TD4

Ex. 1

* bactérie taille $1\,\mu m$ vitesse $30\,\mu m\,s^{-1}$ $\quad Re = \dfrac{\rho U L}{\eta} = \dfrac{10^3 \times 3\,10^{-5} \times 10^{-6}}{10^{-3}} = 3\,10^{-5} \ll 1$ $\Rightarrow$ laminaire

* H qui marche vit $\sim 1\,m\,s^{-1}$ $\quad Re = \dfrac{1 \times 1 \times 1}{10^{-5}} = 10^5$ $\Rightarrow$ turbulent

* nageur vit $\sim 1\,m\,s^{-1}$ $\quad Re = \dfrac{10^3 \times 1 \times 1}{10^{-3}} = 10^6 \gg 1$

* voiture à $130\,km/h = 36\,m/s$ $\quad Re = \dfrac{1 \times 36 \times 1}{10^{-5}} \simeq 4\,10^6 \gg 1$

* chute libre d'1 H F. Baumgartner depuis 1 altitude de 36000 m (2012)

  $v \simeq c = 300\,m/s$ $\quad Re = \dfrac{1 \times 3\,10^2 \times 1}{10^{-5}} = 3\,10^7 \Rightarrow F \not\propto v$

* Airbus $v \lesssim c$ $\quad Re = \dfrac{1 \times 3\,10^2 \times 10}{10^{-5}} = 3\,10^8$

1 seul de ces écoulements est laminaire !

Ex. 2

a) $\vec{u_z}$ $\downarrow$ $\quad \vec{g} = g\vec{u_z}$

z

force de Stokes : on suppose l'écoulement laminaire → à vérifier par la suite avec le nb de Reynolds

$$m\frac{d\vec{v}}{dt} = m\vec{g} - \rho_L V \vec{g} - \underbrace{6\pi\eta R\vec{v}}$$

$$\rho_S V \frac{dv_z}{dt} = \rho_S V g - \rho_L V g - 6\pi\eta R v_z$$

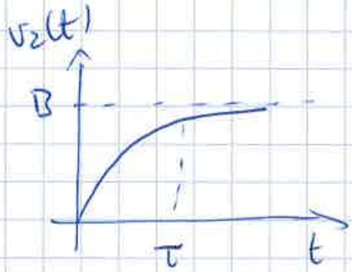
$$\frac{dv_z}{dt} + \frac{6\pi\eta R}{\rho_S V} v_z = \frac{\rho_S - \rho_L}{\rho_S} g \qquad V = \frac{4}{3}\pi R^3 \qquad \frac{6\pi\eta R}{\rho_S V} = \frac{6\pi\eta R}{\rho_S \frac{4}{3}\pi R^3} = \frac{9}{2}\frac{\eta}{\rho_S R^2}$$

$$\frac{dv_z}{dt} + \frac{9}{2}\frac{\eta}{\rho_S R^2} v_z = \frac{\rho_S - \rho_L}{\rho_S} g$$

b) $$v_z = A e^{-\frac{9}{2}\frac{\eta}{\rho_S R^2} t} + \underbrace{\frac{2\rho_S R^2}{9\eta}\,\frac{\rho_S - \rho_L}{\rho_S}\, g}_{B}$$

$$v_z(0) = 0 = A + B \Rightarrow A = -B$$

$$v_z(t) = \frac{2R^2}{9\eta}(\rho_S - \rho_L) g \left(1 - e^{-\underbrace{\frac{9}{2}\frac{\eta}{\rho_S R^2}}_{\frac{1}{\tau}} t}\right)$$



c) $\tau = \dfrac{2\rho_s R^2}{9\eta}$

η en Pa s $= \dfrac{kg\,m\,s^{-2}}{m^2}\,s = kg\,m^{-1}\,s^{-1}$

$[\tau] = \dfrac{kg\,m^{-3}\,m^2}{kg\,m^{-1}\,s^{-1}} = s$ OK

$v(t) \underset{t\to\infty}{\rightarrow} v_{lim} = \dfrac{2}{9}\dfrac{R^2(\rho_s-\rho_L)g}{\eta}$ $\quad \dfrac{m^2\,kg\,m^{-3}\,m\,s^{-2}}{kg\,m^{-1}\,s^{-1}} = m\,s^{-1}$ OK

d) * $\underline{R = 1\,cm}$

$\tau = \dfrac{2}{9}\dfrac{7{,}85\,10^3\,10^{-4}}{1} = 0{,}174\ s$

$v_{lim} = \dfrac{2}{9}\dfrac{10^{-4}(7{,}85\,10^3 - 10^3) \times 9{,}8}{1} = 14{,}9\ 10^{-1} = 1{,}49\ m\,s^{-1}$

$Re = \dfrac{v_{lim}\,R\,\rho_L}{\eta} = \dfrac{1{,}5 \times 10^{-2} \times 10^3}{1} = 15$ → force de Stokes pas valable

* $\underline{R = 1\,mm}$

~~$\tau = 0{,}0$~~ $\tau = 1{,}74\ 10^{-3}\ s = 1{,}74\ ms$

$v_{lim} = 1{,}49\ cm\,s^{-1}$

$Re = \dfrac{1{,}5\ 10^{-2} \times 10^{-3} \times 10^3}{1} = 1{,}5\ 10^{-2} \ll 1$ OK

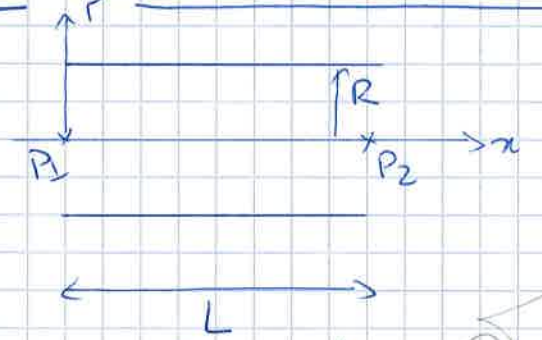
Rem ds de l'eau pr la bille de 1 mm de rayon

$\tau = \dfrac{2}{9}\dfrac{7{,}85\,10^3\,10^{-6}}{10^{-3}} = 1{,}74\ s$

$v_{lim} = \dfrac{2}{9}\dfrac{10^{-6}\ 6{,}85\,10^3\ 9{,}8}{10^{-3}} = 1{,}49 \times 10 = 14{,}9\ m\,s^{-1}$!

$Re = \dfrac{1{,}5 \times 10 \times 10^{-3} \times 10^3}{10^{-3}} = 1{,}5 \times 10^4 \gg 1$! → viscosimètre pas du tout adapté pour l'eau

Ex. ~~1~~ 2 Écoulement de Poiseuille dans un tuyau



à vérifier a ~~posteriori~~ priori !

NS $\rho\left[\cancel{\frac{\partial \vec{v}}{\partial t}} + \cancel{(\vec{v}\cdot\vec{\nabla})\vec{v}}\right] = \cancel{\rho\vec{g}} - \vec{\nabla}P + \eta\Delta\vec{v}$

stat. laminaire neg

$\cancel{\frac{\partial P}{\partial y}} \approx \frac{\|\rho g\|}{\|\nabla P\|} \approx \frac{\rho g}{P/D}$

tuyau arrosage $R = 1\,cm$

$\frac{10^4 \; 2\,10^{-2}}{10^5} \approx 2\,10^{-3} \ll 1$ OK

$$\vec{\nabla}P = \eta\Delta\vec{v}$$

$\vec{v} = v_x(r)\,\vec{u_x}$

$\Delta\vec{v} = \Delta v_x(r)\,\vec{u}_x = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right)\vec{u}_x$

sur $\vec{u}_r$ $\quad \frac{\partial P}{\partial r} = 0 \Rightarrow P(x)$

$\vec{u}_x$ $\quad \frac{\partial P}{\partial x} = \eta\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x(r)}{\partial r}\right) = f(r) \Rightarrow P = \cancel{f(r)}\,x + cste$

$= cste(/x)$

$\cancel{\frac{\partial}{\partial r}\left(\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial P}{\partial r}\right) = 0 \Rightarrow \frac{\partial P}{\partial x}(\cancel{r})}$

$\frac{\partial P}{\partial x} = \frac{P_2 - P_1}{L} = -\frac{\Delta P}{L}$

~~⇒ + sple : gravité nég~~
~~dc P(x) mais P(X)~~

d'où $\frac{\partial P}{\partial x} = \frac{P_2 - P_1}{L} = -\frac{P_1 - P_2}{L} = -\frac{\Delta P}{L}$

$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right) = -\frac{1}{\eta}\frac{\Delta P}{L}$

$\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right) = -\frac{1}{\eta}\frac{\Delta P}{L}\,r$

$r\frac{\partial v_x}{\partial r} = -\frac{1}{2\eta}\frac{\Delta P}{L}r^2 + B$

$\frac{\partial v_x}{\partial r} = -\frac{1}{2\eta}\frac{\Delta P}{L}r + \frac{\cancel{B}^{\,0}}{r}$ sinon $\frac{\partial v_x}{\partial r} \xrightarrow[r\to 0]{} \infty$

$v_x = -\frac{1}{4\eta}\frac{\Delta P}{L}r^2 + C$

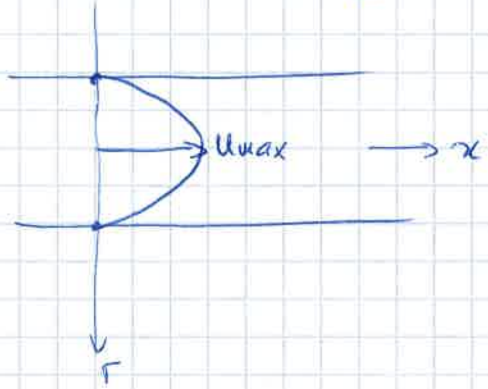
$v_x(R) = 0 = -\frac{1}{4\eta}\frac{\Delta P}{L}R^2 + C \qquad C = \frac{1}{4\eta}\frac{\Delta P}{L}R^2$

$$v_x(r) = \frac{1}{4\eta}\frac{\Delta P}{L}R^2\underbrace{\left(1 - \frac{r^2}{R^2}\right)}_{f(r)}$$

$$f'(r) = -\frac{2r}{R^2} = 0 \text{ en } r = 0$$
$$f''(r) = -\frac{2}{R^2} < 0$$
$\Big\}$ f max en $r = 0$

$$v_x(0) = U_{max} = \frac{1}{4\eta}\frac{\Delta P}{L}R^2$$

$$v_x(r) = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$



→ profil parabolique de Poiseuille ds 1 tuyau cylindrique

b) $$Q_v = \iint \vec{v}\, dS\vec{n} = \iint v_x(r)\vec{u}_x\, dS\vec{u}_x = \int v_x(r)\, 2\pi r\, dr \left(= \pi \times \overbrace{2\int r u(r)\, dr}^{R^2\bar{u}} \right)$$

$$= \int_0^R 2\pi \frac{1}{4\eta}\frac{\Delta P}{L}R^2\left(1 - \frac{r^2}{R^2}\right) r\, dr \qquad \bar{u} = \frac{2}{R^2}\int u(r)\, r\, dr$$

$$= \frac{\pi}{2}\frac{\Delta P R^2}{\eta L}\underbrace{\left[\frac{R^2}{2} - \frac{1}{R^2}\frac{R^4}{4}\right]}_{\frac{R^2}{4}}$$

$$Q_v = \frac{\pi R^4}{8\eta}\frac{\Delta P}{L} \quad \text{Poiseuille}$$

c) $Q_v = 5L\,mn^{-1}$ (remplissage d'1 arrosoir) $= \dfrac{5\,10^{-3}}{60} \simeq 10^{-4}\ m^3 s^{-1}$

$R = 1\,cm$

$\eta = 10^{-3}\ Pa\,s$

$L = 10\,m$

$$\Delta P = \frac{8\eta L}{\pi R^4} Q_v = \frac{8\,10^{-3} \times 10 \times 10^{-4}}{\pi\, 10^{-8}} = 2{,}7\ 10^2\ Pa \ll P_{atm}$$

→ chute de pression négligeable

$$U_{max} = \frac{1}{4\eta}\frac{\Delta P}{L}R^2 = \frac{2.7\,10^2 \times 10^{-4}}{4\,10^{-3} \times 10} = 0{,}7\ m s^{-1}$$ ou $U_{moy} = \dfrac{Q_v}{\pi R^2} = \dfrac{10^{-4}}{3\,10^{-4}} = 0{,}3\ m s^{-1}$

Re ? ~~$Q_v \simeq S \times U_{max}$ en OG~~ ~~$U_{max} = \dfrac{Q_v}{\pi R^2} = \dfrac{10^{-4}}{\pi\, 10^{-4}} \simeq 0{,}3\ m s^{-1}$~~

$$Re = \frac{U_{max} R \rho}{\eta} = \frac{0{,}7 \times 10^{-2} \times 10^3}{10^{-3}} = 7\,10^3 \geq 2\,10^3$$ Poiseuille ~~encore~~ pas applicable !

Ex. 2 bis

gaz = milieu environnant

$$m\frac{d\vec{v}}{dt} = m\vec{g} - \rho_g V\vec{g} - 6\pi\eta_g R\vec{v}$$

$$v_{lim} = \frac{2}{9}\frac{R^2(\rho_l - \cancel{\rho_g})\,g}{\eta_g} = \underset{\sim 10}{\frac{2}{9}}\,\frac{\overbrace{5^2}^{50}\,10^{-12}}{10^{-5}}\,(10^3 - \cancel{1})\times 10 \simeq \cancel{\frac{5\times 10^{-8}}{10^{-5}}} \sim 5\,10^{-3}\ m\,s^{-1}$$

~~5 cm s^{-1}~~

5 mm s^{-1}

$$Re = \frac{5\,10^{-3}\times 5\,10^{-6}}{10^{-5}} \sim 25\,10^{-4}$$

$$\nu_{air} = \frac{10^{-5}}{1} = 10^{-5}\ m^2 s^{-1}$$

$Re = 2\,10^{-3} \ll 1$ laminaire !

$v_{lim} \uparrow$ qd $R \uparrow$

Les gttes tombent mais l'air est ascendant à cause de la ≠ce de T entre le sol (+ chaud) et le haut de l'atm (+ froid)

Si $v_{lim} \leq v_{air}$ les gttes ne tombent pas

$v_{lim} = v_{air}$ dét la hauteur à laquelle se stabilise le nuage

Si $v_{lim} > v_{air}$, il pleut : en pratique cela se produit qd les gttes st assez grosses ($v_{lim} \propto R^2$). Il existe ≠ mécanismes de croissance des gttes ds les nuages

- condensation de la vapeur d'eau à la surface de la gtte
- chocs entre gttes → elles grossissent