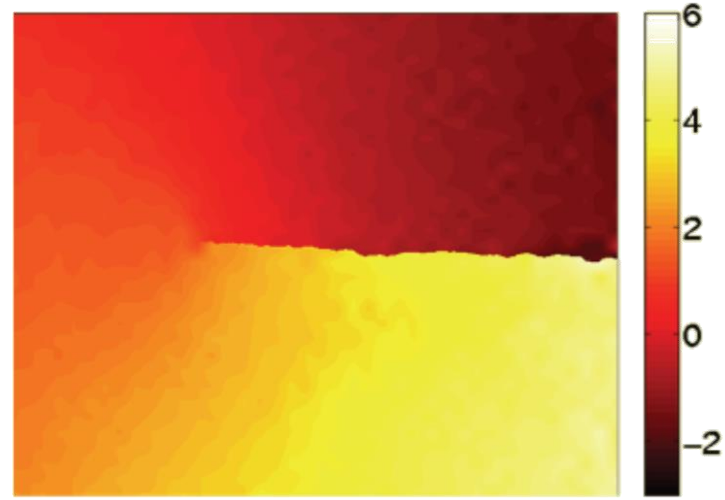
Extended finite element basis

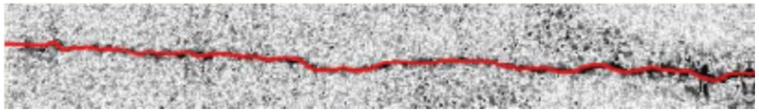
1 pixel = 2.08 µm

[Réthoré *et al.*, 2008, *Int. J. Num. Meth. Eng.* 73 pp. 248-272]

Steel Fatigue Crack







Finite element basis

Extended finite element basis

1 pixel = 2.08 µm

[Réthoré *et al.*, 2008, *Int. J. Num. Meth. Eng.* 73 pp. 248-272]

INTEGRATED DIC

Integrated DIC (I-DIC)

- Crack tailored basis $\underline{\mathbf{v}}(\underline{\mathbf{x}}) = \sum v_i \underline{\boldsymbol{\psi}}_i(\underline{\mathbf{x}})$

$$\underline{\boldsymbol{\psi}}_1(z) = 1$$

$$\underline{\boldsymbol{\psi}}_2(z) = i$$

$$\underline{\boldsymbol{\psi}}_3(z) = iz$$

$$\underline{\boldsymbol{\psi}}_4(z) = (\kappa - 1)z + 2\bar{z}$$

$$\underline{\boldsymbol{\psi}}_5(z) = \sqrt{r}\left[2\kappa e^{i\theta/2} - e^{3i\theta/2} - e^{-i\theta/2}\right]$$

$$\underline{\boldsymbol{\psi}}_6(z) = i\sqrt{r}\left[2\kappa e^{i\theta/2} + e^{3i\theta/2} - 3e^{-i\theta/2}\right]$$

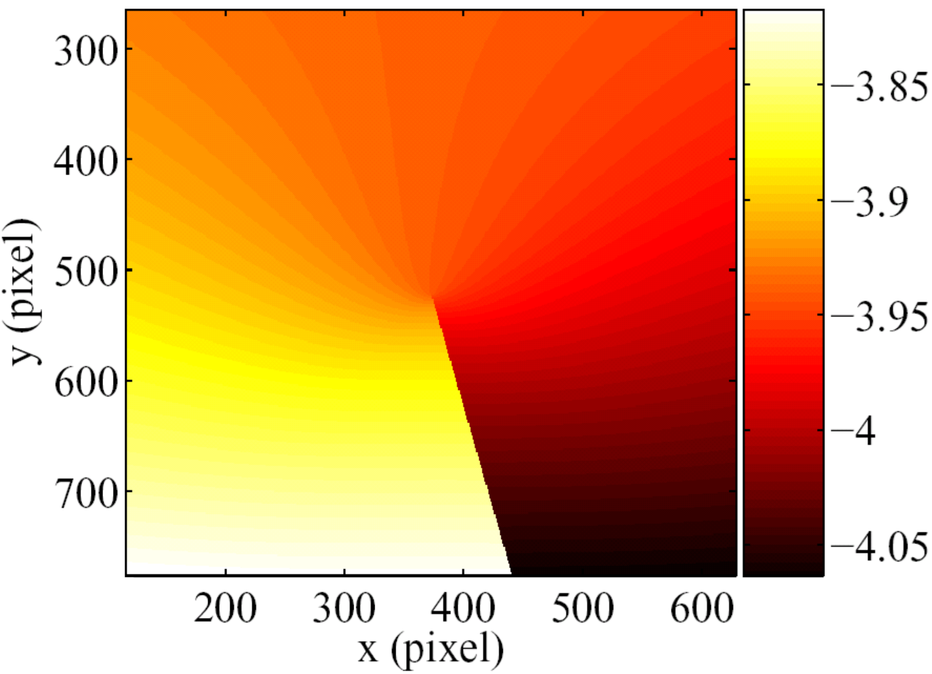
$$\underline{\boldsymbol{\psi}}_7(z) = \sqrt{r^3}\left[2\kappa e^{3i\theta/2} - 3e^{i\theta/2} + e^{-3i\theta/2}\right]$$

$$\underline{\boldsymbol{\psi}}_8(z) = i\sqrt{r^3}\left[2\kappa e^{3i\theta/2} + 3e^{i\theta/2} - 5e^{-3i\theta/2}\right]$$

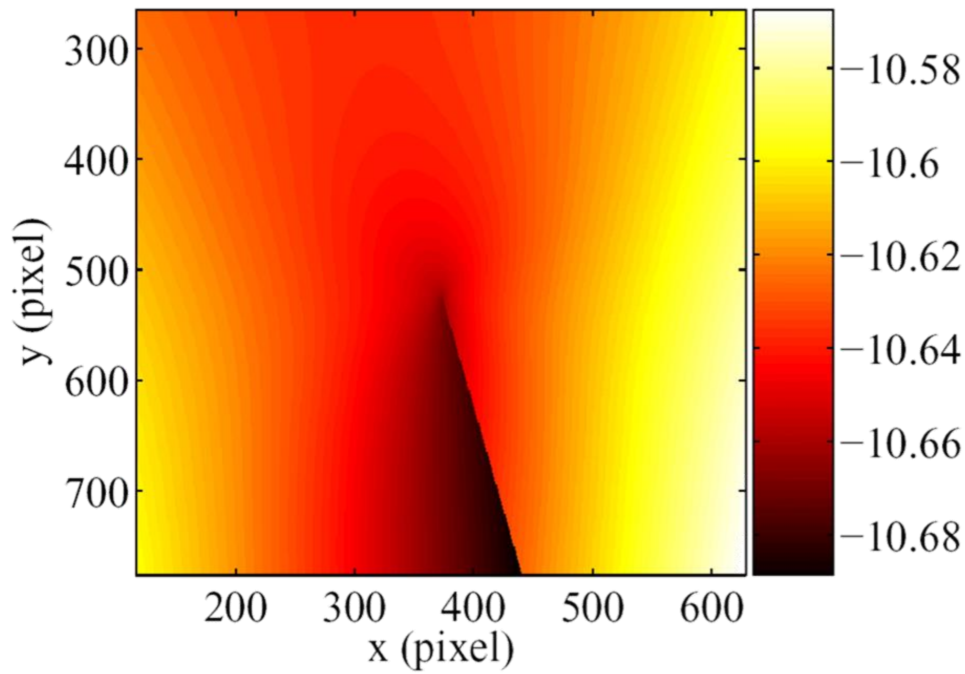
[SR *et al.*, 2006, *Int. J. Fract.* 140 pp. 141-157]

Measurement *and* Identification

Horizontal displacement (pixel)



Vertical displacement (pixel)



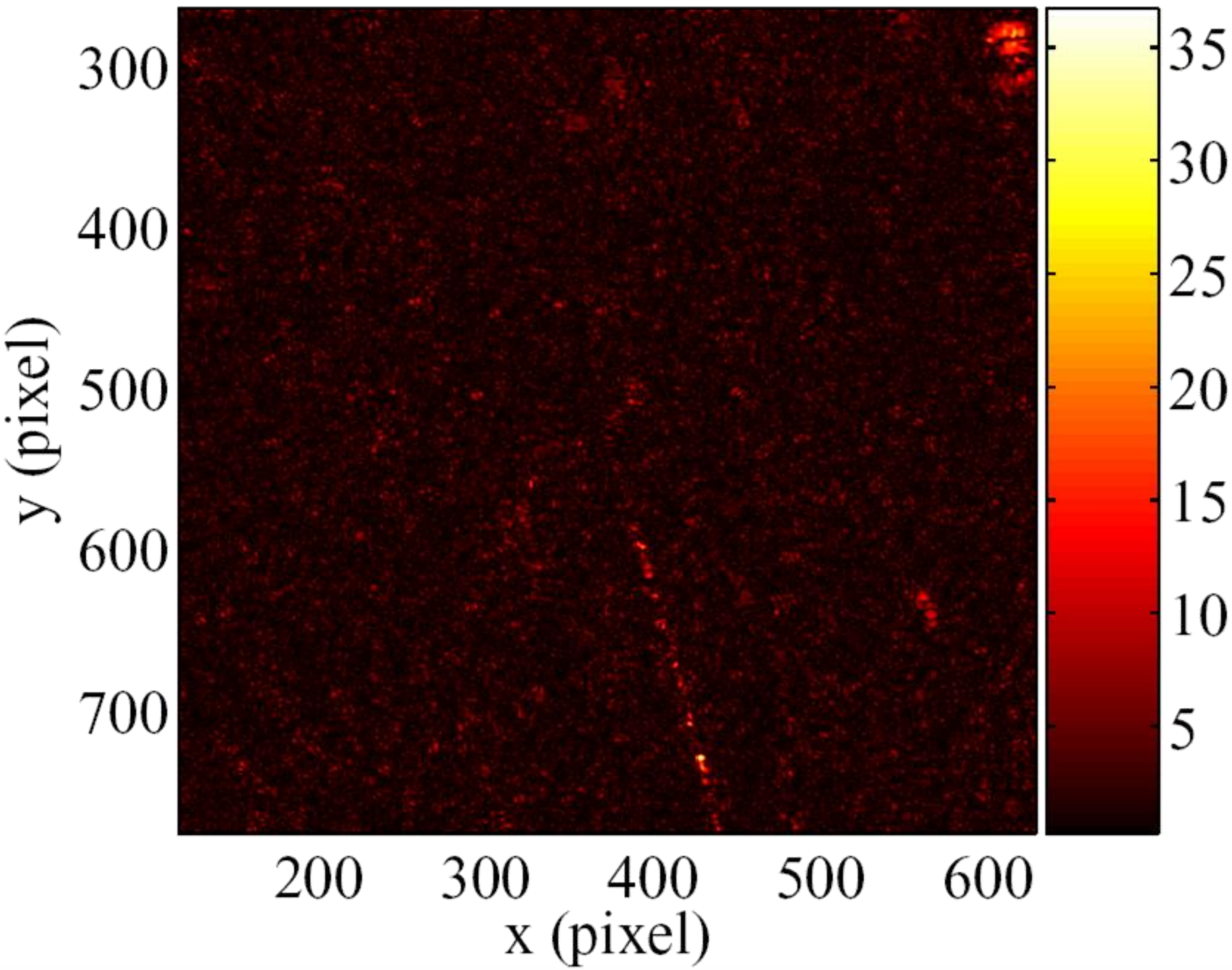
$K_I = 2.8 \pm 0.1$ MPa $m^{1/2}$

$K_{II} = 0.0 \pm 0.1$ MPa $m^{1/2}$

1 pixel ↔ 1.85 μm

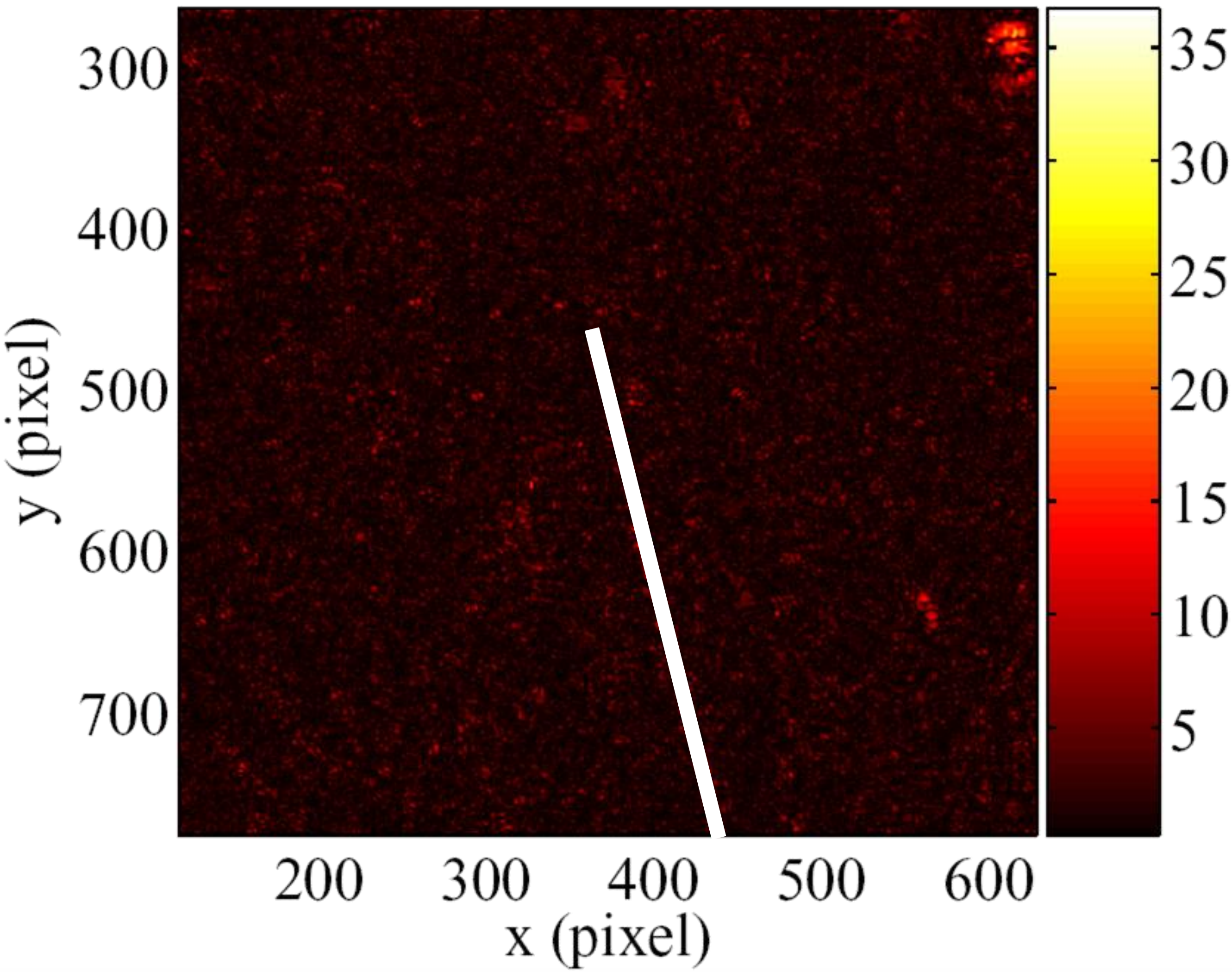
Residuals

$$\rho(\underline{\mathbf{x}}) = \left| g(\underline{\mathbf{x}} + \underline{\mathbf{u}}(\underline{\mathbf{x}})) - f(\underline{\mathbf{x}}) \right|$$



Residuals

$$\rho(\underline{\mathbf{x}}) = \left| g(\underline{\mathbf{x}} + \underline{\mathbf{u}}(\underline{\mathbf{x}})) - f(\underline{\mathbf{x}}) \right|$$



DIC Analysis of AFM Pictures

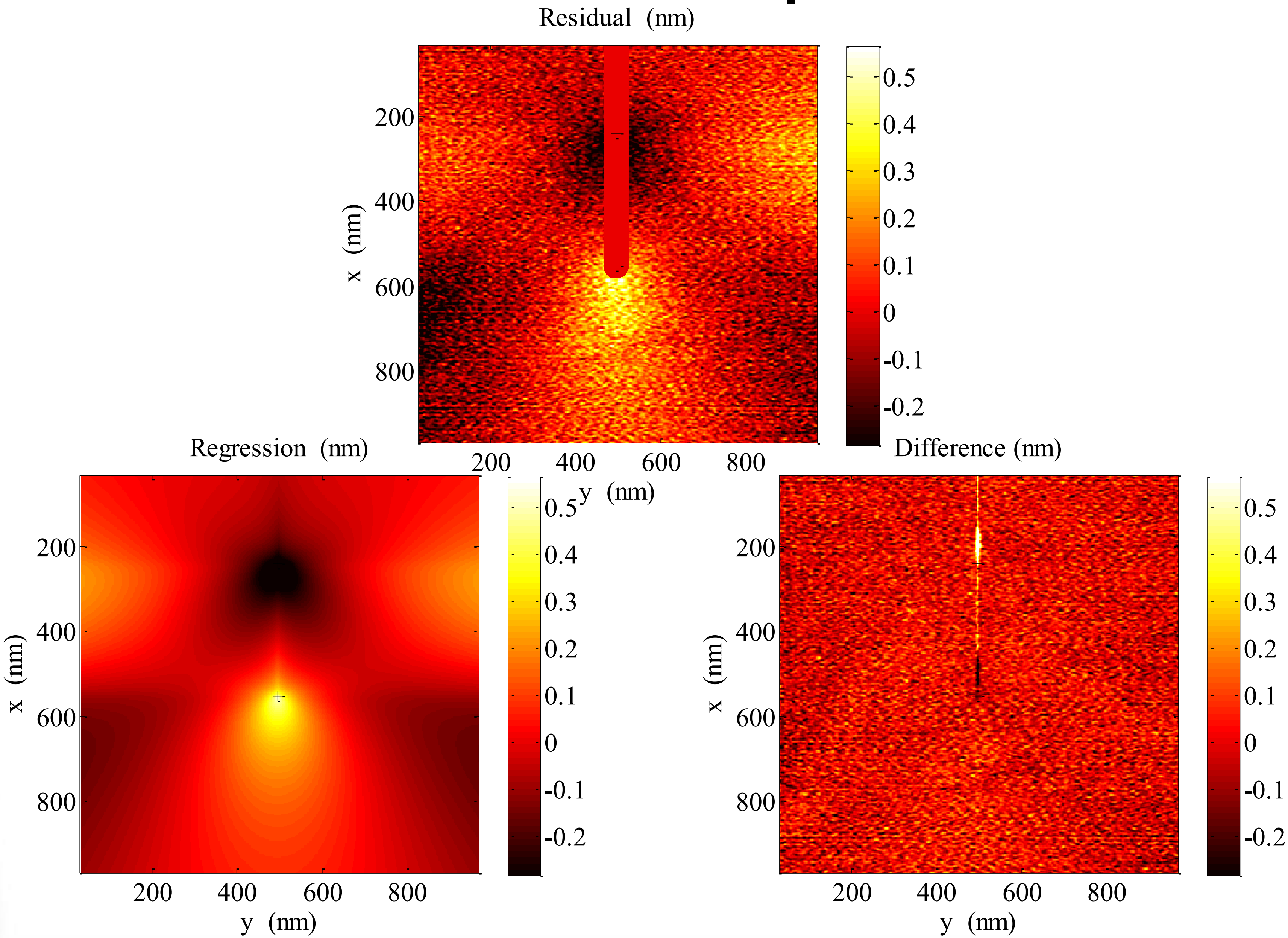
Optical flow conservation has to be generalized to

$$g(x,y) = f(x+u_x(x,y), y+u_y(x,y)) + u_z(x,y)$$

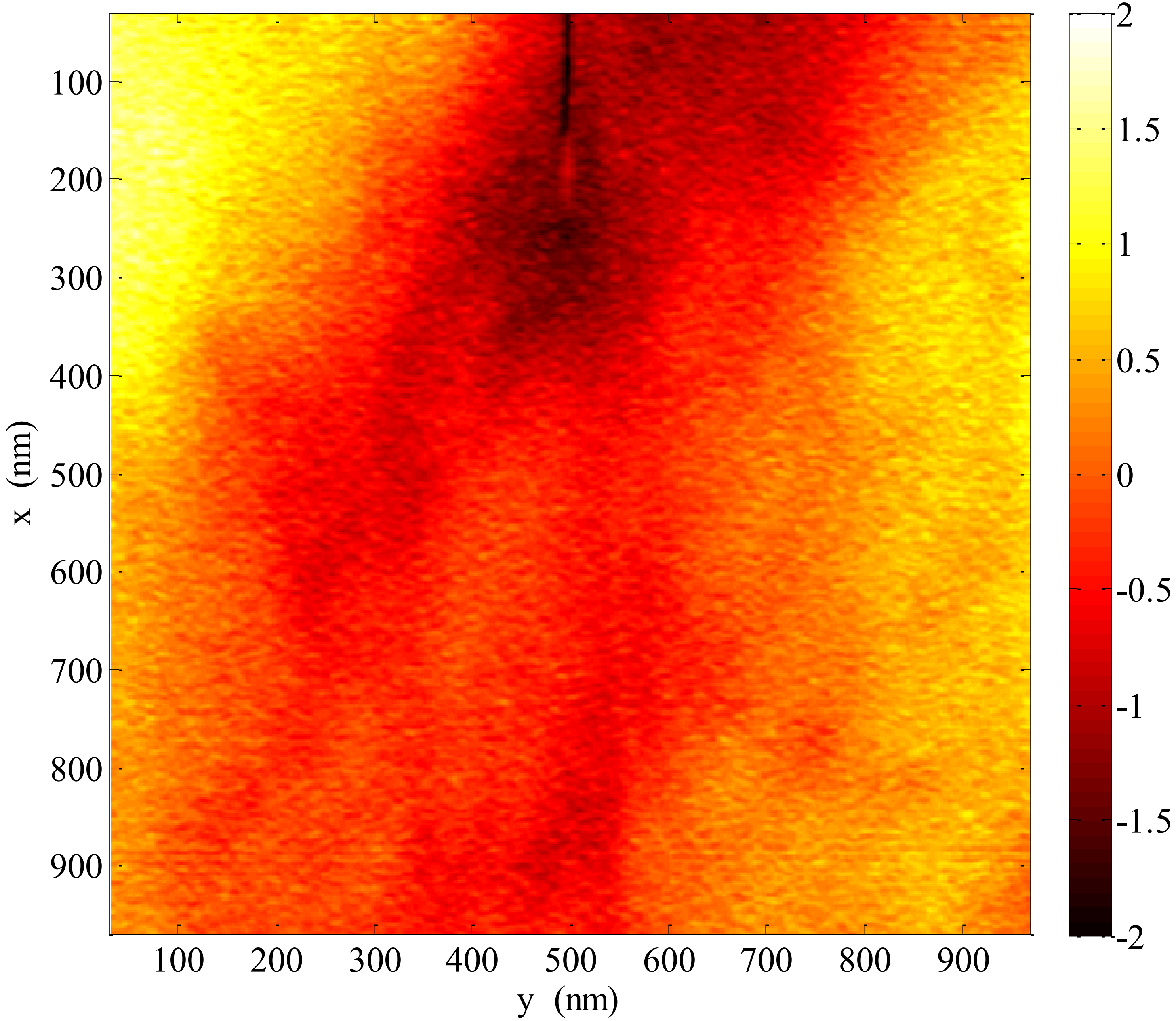
Moreover u_z is not known analytically!

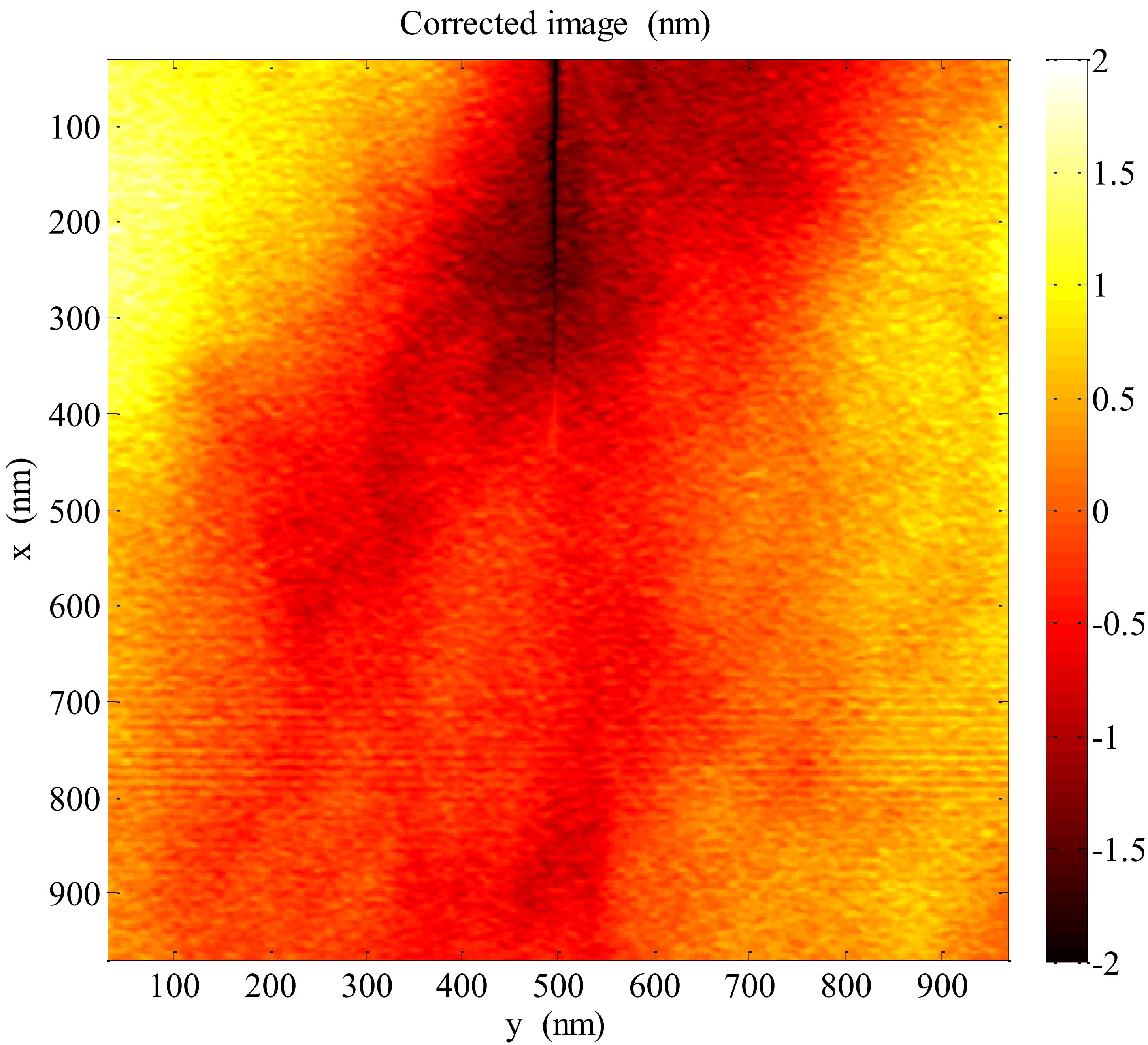
[Han *et al., 2010, Europ. Phys. Lett.* 89 66003]

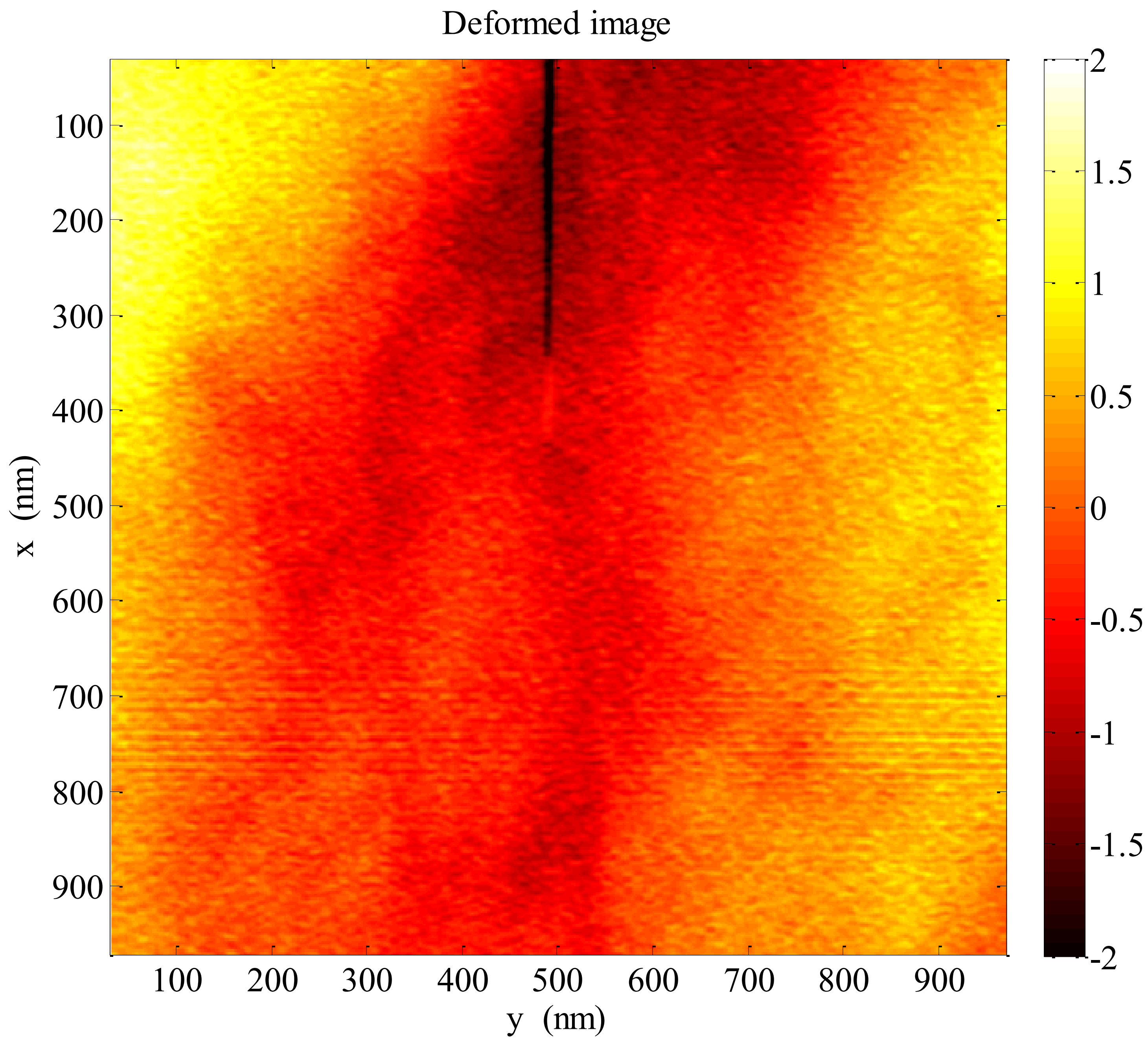
Out of Plane Displacement



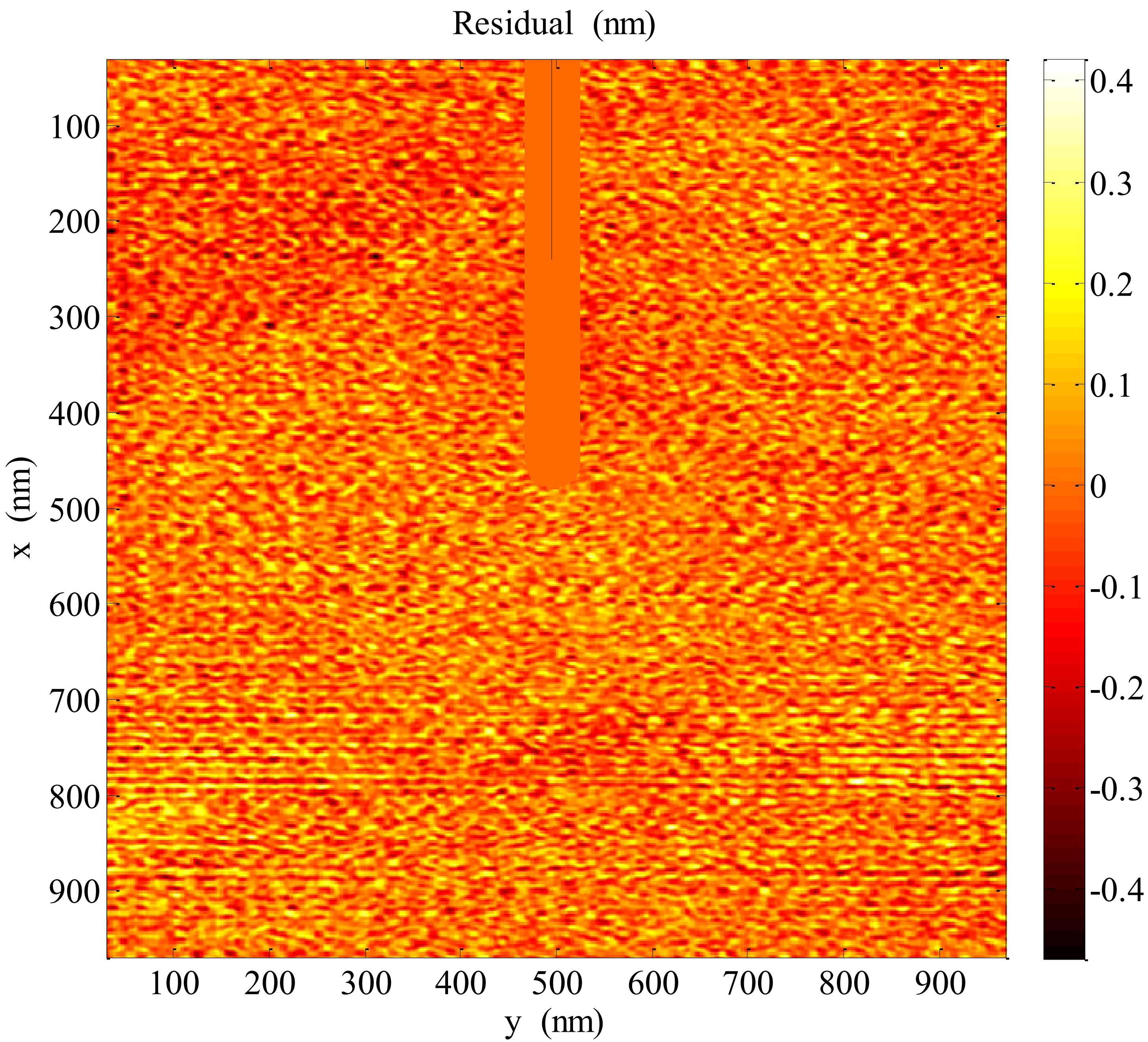
Reference image (nm)



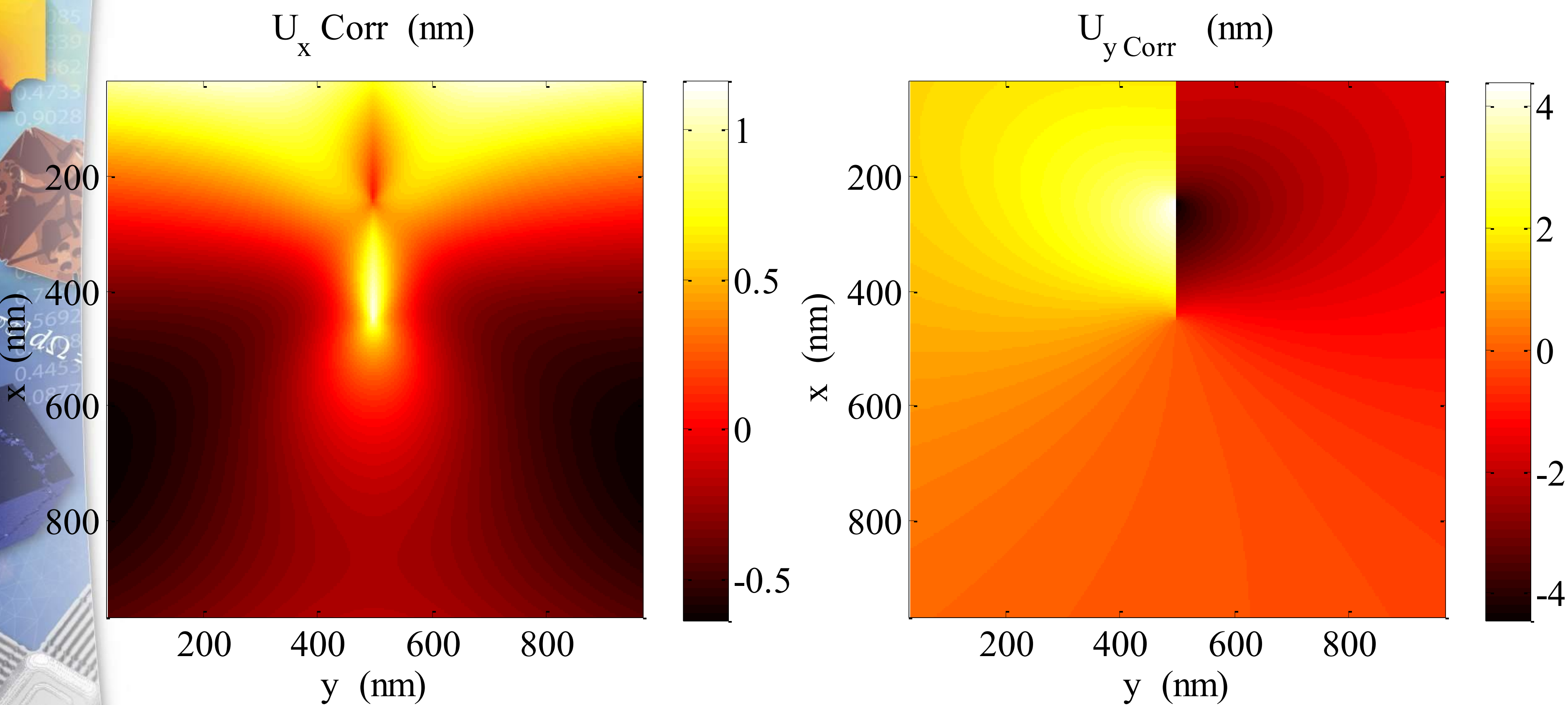

Corrected image (nm)
x (nm)
100
200
300
400
500
600
700
800
900
y (nm)
100
200
300
400
500
600
700
800
900
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2



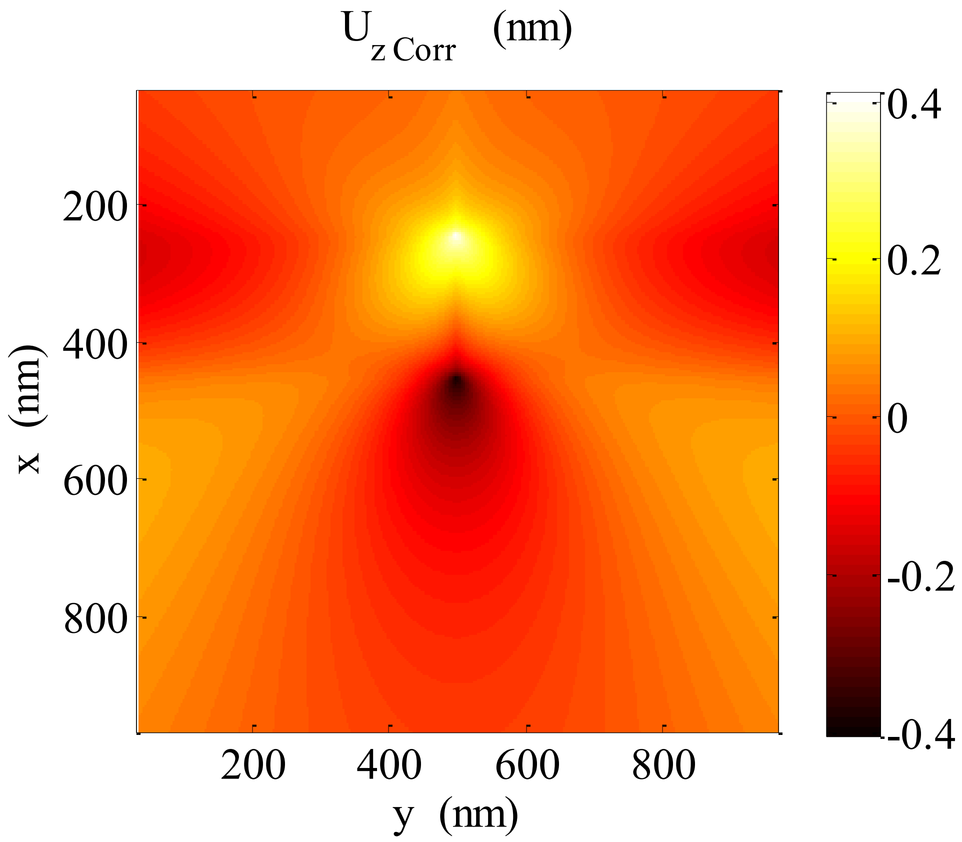
Deformed image
100
200
300
400
500
600
700
800
900
x (nm)
y (nm)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2


Residual (nm)
x (nm)
100
200
300
400
500
600
700
800
900
y (nm)
100
200
300
400
500
600
700
800
900
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4

In-Plane Displacement



Out of Plane Displacement



SIF Estimate

- Finally a number!

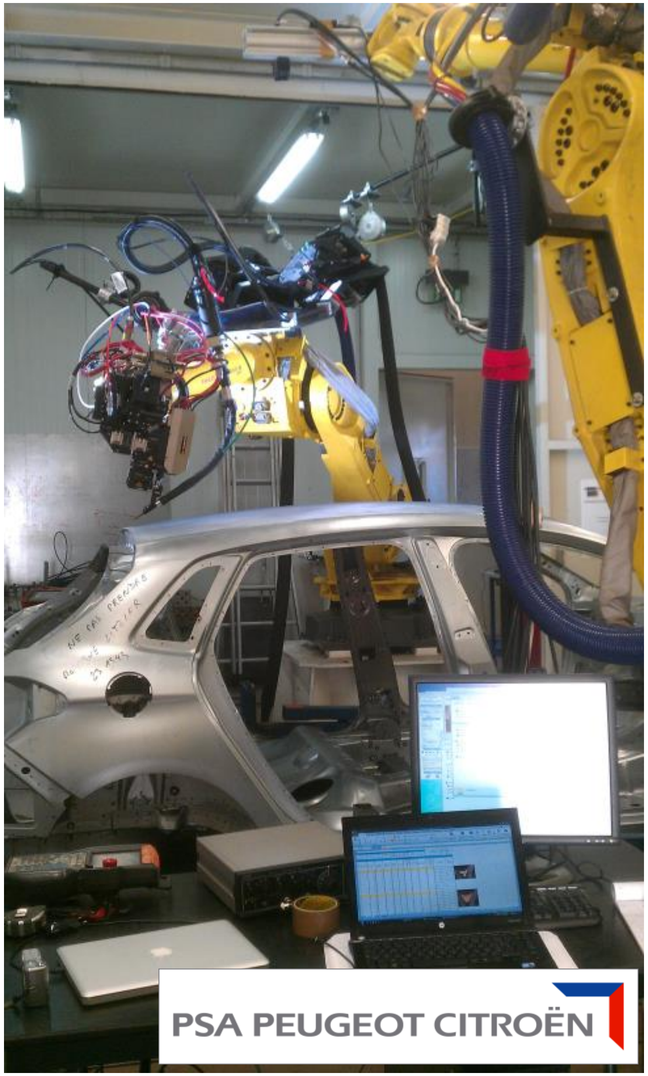
DIC estimate $K_I = 0.40 \pm 0.04$ MPa.m$^{1/2}$

Macro estimate $K_I = 0.39 \pm 0.02$ MPa.m$^{1/2}$

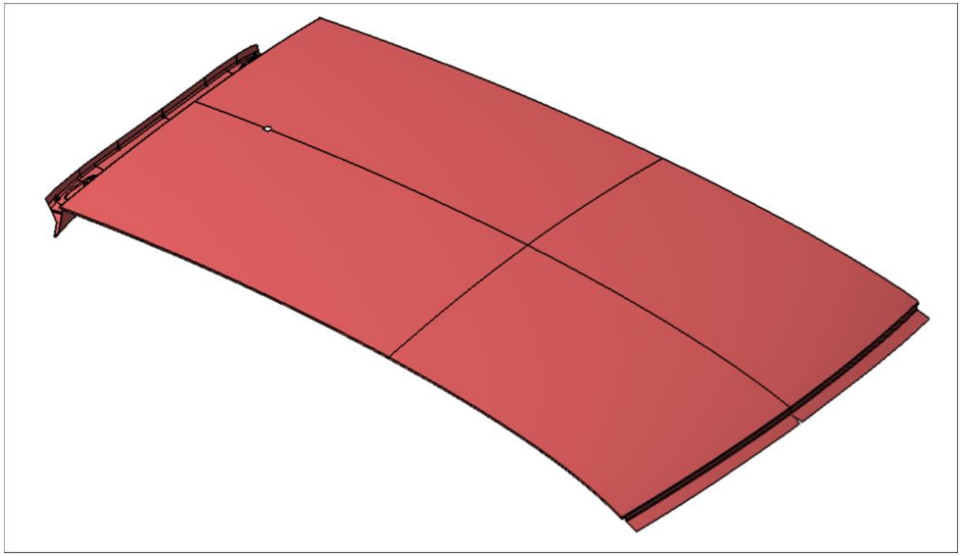
- A similar analysis also provides sensible estimates for 200×200 nm^2 images

GLOBAL STEREO-CORRELATION (TWO CAMERAS)

Geometric Regularization

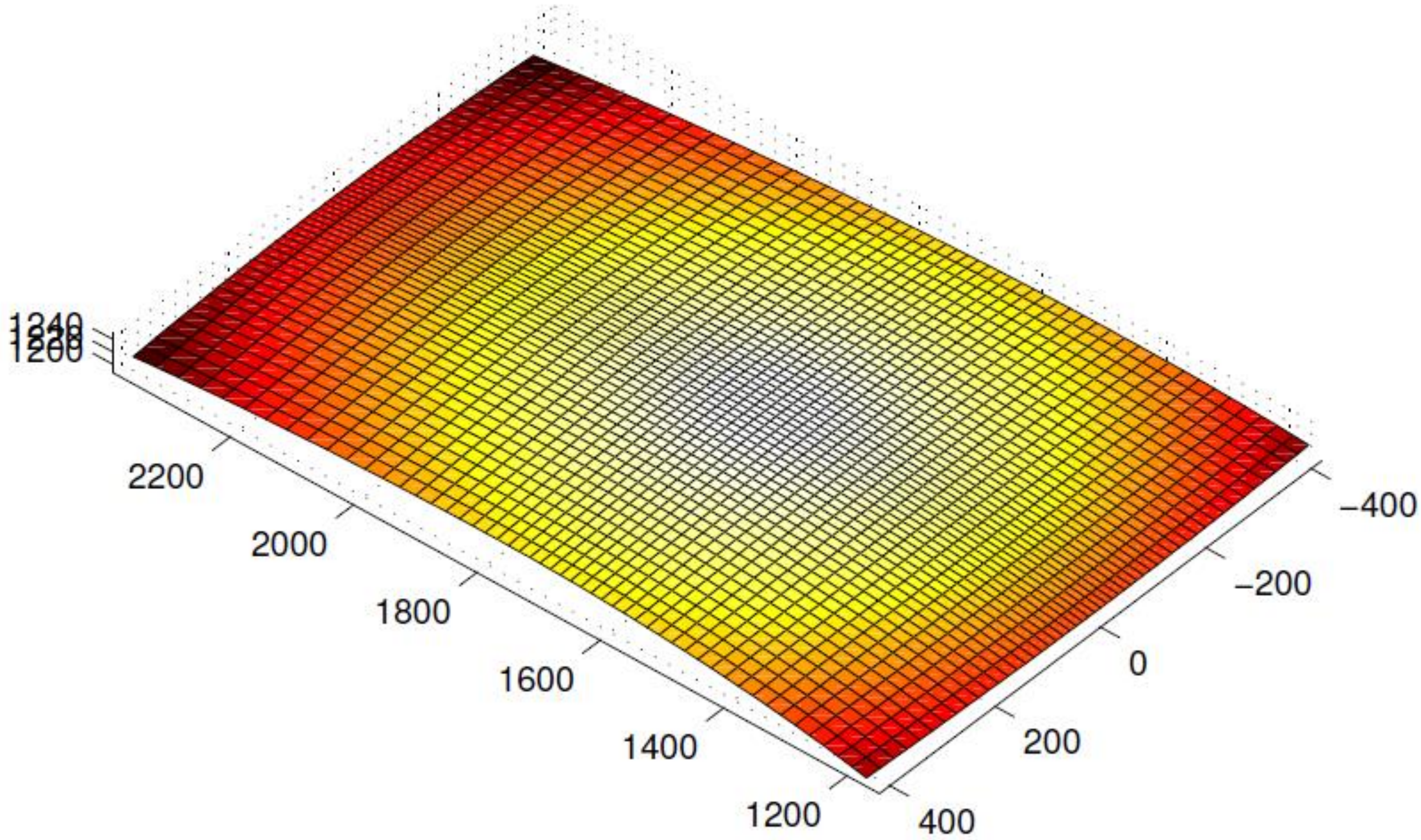


CAD model of observed surface

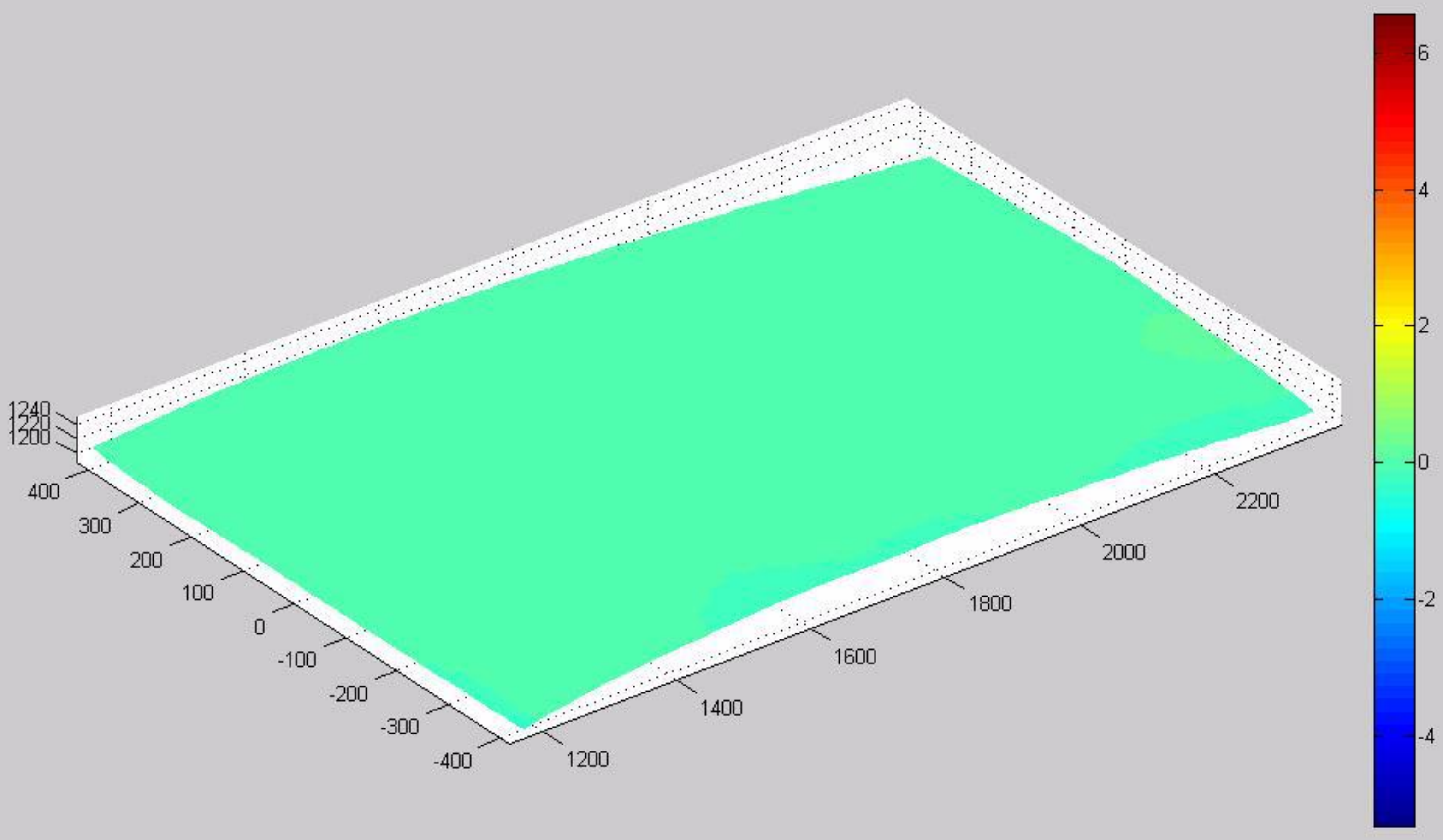


[Beaubier *et al.*, 2014, *Exp. Mech.* 54 pp. 329-341]

Measured 3D shape

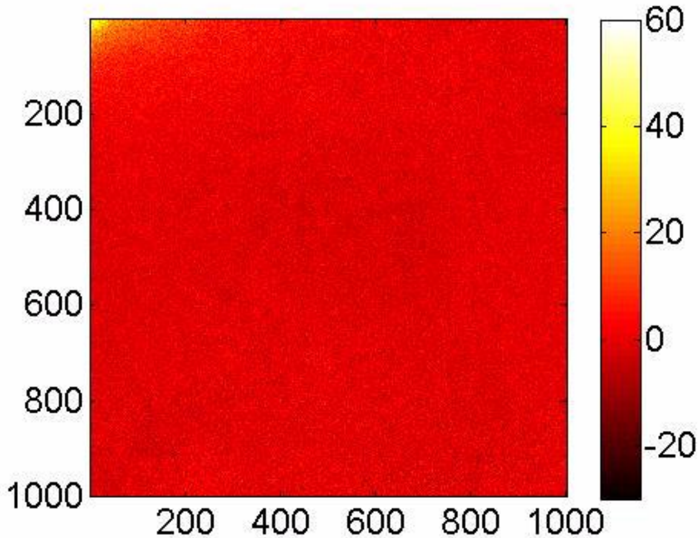


Normal Displacements (mm)

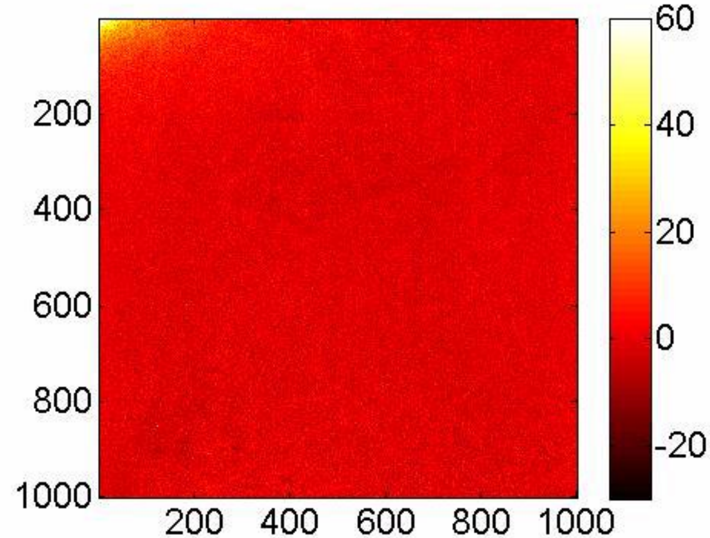


Correlation Residuals (GL)

Left camera



Right camera

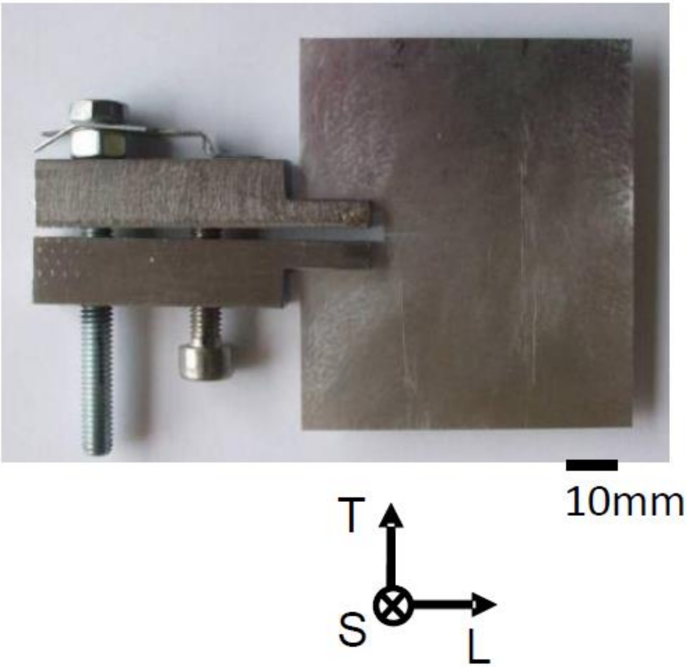


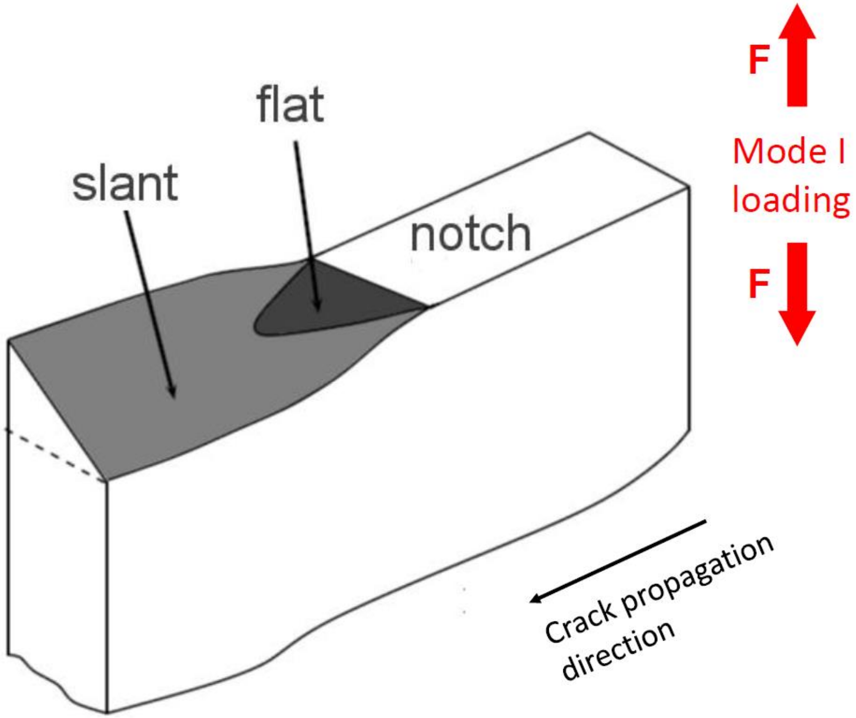
[Dufour *et al*., 2015, *Exp. Mech*. 55(9) pp.1657-1668]

DVC FOR SOLVING OLD PUZZLES

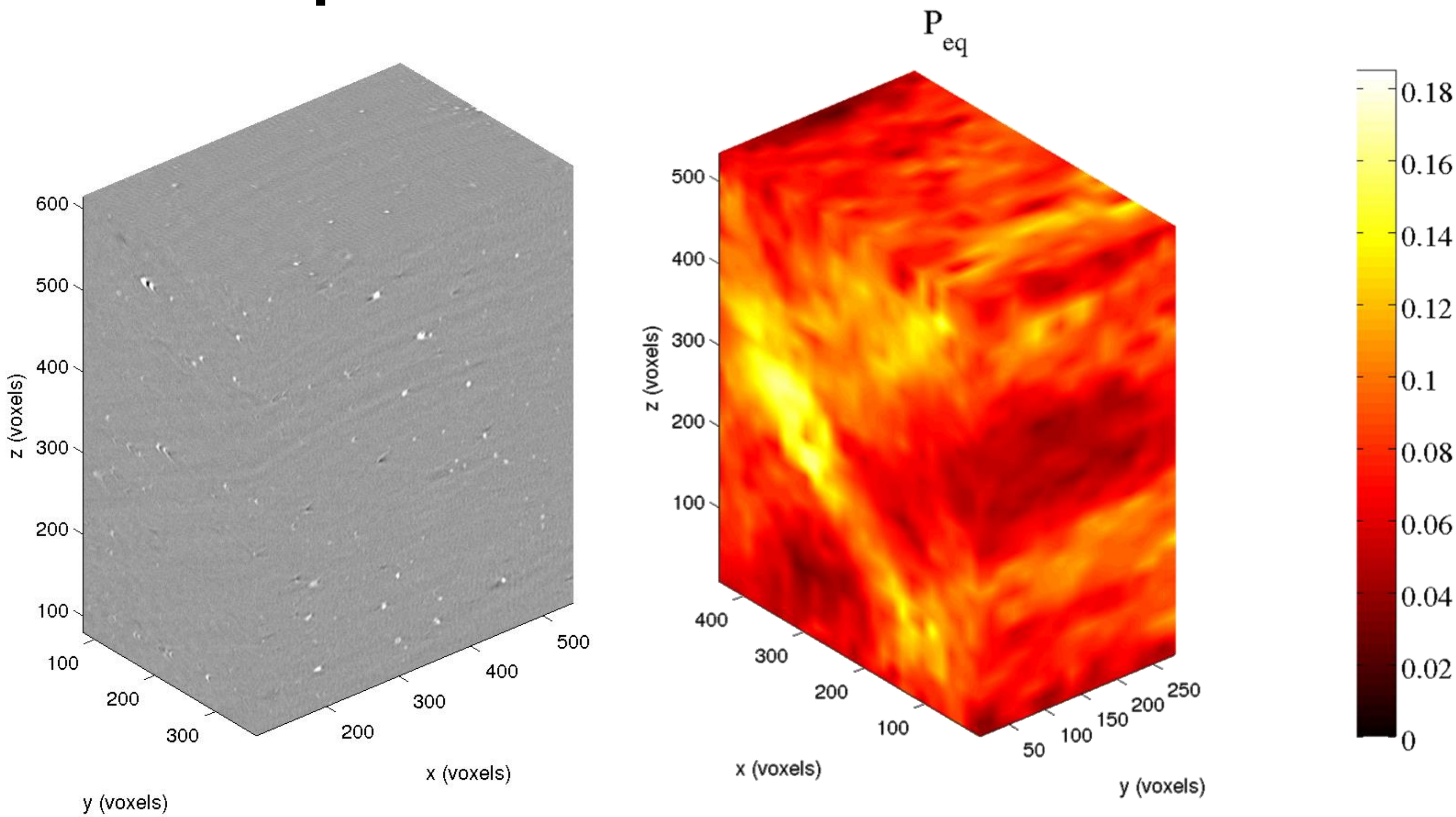
Tearing of Thin AA21xx Sheets

- Laminography with in-situ test
- Flat-to-slant transition





Al Alloy Specimen: Equivalent Strain Field



[Morgeneyer *et al.*, 2014, *Acta Mat.* 69 pp. 78-91]

DIC FOR OTHER APPLICATIONS

DIC Can Be Used For

- Correcting imaging devices
  - Distortion quantification (optical and SEM)
  - AFM scanning biases
- Registering images
  - Stereo pairs to provide 3D shape and motion
  - Stereo pairs with different modalities (e.g., opt + IR)
  - Acquisition with two very different modalities (X-neutron tomographies)

DIC Can Be Used For

- Exploiting / enhancing imaging sensitivity
  - DIC on IR images to measure $\mathbf{u}(\mathbf{x}),\mathrm{T}(\mathbf{x})$
  - Process EBSD maps to track crystal orientation (use quaternion-coded orientation as color levels)
  - Perform elastography from numerically computed kinemati[c basis

[Maynadier *et al.*, 2012, *Exp. Mech*. 52 pp. 241-255]
[Shi *et al.*, 2016, *Meas. Sci. Technol*. 27 095006]

DIC Can Be Used For

- Upgrading imaging to measurement devices
  - Analyze XRD spectra and their evolution
  - HR-EBSD on Kikuchi figures to access crystal elastic strain
- Provide new opportunities
  - Detect defects in woven composites
  - 4D tomography