# **TD4** : Version Space Learning

#### Exercise 1:

### Supervised Learning Problem

We need to classify a mushroom based on two attributes:

- $A_1$ : Cap shape (binary):
  - 0: Convex.
  - 1: Flat.
- $A_2$ : Mushroom smell (ternary):
  - 0: No smell.
  - 1: Flour smell.
  - 2: Moldy smell.

We have the following training set:

$A_1$	$A_2$	Class
1	0	Edible
0	2	Poisonous
1	1	Poisonous
0	1	Edible
1	2	Poisonous

#### List of Possible Hypotheses

A hypothesis h is a pair  $(h_1, h_2)$ , where:

- $h_1 \in \{0, 1, ?\}$  defines the cap shape.
- $h_2 \in \{0, 1, 2, ?\}$  defines the smell.

The hypothesis (?, ?) represents any possible value, while  $(\emptyset, \emptyset)$  is the most specific hypothesis, which covers nothing.

#### Initialization of Version Space

**Question 1** • Give the hypothesis  $S_0$  (the most specific hypothesis) and  $G_0$  (the most general hypothesis).

# Comparison of Hypotheses

**Question 2** • Compare the hypotheses (1, ?), (?, 0), and (?, ?). Are they comparable according to the generalization-specialization relation?

**Question 3** • Identify two hypotheses that are incomparable to each other.

# **Building the Lattice**

**Question 4** • Build the hypothesis lattice by representing the relationships between  $S_0$ ,  $G_0$ , and the intermediate hypotheses. Include the empty set  $\perp$  in the structure.

# Updating Hypotheses

**Question 5** • From the example (1, 0), class = Edible, what hypothesis do you obtain by generalizing  $S_0$ ?

- **Question 6** After receiving a positive example (0, 1), how should you adjust S?
- **Question 7** After receiving a negative example (1, 1), how should you adjust G?

## Version Space Algorithm Process

**Question 8** • Apply the algorithm by processing the examples one by one. Update S and G after each example.

## **Empty Lattice**

**Question 9** • When does the lattice become empty  $(\perp)$  and what does that mean?

**Question 10** • Add two additional examples that lead to the collapse of the lattice  $(\perp)$ . Explain why this happens and its implications for learning.