Statistical Physics

Pascal Simon
Laboratoire de Physique des Solides

pascal.simon@universite-paris-saclay.fr

Where to find information

Directly on the web page of Christophe Texier (in French)
http://lptms.u-psud.fr/christophe_texier/
puis onglet « Enseignement ! Licence ! L3 - Physique statistique »

You will find:

* some summary of the lectures (french)
- The exercices for the tutorials
- The previous problems for exams

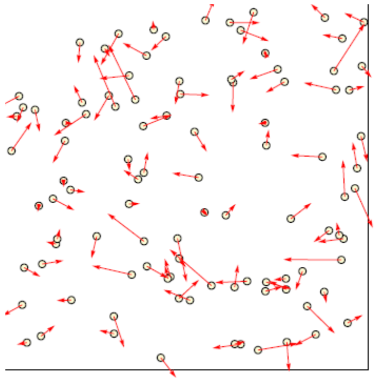
Plan

- Introduction : what is statistical physics and its perimeter
- Ergodicity
- Isolated systems: fundamental postulate and microcanonical ensemble
- Non isolated systems (1): relax constraints between sub-systems
- Non isolated systems (2): Canonical systems
- Application 1 : semiclassical theory of gases
- Application 2 : Thermodynamics of hamonic oscillators
- Non isolated systems (3): Grand-canonical ensemble
- Application 3 : Ising model and paramagnetic-ferromagnetic transition

Use of statistical physics

Describe complex systems with a large number degrees of freedom

A gas of $N \sim 10^{23}$ atoms



Classical description:
N positions & N momenta:
➔ 6N degrees of freedom

Gas, liquid, solid

Magnetism

Etc....

(the limit is your imagination....)

Physics of complexity

Noble prize in Physics 2021

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2021
"for groundbreaking contributions to our understanding of complex physical systems"
with one half jointly to
Syukuro Manabe
Princeton University, USA
Klaus Hasselmann
Max Planck Institute for Meteorology, Hamburg, Germany
"for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming"

and the other half to
Giorgio Parisi
Sapienza University of Rome, Italy
"for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales"

Plan of this chapter

1. Introduction : what is statistical physics ?

2. Existence of a microscopic scale: from micro to macroscales

3. Emergence and universality

Plan of this chapter

1. **Introduction : what is statistical physics ?**
2. Existence of a microscopic scale: from micro to macroscales
3. Emergence and universality

What is statistical physics ?

One of the most important theory in physics with a huge range of applications.

GOAL: stat phys allows to infer using **probabilities** the properties of a system with a large number of constituents from the **existing laws** governing its constituents.

In short, stat mechs allows you to go from microscopic to the macroscopic world
i.e. from the building blocks to the whole when N is large typically ~ 10^{23}

Stat. Phys **uses the existing laws** such as mechanics, relativity, quantum mechanics, electromagnetism, etc.

Applications

1. Thermodynamics of gas and fluids

2. Solid state physics (vibrations of a crystal, magnetic properties, electronic properties, et.)

3. Thermodynamics of radiation (gas of photons)

4. Astrophysics&cosmology (white dwarfs, neutron stars, cosmic radiations, black hole physics, etc.)

5. Sof matter (polymers, foams, liquid crystals, etc.)

6. Bio-physics (interacting cells, flock of birds fishes…)

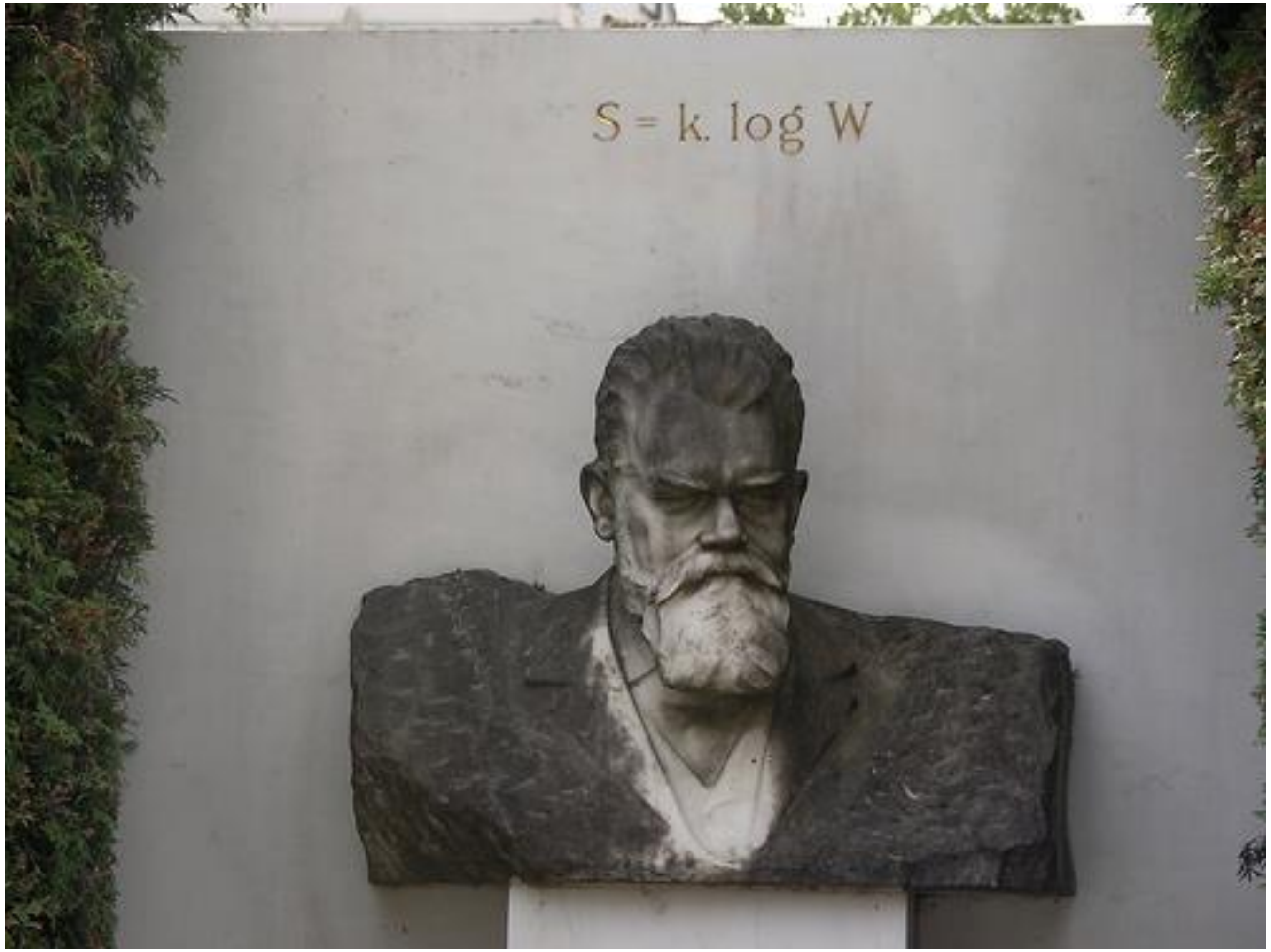
7. Plasma physics

8. Networks :neural networks, web, social networks, voters, epidemy

9. Finance: econophysics &market modeling

A bit of History

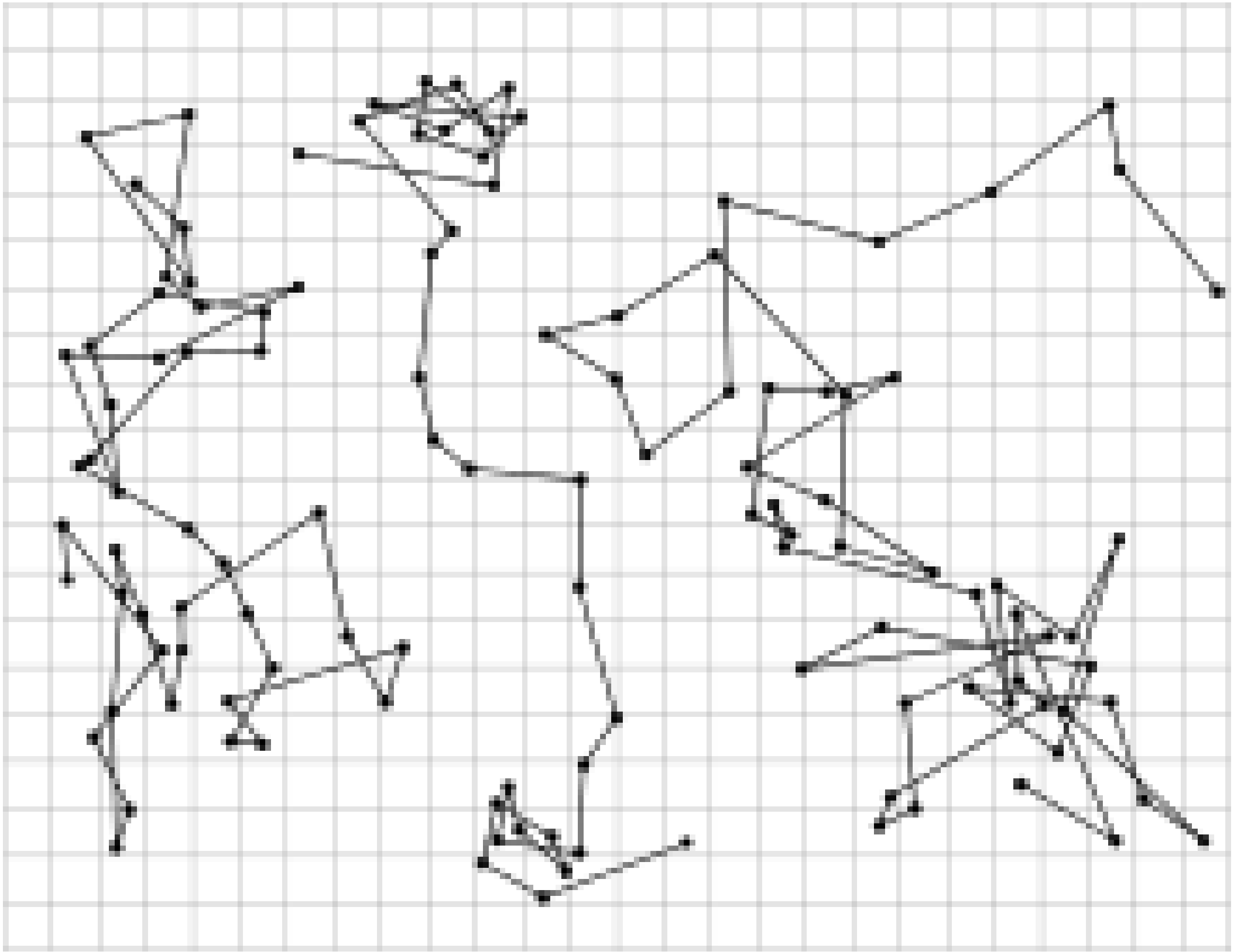
1727 : Bernouilli proposed a kinetic interpretation of pression du to molecules
1820 : The kinetic theory of gases : Herapath and Joule (1821).
1847 : 1st principle of thermodynamics: equivalence between work and heat : J. Joule.
1854 : 2nd principe of thermodynamics: Clausius shows that $\oint dQ/T \geq 0$ for a cycle. Notion of entropy.
1857 : Clausius recovers the law of perfect gas using the concepts of partical in motion
1860 : Maxwell distribution of velocities (kinetic theory of gases)
1866 : Influenced by Maxwell, Boltzmann understands that statistical concepts have to be used.
1867 : Maxwell deamon
1868-1871 : Boltzmann extends the kinetic theory of gases and proposed the equipartition theorem
1872 : Boltzmann famous equation:


S = k. log W

History

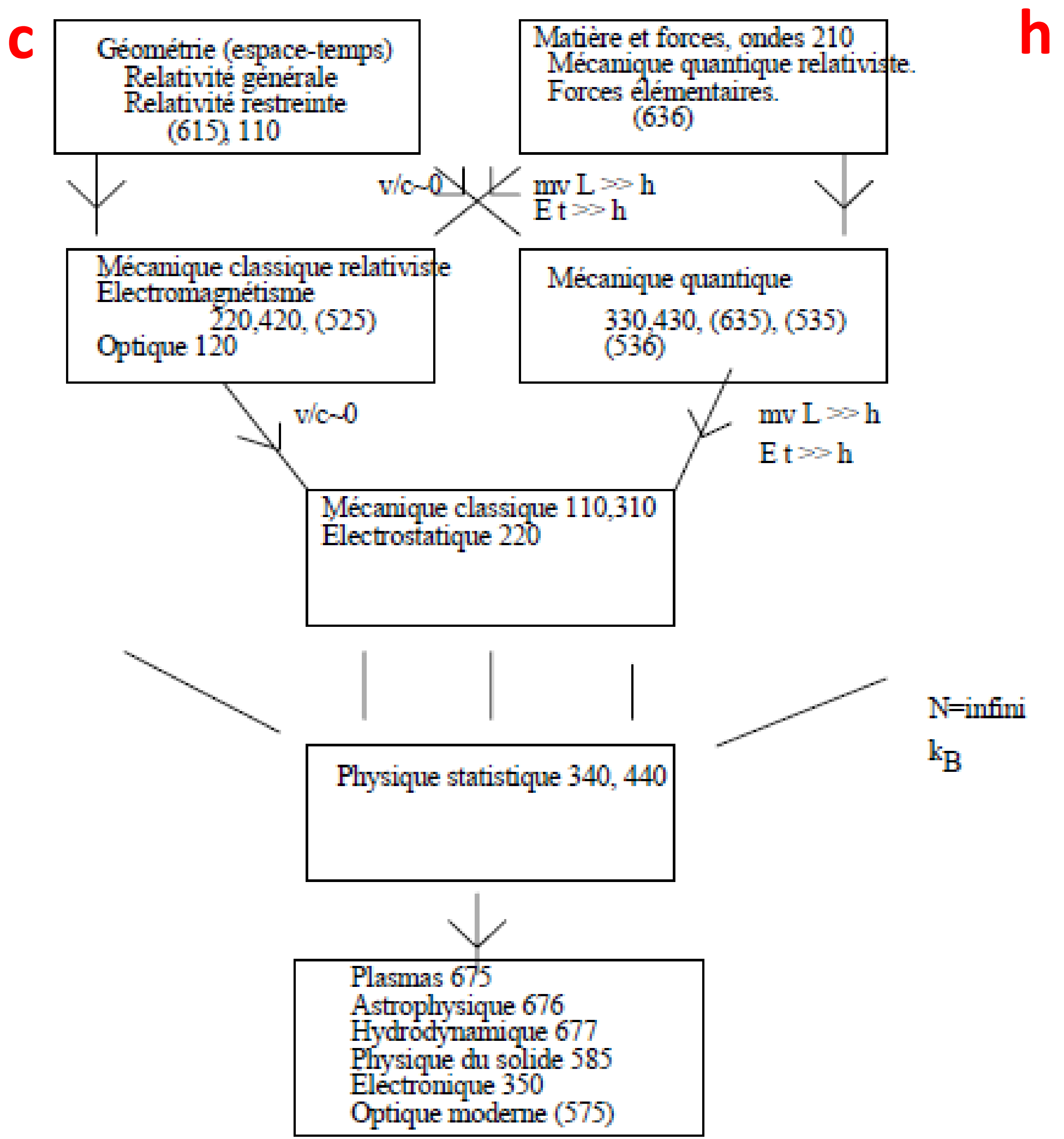
1872 : Boltzmann H theorem (extends notion of entropy out-of-equilibrium). The idea of irreversibility is introduced on solid grounds!
1874-1876 : Paradox of irreversibility (Thomson & Loschmidt) : how reversible equations can give rise to irreversible macroscopic behaviours ?
1875 : Gibbs Paradox about undistinguishability
1900 : Planck laws about thermodynamica of radiation: major breakthrough which will give birth to quantum mechanics.
1907 : Einstein developped the first theory of specific heat in solids
1908: Perrin's experiment proving the existence of atoms by brownian motion
1912 : Sackur (1880-1914) and Tetrode (1895-1931) proposed (independently) the entropy of a monoatomic gas
1924 : Bose-Einstein statistics
1926 : Using Pauli's ideas, Fermi and Dirac proposed (independently) the Fermi-Dirac statistics
1937 : Phenomenological theory of second order phase transitions by Landau.
- theory of Superfluidity by Landau (Nobel in 1962)
1939 : spin-statistics theorem (Pauli, Fierz et Belinfante)
1944 : Exact solution of the 2D Ising model by Lars Onsager (Nobel in chemistry 1968)
1948 : Information theory by Shannon & Weaver
Years 70s: Critical phenomena and universality of phase transitions (Wilson…)
Years 80s: new tools to describe disordered systems (Parisi….)
……..

Brownian motion

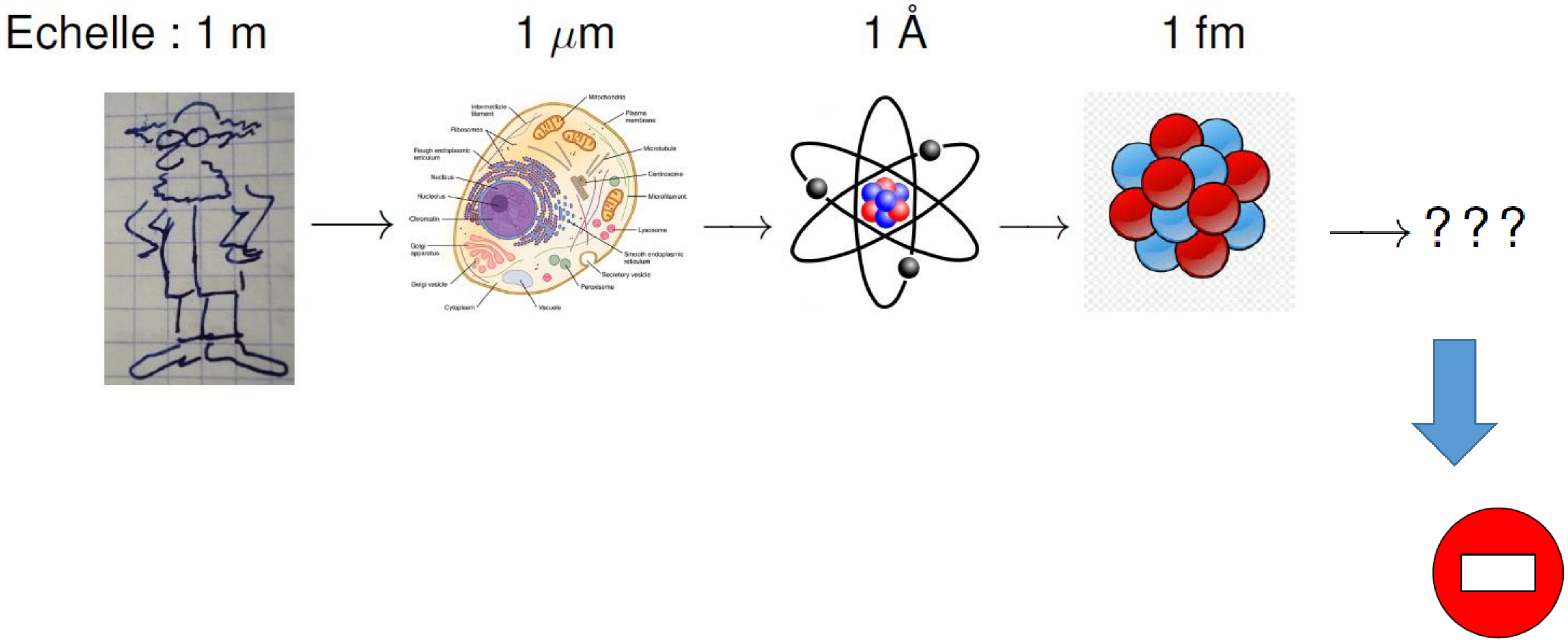


Reproduced from the book of Jean Baptiste Perrin, Les Atomes, three tracings of the motion of colloidal particles of radius 0.53 µm, as seen under the microscope, are displayed. Successive positions every 30 seconds are joined by straight line segments (the mesh size is 3.2 µm).[3]

How to position statistical mechanics ?



A word about reductionism

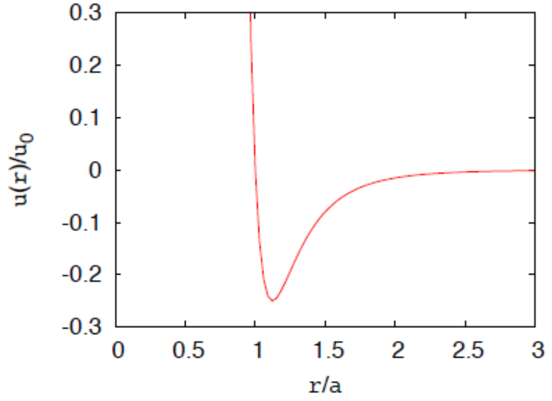


Statistical physics completely explodes the myth of reductionism: describes physics by its ultimate smallest (most elementary) constituents does not help to understand the macroscopic behaviour

Does not answer the question: how systems organize at the macroscopic behaviour!

Example: the classical fluid

Microscopic scale:



Van der Waals 2-body interaction

+ Newton laws

Macroscopic scale:

Different states of Matter despite the equations are the same At the micro level!

Much richer behaviour at the macro scale and the explanation is not uniquely in the elementary level

Elementary

$\neq$

Fundamental

Plan of this chapter

1. Introduction : what is statistical physics ?

2. **Existence of a microscopic scale: from micro to macroscales**

3. Emergence and universality

Existence of a microscopic scale

In our everyday life, we live with macroscopic objects. However, we know that we are made of atoms.

Feynman's most important sentence legacy:

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words?
I believe it is the atomic hypothesis that *all things are made of atoms — little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.*

In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

Micro vs macro

- Discrete
- Small nb of laws and postulates
- Random processes
- Reversible

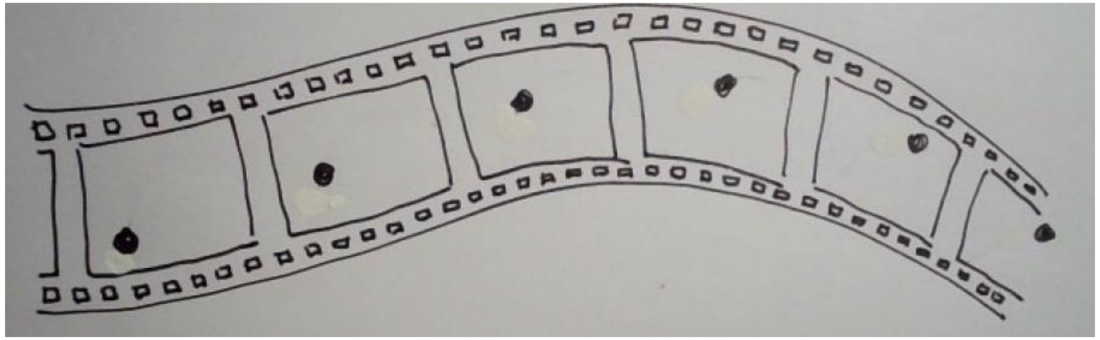
- Continuous
- Complexity, diversity, different states of matter
- Fully deterministic
- Irreversible

How could it be that different ?

P. W. Anderson: ``More is different''

Newton mechanics: reversibility of laws

$$t \to -t$$

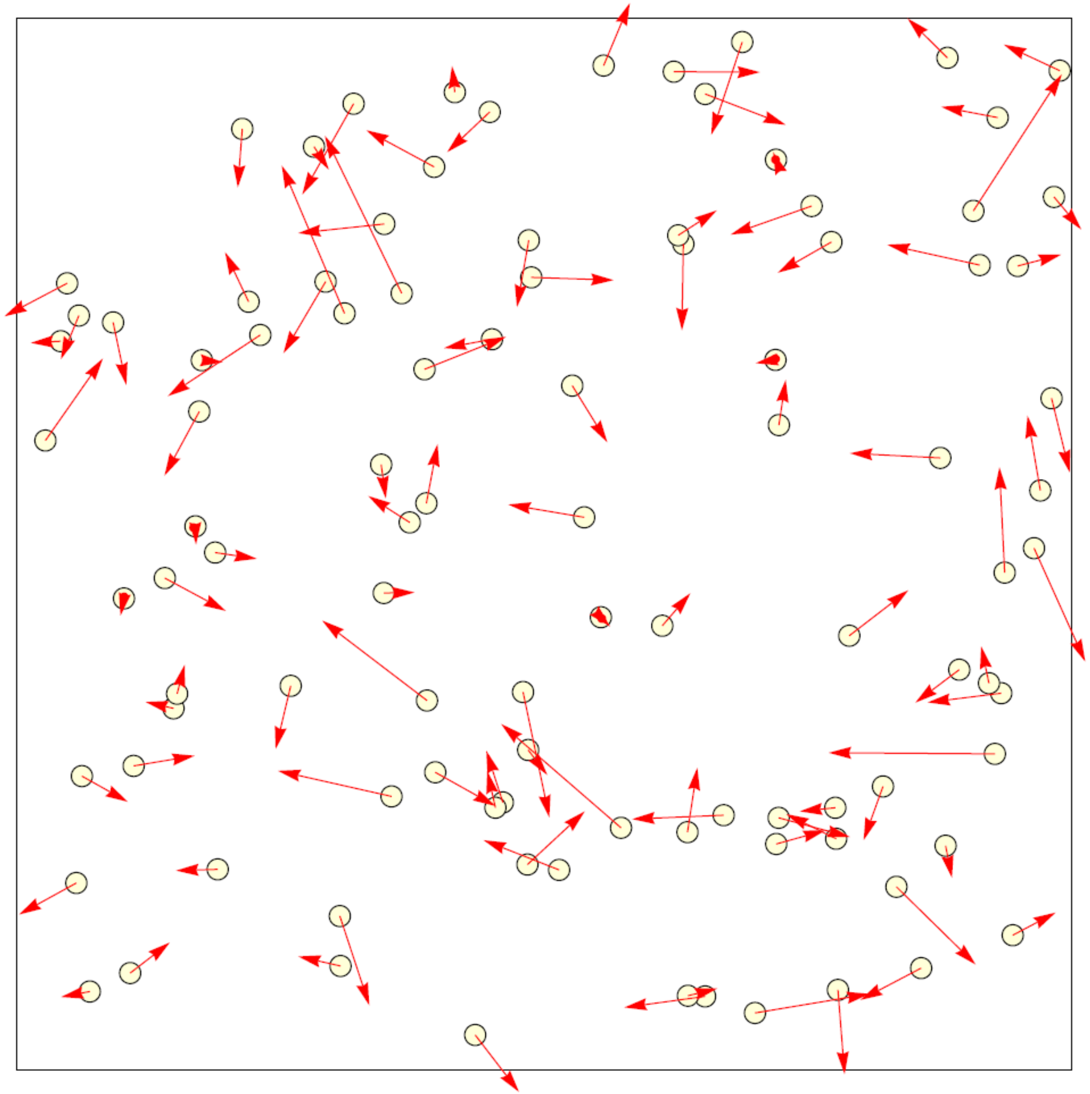


At the macroscopic scale: irreversibility prevails

$$t \to -t$$

GOAL: stat phys allows to infer using the tools of **probabilities** the properties of a **macroscopic** system (i.e. with a large number of constituents) from the **existing laws** which govern governing its constituents at the **microscopic** scale

Gas of atoms

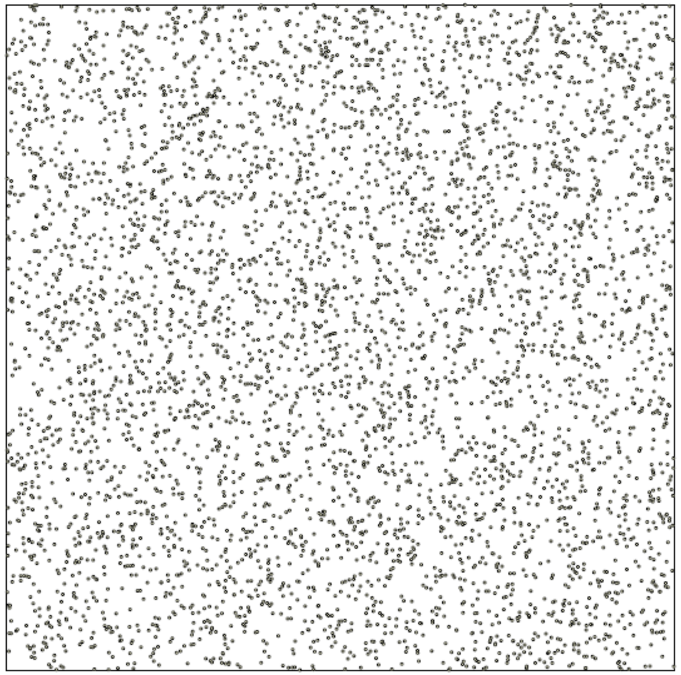


$N = 100$

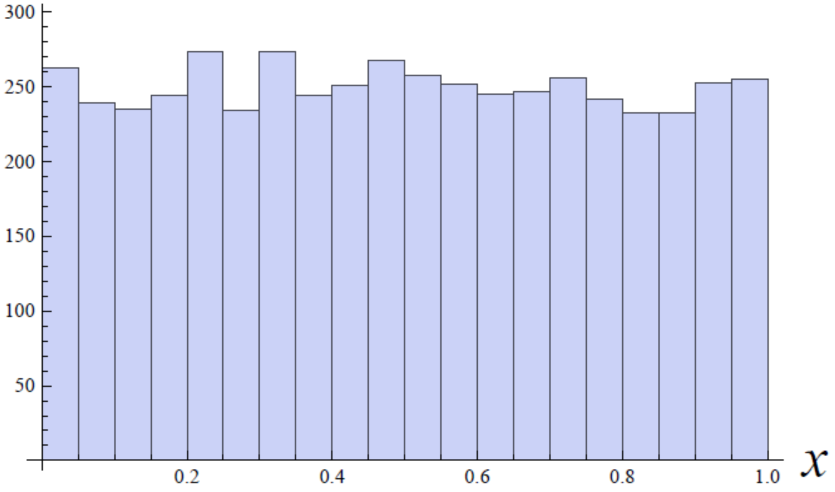
Microscopic informations

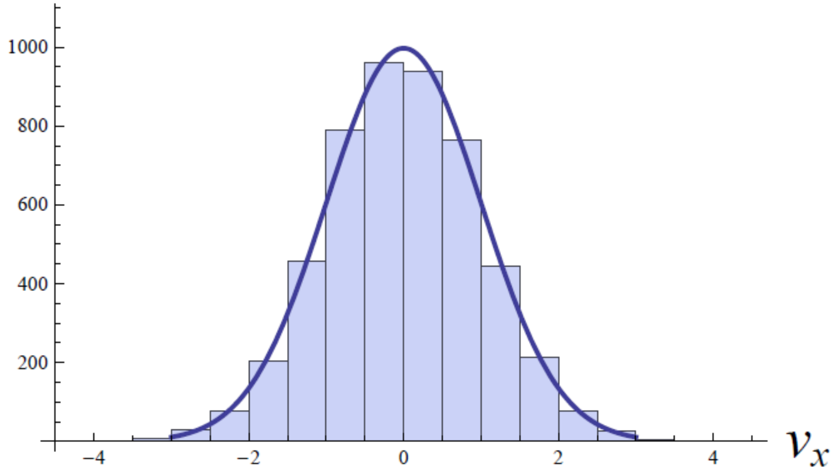
x	y	v_x	v_y
0.3481339673503039	0.8044479514019125	-1.6296637340939009	-0.17330986044336907
0.943959404658679	0.12813193244499477	0.812806796438423	0.4300848998593466
0.17276198311576696	0.6261067852573887	-0.6582154645152644	-1.2174608385147105
0.46055124775184053	0.25442041272746563	-0.3136164735855665	0.9805970397833481
0.14006225144553186	0.4677371375069985	1.1558487545104825	-0.6238148235750349
0.8336582975597342	0.17417228232681237	1.501374679233153	-1.4775425142866758
0.23061053136019805	0.698905388433749	-1.749994817332023	-1.161582192751426 3
0.28152597239575927	0.8779305189530922	0.2799065195854414	-0.43495647484314803
0.3880555392247187	0.5388695376531125	0.2113665245174106	1.1238928271991324
0.14832414880233502	0.34385509840020334	-0.5277070498948921	0.1209178424866921
0.631693817285746	0.7853888857849345	-0.027342016693069866	-1.6394823744925753
0.3190611252791815	0.9188073776746031	-0.9714339552399226	-1.724040289571618
0.8932578286677433	0.38063091951769534	-0.8304478190721315	-0.6262481498578175
0.10135076366400786	0.4483960566658738	-0.06550359758787075	-0.32446623404654185
0.35500080543687873	0.260118513169014	-2.151790560949367	0.48629349837750263
0.43434361533645216	0.03327026946260059	0.8285427426639329	-1.0806565165683202
0.345202946479975 5	0.575631192655435 6	0.08811995940081274	-0.6401668087319606
0.2194985114865977	0.7304919014957585	-0.4441203934946385	0.9397507457922605
0.5263677711231913	0.6507220113082153	0.6631350569283433	-1.0754642145451174
0.7533116033395919	0.8215267499071246	-1.533178003185469	-0.5415114583147917
0.8209626024671028	0.2695908503497251	-0.3667364317997753	-1.0200883928271705
0.9363113893569659	0.4729287651212255	-0.0996402786924323	2.0030823254737427
0.7190262322540573	0.675189660956744	-0.3731323756784452	-0.061619276784647035
0.30667318823449286	0.09886265068786715	-0.8003770847863317	0.18159697511938094
0.7889985590436193	0.41293029635677314	1.1180665545444195	0.8636770689801199
0.3886336046839143	0.3513149821106487	-1.9844626370138359	1.5183695574966924
0.9641468998506606	0.4961831470138043	0.9689175990486405	-2.153546125089086
0.17519735461277697	0.674871435792221	0.3957099296830754	-0.0371161780282621 3
0.3954130010879797	0.8587480142599067	-1.2838772712715474	0.695567687130941
0.9885170153801843	0.9502569355989523	-1.059074848106663	0.49485599883142606
0.19023891689654704	0.1907588855761544	-0.7885344280035872	-0.023389462177139966
0.11053976986827152	0.28912558974990143	1.1557251848827483	0.19367841599587346
0.04143574189148591	0.6929788771263379	-0.5676269678772149	-0.030961911748494567
0.1691189347327009	0.516941813701161	0.010840189334410156	-0.3457376241828511 4
⋮	⋮	⋮	⋮

Statistical informations



$N = 5\,000$





Thermodynamical versus microscopic informations

Microscopic:

$\sim N \log_{10} N$ **with** $N \sim 10^{23}$

Thermodynamics: Only a few variables

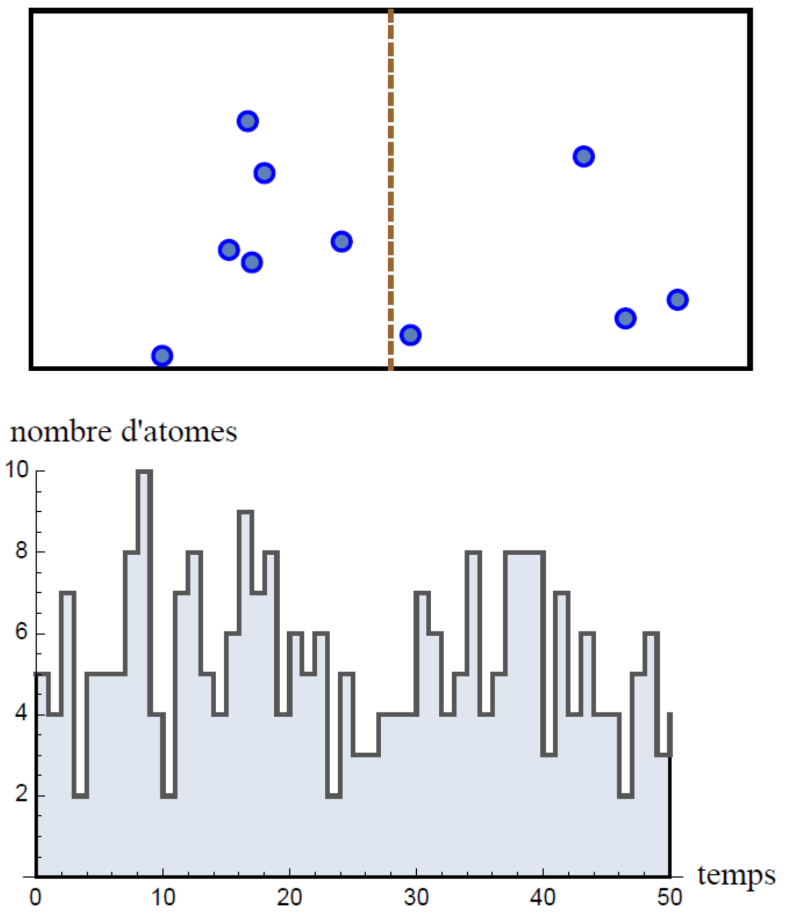
Density: n=N/V

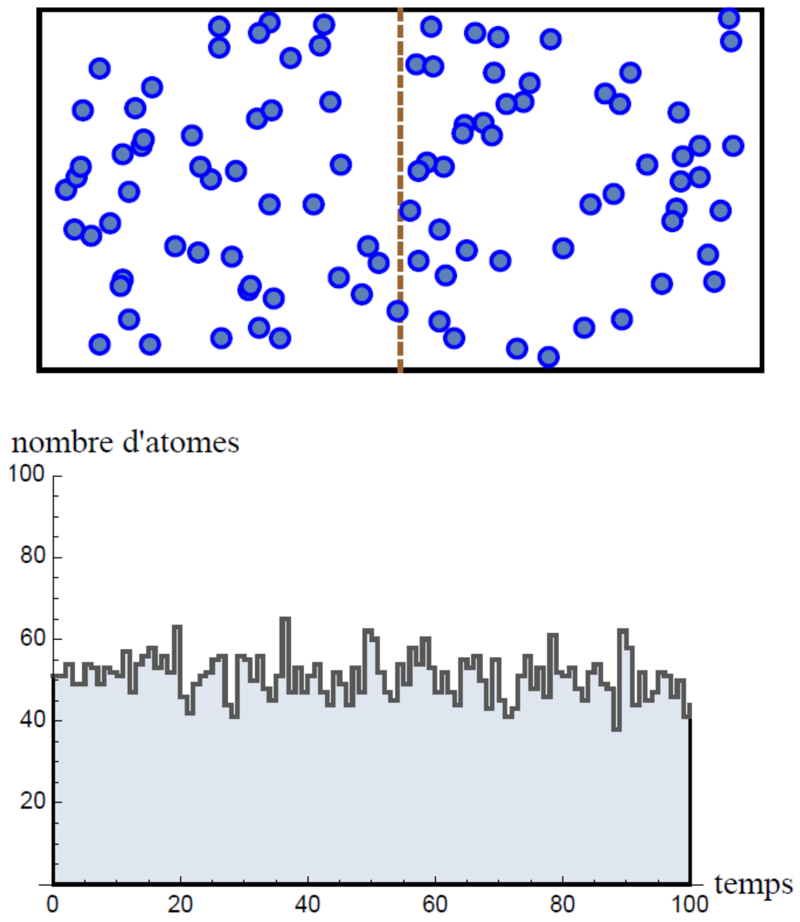
pressure: P

Temperature: T

How to go from N~10^{23} to only a few variables ?

At macroscopic equilibrium: $N_{left}(t)$





The **relative** fluctuations of $N_{left}(t)$ go as $1/\sqrt{N(t)}$ $\longrightarrow$ **0 as N $\longrightarrow$ ∞**

➡ Fluctuations become negligible

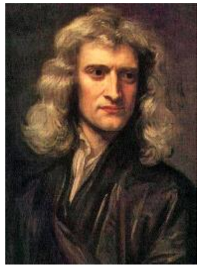
Elementary vs collective

Microscopic → Macroscopic
Elementary → Collective

Classical or quantum mechanics are obviously correct, however do not provide **the ``right language''** to analyze and understand collective phenomena involving large $N \sim 10^{23}$ degrees of freedom.

The ``right language'' of statistical mechanics necessarily involves concept of probabilities

Elementary dynamics	+	Statistical physics	=	Macroscopic description



About thermodynamics ?

Relation between thermodynamics and statistical physics :

Thermodynamics:

This is an axiomatic theory, which is purely phenomenological (i.e. based on observation at the macroscopic scale) allows to describe macroscopic states.

Statistical physics:

This is a theory which allows to build «**microscopic models** » based on the description at the elementary scale

Statistical physics is far more ambitious as it is much more general (not only gases!), it has a predictive power and is actually able to found thermodynamics !

Plan of this chapter

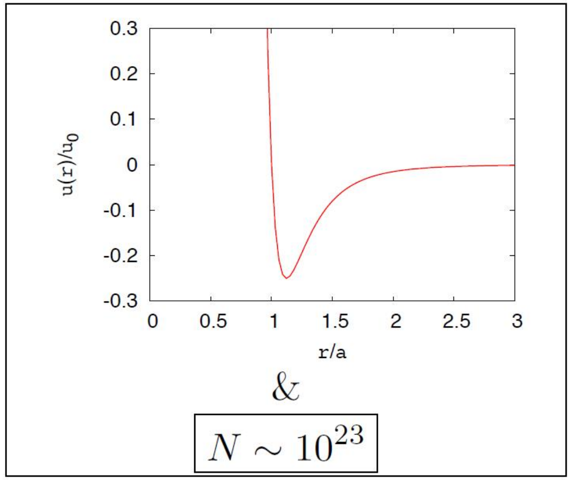
1. Introduction : what is statistical physics ?

2. Existence of a microscopic scale: from micro to macroscales

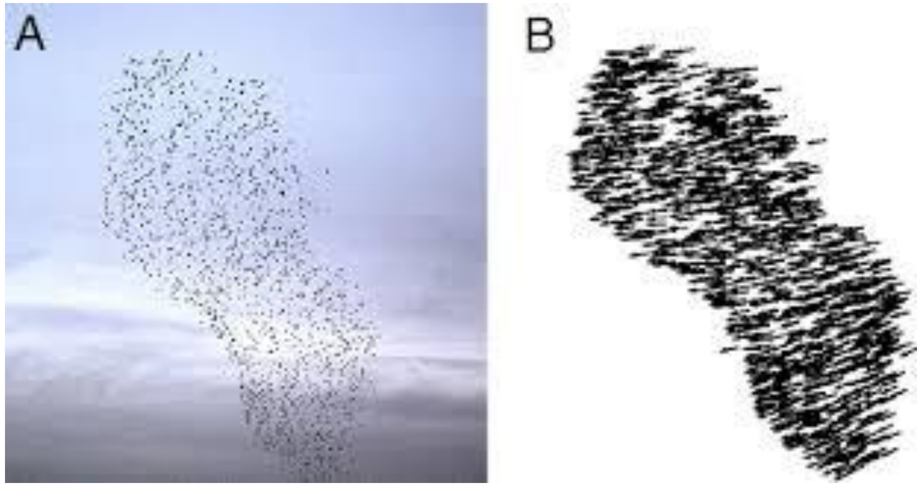
3. **Emergence and universality**

We have seen that the reductionist approach (top-to-bottom) is fully useless to describe a macroscopic set of entitites.

Statistical physics is instead a bottom-up approach with the idea that the macroscopic behavior emerges.



Emergence of a collective behaviour in a flock of birds



Emergence & universality

At the macroscale, there are emergence of a plethora of new (sometimes exotic) collective phenomena which are totally unpredictable with the sole understanding of the dynamics at the microscale.

Examples: superconductivity, fractionalization of charge, new exotic quasi-particles with non-Abelian character (such as Majorana quasi-particles), etc.

Macroscopic collective phenomena are usually almost insensitive to microscopic details (therefore nature becomes somehow much simpler and nicer!)

➡ **UNIVERSALITY**

Each scale demands its own level of analysis: there is a kind of **scale decoupling**

Examples: In particle physics, asymptotic freedom of QCD, magnetic impurity in a metal, etc.

P. W. Anderson, « More is different », Science 177 (1972)
(Philipp W. Anderson (1923-2020), prix Nobel 1977)