IMAGERIE DE CONTRASTE DE PHASE

1.1 - $h_{Fresnel}(k_x, z) = e^{ikz} \exp\left[-i \frac{z}{2k} k_x^2\right]$ (*)

On a vu en cours : $F(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right) \longleftrightarrow F(\omega) = \sqrt{2\pi}\,\sigma \exp\left(-\frac{\omega^2\sigma^2}{2}\right)$

Dans (*), $\sigma^2 = \frac{iz}{k} \Rightarrow \sigma = e^{i\pi/4}\sqrt{z/k}$

Donc $h_{Fresnel}(x, z) = \frac{1}{\sqrt{2\pi}} e^{-i\pi/4}\sqrt{\frac{k}{z}} \exp\left[-\frac{x^2 k}{2iz}\right] e^{ikz}$

$$\boxed{h_{Fresnel}(x,z) = \frac{e^{-i\pi/4}}{\sqrt{\lambda z}} e^{ikz} \exp\left(i\frac{k x^2}{2z}\right) \qquad H = \frac{e^{-i\pi/4}}{\sqrt{\lambda z}}}$$

1.2 - $\boxed{\mathcal{E}_B(k_x) = \mathcal{E}_A(k_x)\, h_{Fresnel}(k_x, f) = \mathcal{E}_A(k_x)\, e^{ikf} \exp\left(-\frac{if}{2k} k_x^2\right)}$

1.3 - $\boxed{\mathcal{E}_C(x) = \mathcal{E}_B(x)\, t_f(x)}$

1.4 - $\mathcal{E}_D(x) = h_{Fresnel}(x, f) * \mathcal{E}_C(x) = h_{Fresnel}(x,f) * \left(\mathcal{E}_B(x)\, t_f(x)\right)$

$$= \frac{e^{-i\pi/4}}{\sqrt{\lambda f}} e^{ikf} \int_{-\infty}^{\infty} dx'\, \mathcal{E}_B(x')\, t_f(x') \exp\left[i\frac{k}{2f}(x-x')^2\right]$$

$$= \frac{e^{-i\pi/4}}{\sqrt{\lambda f}} e^{ikf} \exp\left(i\frac{kx^2}{2f}\right) \int_{-\infty}^{\infty} dx' \cancel{\exp\left(-\frac{ikx'^2}{2f}\right)} \mathcal{E}_B(x') \cancel{\exp\left(\frac{ikx'^2}{2f}\right)} \exp\left(-\frac{ikxx'}{f}\right)$$

$$\boxed{\mathcal{E}_D(x) = \frac{e^{-i\pi/4}}{\sqrt{\lambda f}} e^{ikf} \exp\left(\frac{ikx^2}{2f}\right) \mathcal{E}_B\left(k_x = \frac{kx}{f}\right)}$$

1.5. $$\mathcal{E}_D(x) = \frac{e^{-i\pi/4}}{\sqrt{\lambda f}}\, e^{ikf} \exp\left(i\frac{kx^2}{2f}\right) \mathcal{E}_A\left(k_x = \frac{kx}{f}\right) e^{ikf} \exp\left(-\frac{if}{2k}\frac{k^2x^2}{f^2}\right)$$

simplification

$$\boxed{\mathcal{E}_D(x) = \underbrace{\frac{e^{-i\pi/4}}{\sqrt{\lambda f}}}_{H}\, e^{2ikf}\, \mathcal{E}_A\left(k_x = \frac{kx}{f}\right)}$$

1.6 $$e^{i\phi(x)} \simeq 1 + i\phi(x) - \frac{\phi(x)^2}{2} = 1 + i\phi_0 \cos\frac{2\pi x}{p} - \frac{1}{2}\phi_0^2 \cos^2\frac{2\pi x}{p}$$

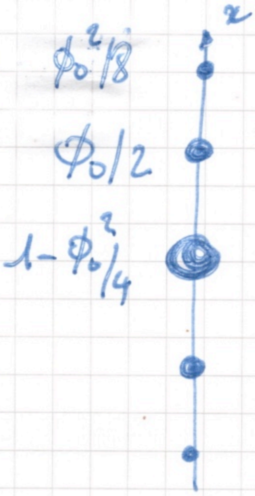
$$\boxed{e^{i\phi(x)} = \underbrace{1 - \frac{\phi_0^2}{4}}_{K} + \underbrace{i\phi_0}_{L} \cos\frac{2\pi x}{p} \underbrace{- \frac{\phi_0^2}{4}}_{M} \cos\frac{4\pi x}{p}}$$

1.7 $$\mathcal{E}_A(x) = \mathcal{E}_0\left(1 - \frac{\phi_0^2}{4}\right) + i\phi_0 \mathcal{E}_0 \frac{1}{2}\left(e^{i2\pi x/p} + c.c.\right) - \frac{\phi_0^2}{8}\mathcal{E}_0\left(e^{i4\pi x/p} + c.c\right)$$

$$\boxed{\begin{aligned}\mathcal{E}_A(k_x) = {} & 2\pi \mathcal{E}_0\left(1 - \frac{\phi_0^2}{4}\right)\delta(k_x) \\ & + i2\pi \mathcal{E}_0 \frac{\phi_0}{2}\left(\delta\left(k_x - \frac{2\pi}{p}\right) + \delta\left(k_x + \frac{4\pi}{p}\right)\right) \\ & - 2\pi \mathcal{E}_0 \frac{\phi_0^2}{8}\left(\delta\left(k_x - \frac{4\pi}{p}\right) + \delta\left(k_x + \frac{4\pi}{p}\right)\right)\end{aligned}}$$

1-8 D'après la question 1.5, on a sur l'écran placé dans le plan de Fourier :

Amplitude $\propto$



$\Delta x = \frac{f}{k}\frac{2\pi}{p} = \frac{\lambda f}{p} = 1$ cm

1.9 _ Une distribution uniforme car $|e^{i\phi(x)}|^2 = 1$

1.10 _ La fente coupe les ordres ± 2

$$\boxed{\mathcal{E}_q(x) = \mathcal{E}_0\left(1-\frac{\phi_0^2}{4}\right) + i\phi_0\,\mathcal{E}_0 \cos\frac{2\pi x}{p}}$$

1.11 - $\boxed{I_q(x) = I_0\left[\left(1-\frac{\phi_0^2}{4}\right)^2 + \phi_0^2\cos^2\frac{2\pi x}{p}\right]}$

$\boxed{\text{Période} = p/2}$

$$\boxed{C = \frac{\phi_0^2}{2\left(1-\phi_0^2/4\right)^2} \simeq \frac{\phi_0^2}{2} = 5\times 10^{-5}}$$

1.12 $\mathcal{E}_q(x) = i\mathcal{E}_0\left(t\left(1-\phi_0^2/4\right) + \phi_0\cos\frac{2\pi x}{p}\right)$

$$I_q(x) = I_0\left[t^2\left(1-\phi_0^2/4\right)^2 + \phi_0^2\cos^2\frac{2\pi x}{p} + 2t\left(1-\phi_0^2/4\right)\phi_0\cos\frac{2\pi x}{p}\right]$$

$$\boxed{I_q(x) \simeq I_0\left(t^2 + 2t\phi_0\cos\frac{2\pi x}{p}\right)}$$

$\boxed{\text{Période} = p}$

$$\boxed{C = \frac{4t\phi_0}{2t^2} = \frac{2\phi_0}{t} = 0{,}2}$$

1.13. Ce système sert à imager les petites aberrations de front d'onde, dues par exemple à la propagation d'une onde plane dans un milieu turbulent (= strioscopie).