# Chap I: Introduction

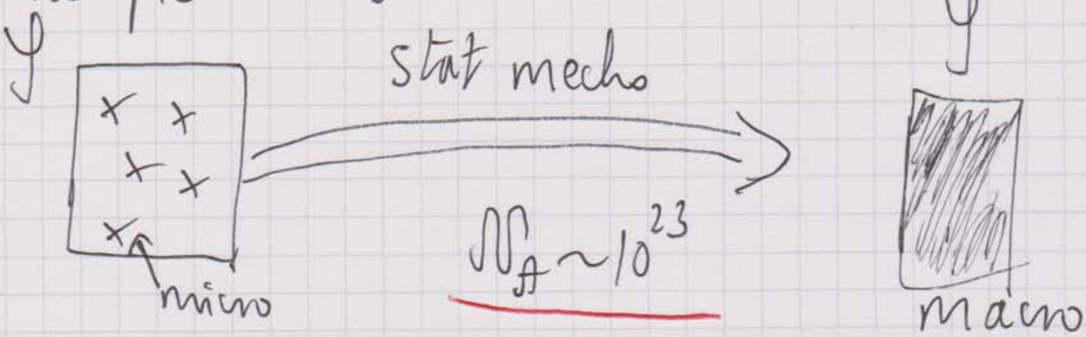


## I) What is statistical mechanics?

- One of the most important theory in physics!
  starts around 1850 and still developping

**Goal:** Stat mechs allows to infer using probabilities the properties of a system with a large number of constituents from the existing laws governing its constituents

In short, stat mechs allows ~~you~~ to go from microscopic to macroscopic world!




Stat mechs uses the laws of mechanics, relativity, Quantum mechanics to explain the collective behaviour of a set of N entities when N becomes large. Typically $N \sim 10^{23}$

Because N is large, stat mechs uses notions of probabilities

Applications; (non exhaustive list)

1) – Thermodynamics of gas and fluids

2) – Solid state physics
   - vibrat° of a crystal
   - Electronic properties
   - Magnetic properties
   - etc....

3) Thermodynamics of radiation (gas of photons)
4) Astrophysics (white dwarfs, neutron stars, ...)
5) soft matter (polymers, foams, liquid crystals, etc... )
6) Plasma physics
7) Bio-physics ⇒ how thousand of birds fly together

BUT ALSO

- Networks (neural networks, web, etc...)
- Social networks, epidemy propagation, voters
- Finance : markets modeling
- ... Basically, it applies everywhere <u>when N is large!</u>

<u>How to position Stat mechs with respect to other sciences?</u>

Fathers: Clausius, Maxwell, <u>Boltzmann</u> (2nd half of 19 century)

Boltzmann $S = k_B \ln \Omega$

↗ entropy ↖ statistical quantity

$k_B$ : Boltzmann constant → a new fondamental constant?

$k_B \simeq 1{,}380649 \; 10^{-23} \; J K^{-1}$ ↳ we will see

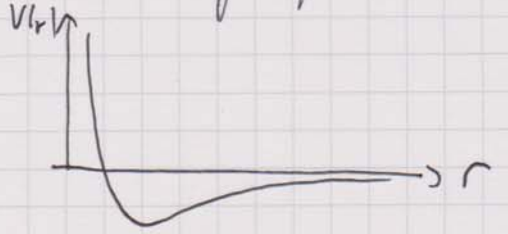
other important contributors: Gibbs, Einstein, ...

## II) Existence of a microscopic scale

In our everyday life, we live with macroscopic objects
However, we __know__ that we are made of atoms (not the case in 1870)

Feynman most important sentence:
"Everything is made of atoms, small particles moving, attracting within eachother at intermediate distance and repulsing at short distance"

=> A sentence to be left for humanity if everybody dies




Put N particles in such potential.
=> lot of physics

However, we learnt about atoms about early 20$^s$ century.

__2 scales:__

| MICRO | | MACRO |
|---|---|---|
| • Discrete | ⟷ | • Continuous |
| • small nb of laws and postulates | ⟷ | • Complexity, diversity ≠ states of matter |
| • Random processes | ⟷ | • Deterministic |
| • Reversible | ⟷ | • Irreversible |

How could it be that different?

Ph Anderson: "MORE IS DIFFERENT"

Stat mech: A fundamental theory with a particular status
≠ reductionism approach

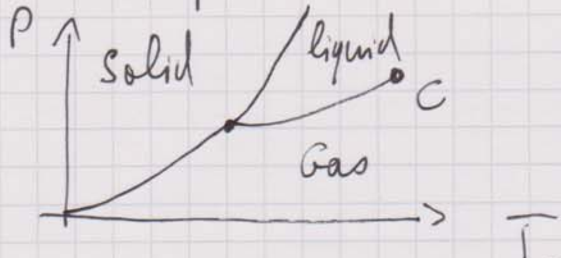
# III) Emergence & universality

Ex 1 : consider N particles in a VdW potential

$$U(r) = U_0 \left( \left(\frac{r_0}{r}\right)^6 - \left(\frac{r_0}{r}\right)^{12} \right)$$

Same atoms, same laws of mechanics

However ≠ macroscopic behavior




This tells us that the physics at the macroscopic scale is INDEPENDENT of the physics at the microscopic scale!

⇒ scale decoupling

≠ mythm of reductionist approach

⇒ the theory of strings & quarcks will not tell you the behaviour of interacting molecules

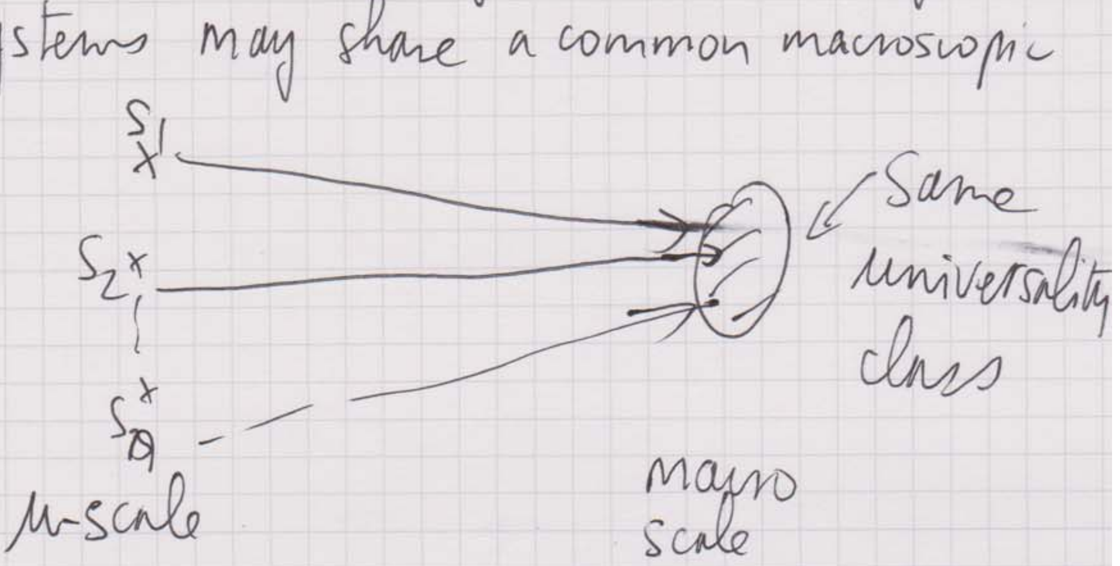
⇒ Fondamental aspect of stat mechs is encoded into the path from micro to macro

Emergence of collective behaviour at the macro scale difficult to predict at the micro-scale

⇒ sometimes behaviour at the macro more universal and simpler ⇒ beautiful

Ex 2 Critical properties of liquid-gas transito are similar to the para-ferromagnet transition

Indeed due to scale decoupling, two drastically different systems may share a common macroscopic behaviour.




**Ex 3: Central limit theorem**

We consider $N$ random variables $X_1, \ldots, X_N$. Assume

i) They are statiscally independent

ii) They all follow the same distribution function $p(x)$

iii) We only assume $\langle X_n \rangle$ and $\langle X_n^2 \rangle$ are finite.

Question: What is the distribution of $S = \sum_{i=1}^{N} X_i$ ?

$$P_N(S) \underset{N \gg 1}{\simeq} \frac{1}{\sqrt{2\pi N \operatorname{var} X}} \exp\left\{ - \frac{(S - N\langle X \rangle)^2}{2N \operatorname{var} X} \right\}$$

In the large N limit, S tends towards a Gaussian distribut° of mean $\langle S \rangle = N \langle X \rangle$ and standard deviat° $\sigma_S = \sqrt{N}\,\sigma_X$

$\Rightarrow$ Independent of $p(x)$!

$\Rightarrow$ Emergence of a new universal law in the large N limit independent of microscopic details