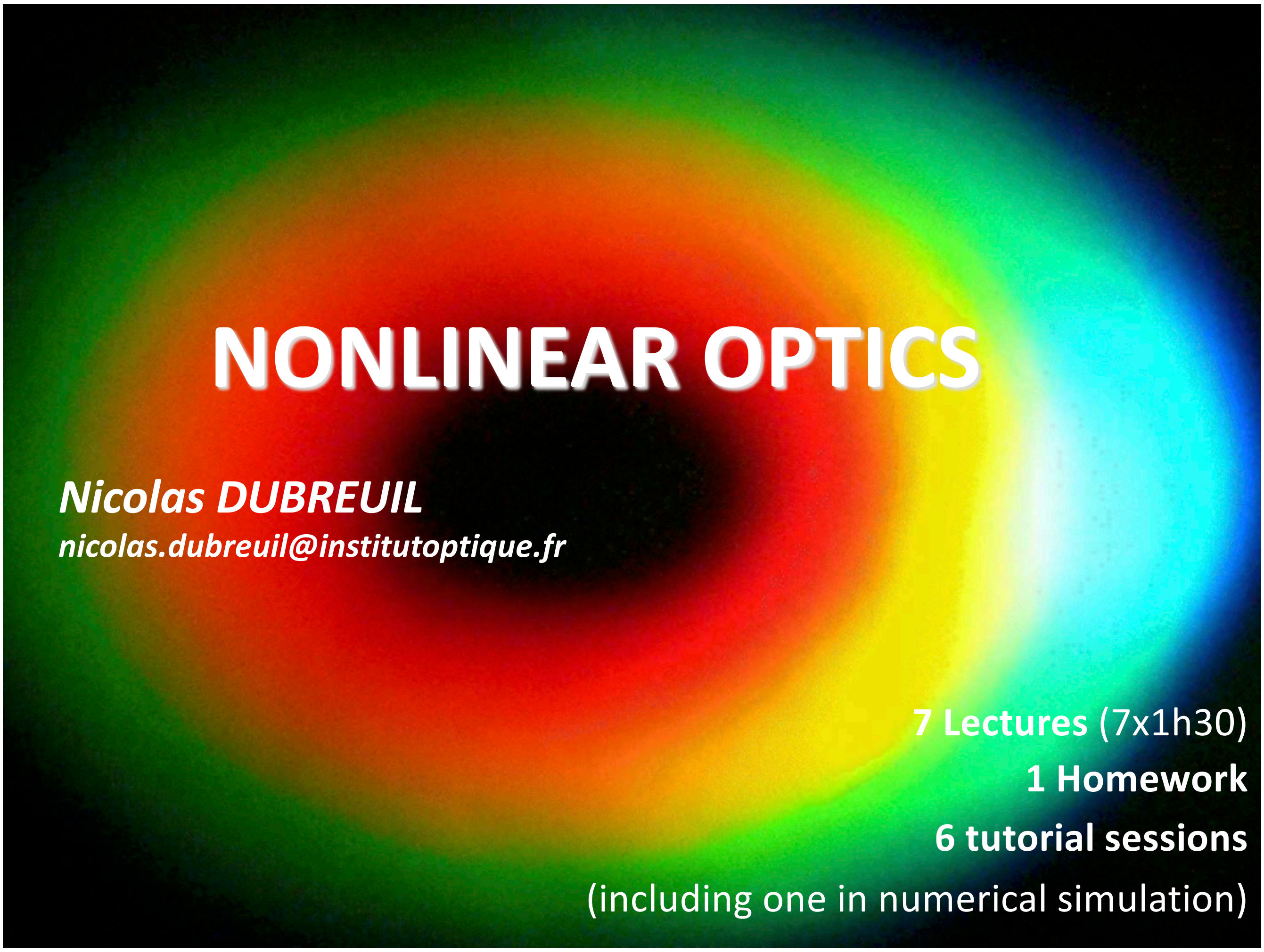


Lecture 4 /7 : learning outcomes

By the end of this lecture, students will be able to...

- Evaluate nonlinear interaction performances/efficiencies under approximations that should be specified, explained and justified (T2)

By the end of this lecture, students will be skilled at...

- deriving and solving the nonlinear wave equation in a parametric situation under the undepleted pump approximation (S3)
- Determining the phase matching conditions for a given nonlinear interaction and achieving/fulfilling this condition by exploiting birefringence properties of materials (S2)

By the end of this lecture, students will understand ...

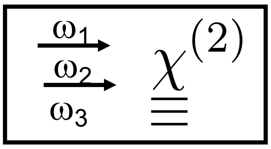
- Nonlinear optics is an essential tool to create novel optical frequencies(U4) generated through the interaction of incident beams within nonlinear materials
- Nonlinear effects are subject to phase matching conditions (U5)

Lecture 4 - Content

- **2nd ORDER NONLINEARITIES**
  - **The Manley-Rowe Relations**
  - **Three wave mixing in $\chi^{(2)}$ materials**
  - **Second Harmonic Generation**
    - **Phase matching consideration**
    - **Reminder : propagation in a linear anisotropic material**
    - **Phase matching condition in birefringent materials**
  - Frequency generation - Parametric processes
    - optical parametric fluoresence and amplification
    - optical parametric oscillation : OPO
  - Quasi-phase matched materials

1- The Manley-Rowe relations

General description of the **NL interaction of 3 waves** @ ω_1, ω_2, and ω_3 (with $\omega_3 = \omega_1 + \omega_2$)**in a <u>2nd order NL</u> <u>lossless</u> medium** :



$$I = 2nc\epsilon_0|A|^2$$

$$\begin{cases} \dfrac{dA_3(z)}{dz} = \dfrac{\imath\omega_3}{2\epsilon_0 n_3 c} \boldsymbol{e}_3 \cdot \boldsymbol{P}_{NL}(z, \omega_3 = \omega_1 + \omega_2) e^{-\imath k_3 z}, \\ \dfrac{dA_2(z)}{dz} = \dfrac{\imath\omega_2}{2\epsilon_0 n_2 c} \boldsymbol{e}_2 \cdot \boldsymbol{P}_{NL}(z, \omega_2 = \omega_3 - \omega_1) e^{-\imath k_2 z}, \\ \dfrac{dA_1(z)}{dz} = \dfrac{\imath\omega_1}{2\epsilon_0 n_1 c} \boldsymbol{e}_2 \cdot \boldsymbol{P}_{NL}(z, \omega_1 = \omega_3 - \omega_2) e^{-\imath k_1 z}. \end{cases} \Rightarrow \begin{cases} \dfrac{dI_3(z)}{dz} = +\imath\epsilon_0\omega_3\chi^{(2)}_{eff} A_1 A_2 A_3^\star e^{\imath\Delta k z} + C.C., \\ \dfrac{dI_2(z)}{dz} = -\imath\epsilon_0\omega_2\chi^{(2)}_{eff} A_1 A_2 A_3^\star e^{\imath\Delta k z} + C.C., \\ \dfrac{dI_1(z)}{dz} = -\imath\epsilon_0\omega_1\chi^{(2)}_{eff} A_1 A_2 A_3^\star e^{\imath\Delta k z} + C.C., \end{cases}$$

$$\begin{aligned} \chi^{(2)}_{eff} &= 2\ \boldsymbol{e}_3 \cdot \underline{\underline{\chi}}^{(2)}(\omega_3; \omega_1, \omega_2) \boldsymbol{e}_1 \boldsymbol{e}_2 \\ &= 2\ \boldsymbol{e}_2 \cdot \underline{\underline{\chi}}^{(2)}(\omega_2; \omega_3, -\omega_1) \boldsymbol{e}_3 \boldsymbol{e}_1 \\ &= 2\ \boldsymbol{e}_1 \cdot \underline{\underline{\chi}}^{(2)}(\omega_1; \omega_3, -\omega_2) \boldsymbol{e}_3 \boldsymbol{e}_2. \end{aligned}$$

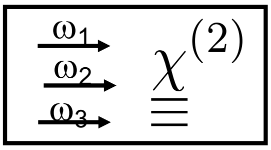
Lossless material :
- Full permutation between the indices of the susceptibility tensor
- Purely real quantities

and reminding that $\omega_3 = \omega_1 + \omega_2$

$$\Rightarrow \boxed{\frac{dI_3(z)}{dz} + \frac{dI_2(z)}{dz} + \frac{dI_1(z)}{dz} = 0}$$

1- The Manley-Rowe relations

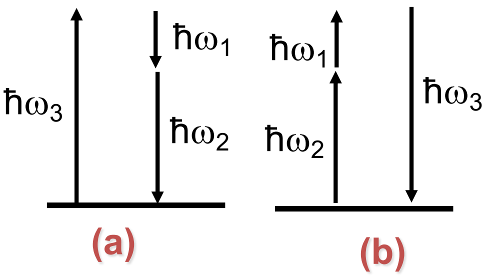
General description of the **NL interaction of 3 waves** @ ω_1, ω_2, and ω_3 (with $\omega_3 = \omega_1 + \omega_2$) **in a <u>2nd order NL</u> <u>lossless</u> medium :**



$$\frac{dI_3(z)}{dz} + \frac{dI_2(z)}{dz} + \frac{dI_1(z)}{dz} = 0,$$

$$\frac{d(I_1/\omega_1)}{dz} = \frac{d(I_2/\omega_2)}{dz} = -\frac{d(I_3/\omega_3)}{dz}$$

$$\frac{d(N_1+N_3)}{dz} = \frac{d(N_2+N_3)}{dz} = \frac{d(N_1-N_2)}{dz} = 0$$

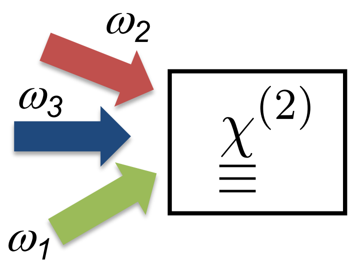


creation/annihilation

The annihilation ***(a)*** (or the creation ***(b)***) of one ω_3 photon must be accompanied by the creation ***(a)*** (or the annihilation ***(b)***) of photon **<u>and</u>** one ω_2 photon

<u>Consistent with quantum-mechanical interpretation</u>

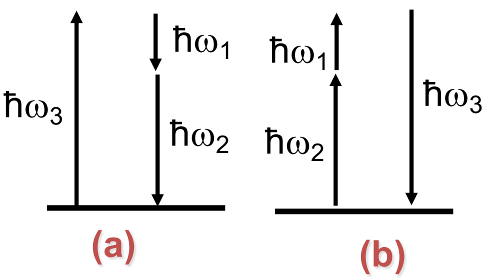
2 - Three-wave Mixing Interactions in a $\chi^{(2)}$ material



Nonlinearities in $\chi^{(2)}$ leads to 3-wave mixing interactions. They are governed by :

- **Energy conservation law**

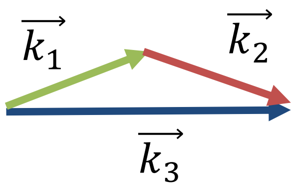
$$\hbar\omega_1 + \hbar\omega_2 = \hbar\omega_3$$



The annihilation ***(a)*** (or the creation ***(b)***) of one ω_3 photon must be accompanied by the creation ***(a)*** (or the annihilation ***(b)***) of photon **<u>and</u>** one ω_2 photon

Consistent with quantum-mechanical interpretation

- **Phase-matching condition = Law of conservation of momemtum**



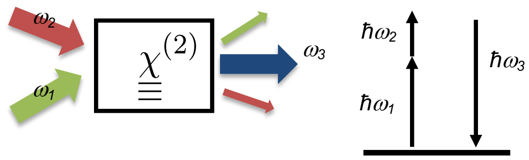
$$\vec{k_1} + \vec{k_2} = \vec{k_3}$$

(Important : vectorial relation)

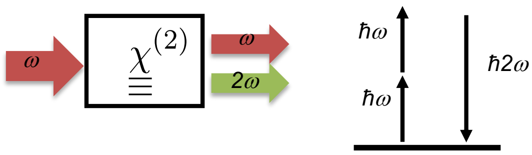
2 - Three-wave Mixing Interactions in a $\chi^{(2)}$ material

Nonlinearities in $\chi^{(2)}$ leads to 3-wave mixing interactions :

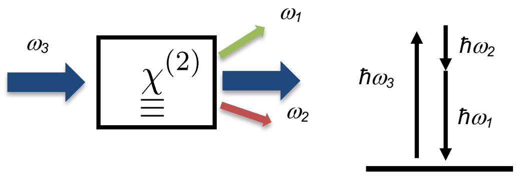
Sum-Frequency Generation (SFG)



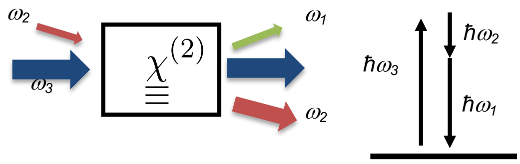
Second Harmonic Generation (SHG)



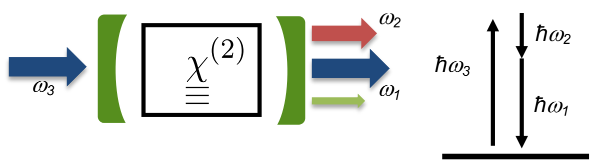
Optical Parametric Fluoresence



Optical Parametric Amplification (APO)



Optical Parametric Oscillation (OPO)



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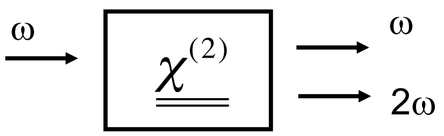
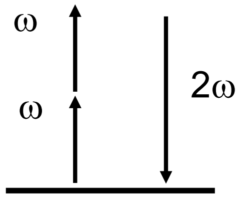
2nd Harmonic Generation

Assumption : Lossless medium





1. Undepleted pump approximation regime

$A_\omega(z)$=Cste

$$\begin{cases} \dfrac{dA_\omega(z)}{dz} = 0 \\ \dfrac{dA_{2\omega}(z)}{dz} = \dfrac{\imath(2\omega)}{2\epsilon_0 n_{2\omega} c} \boldsymbol{e}_{2\omega} \cdot \boldsymbol{P}_{NL}(z, 2\omega) e^{-\imath \boldsymbol{k}_{2\omega} \cdot \boldsymbol{z}} \end{cases}$$

$$\boldsymbol{P}_{NL}(z, 2\omega) = \epsilon_0 \underline{\underline{\chi}}^{(2)}(2\omega; \omega, \omega) \boldsymbol{E}(z, \omega) \boldsymbol{E}(z, \omega) = \epsilon_0 \underline{\underline{\chi}}^{(2)}(2\omega; \omega, \omega) \boldsymbol{e}_\omega \boldsymbol{e}_\omega A_\omega^2(z) e^{\imath 2\boldsymbol{k}_\omega \cdot \boldsymbol{z}}$$

$$\rightarrow \quad \frac{dA_{2\omega}(z)}{dz} = \frac{\imath(2\omega)}{2n_{2\omega} c} \chi_{\text{eff}}^{(2)} \; A_\omega^2(z) e^{\imath \Delta \boldsymbol{k} \cdot \boldsymbol{z}}$$

Wavevector missmatch:

$$\Delta \boldsymbol{k} = 2\boldsymbol{k}_\omega - \boldsymbol{k}_{2\omega}$$

Effective nonlinear susceptibility

$$\chi_{\text{eff}}^{(2)} = \boldsymbol{e}_{2\omega} \cdot \underline{\underline{\chi}}^{(2)}(2\omega; \omega, \omega) \boldsymbol{e}_\omega \boldsymbol{e}_\omega$$

2nd Harmonic Generation

1. Undepleted pump approximation regime

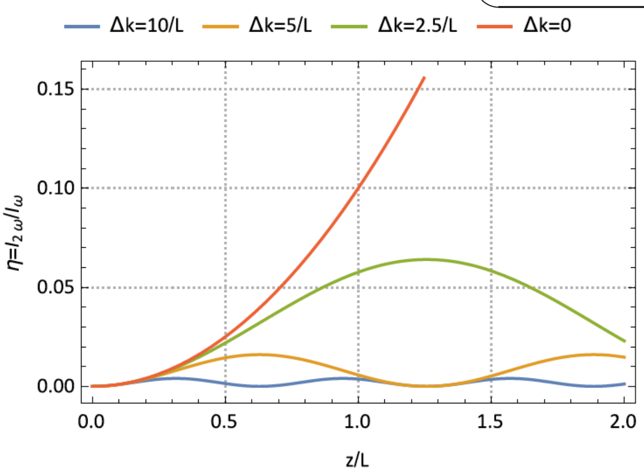
Solution : Intensity evolution

$$I_{2\omega}(z) = \frac{(2\omega)^2}{2\epsilon_0 n_\omega^2 n_{2\omega} c^3} \left|\chi_{\text{eff}}^{(2)}\right|^2 \sin^2\left(\frac{\Delta k}{2} z\right) \frac{I_\omega^2}{(\Delta k)^2} = \frac{(2\omega)^2}{8\epsilon_0 n_\omega^2 n_{2\omega} c^3} \left|\chi_{\text{eff}}^{(2)}\right|^2 \; \text{sinc}^2(\Delta k z/2) \; I_\omega^2 \; z^2.$$

Field intensity :
$I = 2nc\epsilon_0 |E_0|^2,$

SHG efficiency :

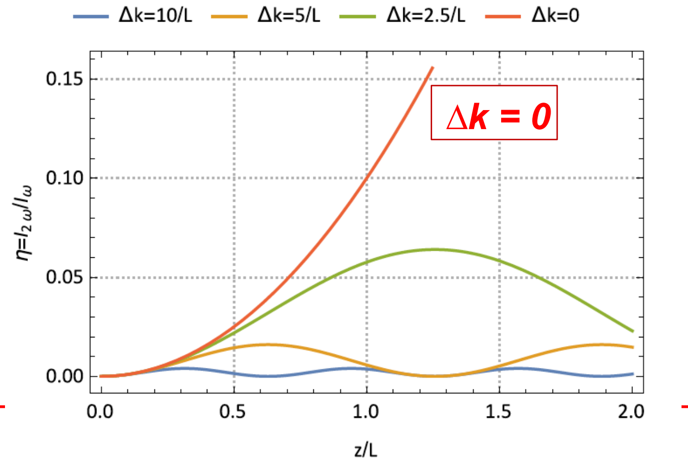
$$\eta_{\text{SHG}} = \frac{I_{2\omega}}{I_\omega} = \frac{(2\omega)^2}{8\epsilon_0 n_\omega^2 n_{2\omega} c^3} \left|\chi_{\text{eff}}^{(2)}\right|^2 \; \text{sinc}^2(\Delta k z/2) \; I_\omega \; z^2.$$



2nd Harmonic Generation

1. Undepleted pump approximation regime

SHG efficiency :



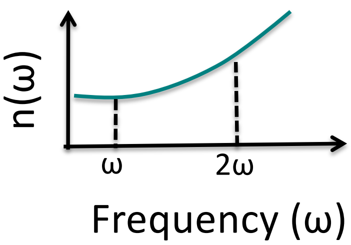
Conclusions :

- Non-phasematched situation : generation @ 2ω occurs on a typical length $L_{coh}=\pi/\Delta k$, called ***coherent buildup length (coherence length)***
- Intensity @ 2ω is proportional to $I_\omega^2/\Delta k^2$
- The conversion efficiency $I_{2\omega}/I_\omega$ is proportional to I_ω *(need to focus the beam @ ω to increase the efficiency)*
- ***Efficiency Max. :*** *requires to fulfill the* ***phasematching condition: Δk=0***

Phase matching condition

About the difficulty to achieve the phase matching condition :

In general, the refractive index for lossless materials shows a NORMAL DISPERSION : the refractive index is increasing with the frequency



Case of 2nd Harm. Gen. :

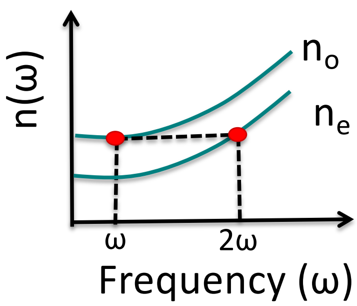
Phase matching condition implies :

$n(\omega)=n(2\omega)$

IMPOSSIBLE !

The most common procedure for achieving the phase matching condition :

Use of birefringence properties of crystals.

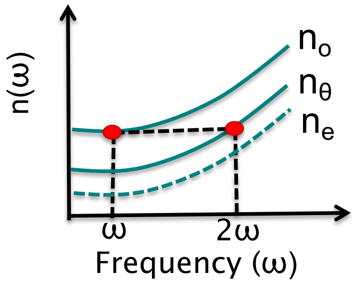


Phase matching condition

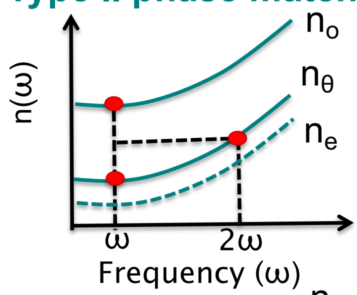
The most common procedure for achieving the phase matching condition :

➜ *Use of birefringence properties of crystals.*

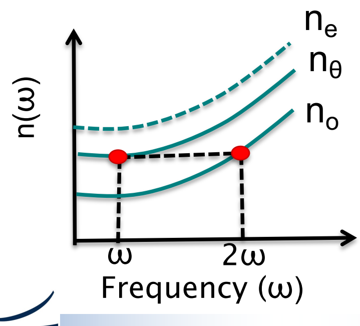
Type I phase matching

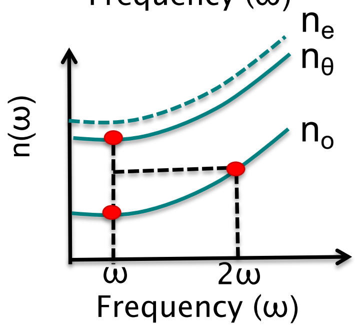


Type II phase matching



Negative uniaxial crystal





Positive uniaxial crystal

Reminder : $n_e = n_{\theta=\pi/2}$

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Linear Wave Eq. in an anisotropic material

- **Maxwell's equations**

no free charges, no free currents, nonmagnetic

$$\begin{cases} \nabla \times \boldsymbol{\mathcal{E}} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{H} = \dfrac{\partial \boldsymbol{\mathcal{D}}}{\partial t} \end{cases}$$

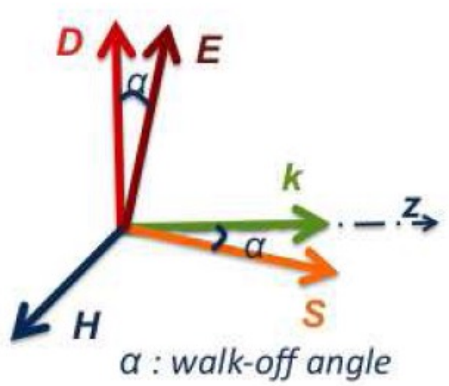
$$\nabla \cdot \boldsymbol{\mathcal{D}} = 0 \;\Rightarrow\; \nabla \cdot \mathbf{D} = \imath \mathbf{k} \cdot \mathbf{D} = 0$$

$$\nabla \cdot \boldsymbol{B} = 0, \;\Rightarrow\; \nabla \cdot \mathbf{H} = \imath \mathbf{k} \cdot \mathbf{H} = 0$$

$\Rightarrow$ $\mathbf{k} \perp \mathbf{D}$, $\mathbf{k} \perp \mathbf{H}$

$$\boldsymbol{D}(\omega) = \underline{\underline{\epsilon}}\, \boldsymbol{E} \qquad \boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Poynting vector : $\mathbf{S} = \mathbf{E} \times \mathbf{H}$



$$\vec{k} \perp \vec{D}$$

$$\vec{S} \perp \vec{E}$$

α : walk-off angle

Linear Wave Eq. in an anisotropic material

Wave equation

In the time domain

$$\nabla \times \nabla \times \boldsymbol{E}(t) + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

In the frequency domain

$$\nabla \times \nabla \times \boldsymbol{E}(\omega) - \frac{\omega^2}{c^2}\boldsymbol{E}(\omega) = \omega^2 \mu_0 \boldsymbol{P}(\omega)$$

The complex field amplitude of the plane wave is defined as

$$\boldsymbol{E}(\omega) = E(\omega) e^{\imath \boldsymbol{k}\cdot\boldsymbol{r}} \boldsymbol{e}.$$

Polarization vector

Linear Wave Eq. in an anisotropic material

Wave equation $\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{e}) + \frac{\omega^2}{c^2} \underline{\underline{\epsilon}}(\omega) \boldsymbol{e} = 0$

$\vec{k} = |k| \vec{s}$

$\vec{s} = (s_x, s_y, s_z)$
Unit vector

$\boldsymbol{D}(\omega) = \underline{\underline{\epsilon}}\, \boldsymbol{E}$

$\underline{\underline{\epsilon}}(\omega) = \epsilon_0 \left(1 + \underline{\underline{\chi}}^{(1)}(\omega)\right)$

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

New coordinate system

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

Principal axis system

Fresnel's equation

$$\frac{s_x^2}{n^2 - n_x^2} + \frac{s_y^2}{n^2 - n_y^2} + \frac{s_z^2}{n^2 - n_z^2} = \frac{1}{n^2}$$

(in the principal axis system)

n_x, n_y, n_z are the principal refractive indices

For any direction of propagation, two waves can propagate (independently) with phase velocity $v_1 = c/n_1$ and $v_2 = c/n_2$ where n_1 and n_2 are solutions of the Fresnel's equation

Linear Wave Eq. in an anisotropic material

The index of ellipsoid : method to determine the refractive indices and the related directions of the electric fields

The electric energy density stored in a medium $w_e = \frac{1}{2} \boldsymbol{E} \cdot \boldsymbol{D} = \frac{1}{2} \sum_{j,k} E_k \epsilon_{kj} E_j$

$$2w_e = \epsilon_{XX} E_X^2 + \epsilon_{YY} E_Y^2 + \epsilon_{ZZ} E_Z^2 + 2\epsilon_{YZ} E_Y E_Z + 2\epsilon_{XZ} E_X E_Z + 2\epsilon_{XY} E_X E_Y$$

➜ **The surface of constant energy forms an ellipsoid** $\epsilon_{XX}, \epsilon_{YY}, \epsilon_{ZZ} > 0$
In a lossless material

➜ **In the principal dielectric axes**, the equation is reduced to $2w_e = \epsilon_{xx} E_x^2 + \epsilon_{yy} E_y^2 + \epsilon_{zz} E_z^2$

The constant energy surfaces in the space (D_x, D_y, D_z) form ellipsoids defined by:

$$2w_e = \frac{D_x^2}{\epsilon_{xx}} + \frac{D_y^2}{\epsilon_{yy}} + \frac{D_z^2}{\epsilon_{zz}}$$

New variable : $\vec{r} = \vec{D} \sqrt{2 w_e \epsilon_0}$

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

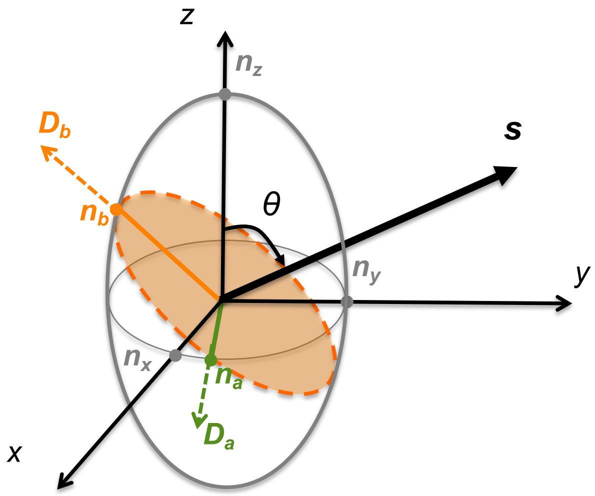
It defines the index of ellipsoid equation that is used to determine, for any direction of propagation, the two refractive indices and the associated direction for $\vec{D}$

Linear Wave Eq. in an anisotropic material

The index of ellipsoid : $\vec{r} = \vec{D}\sqrt{2w_e\epsilon_0}$

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

It defines the index of ellipsoid equation that is used to determine, for any direction of propagation, the two refractive indices and the associated direction for $\vec{D}$



The two axes of the ellipse formed by the intersection between the plane perpendicular to $\vec{s}$, the direction of propagation, and the ellipsoid, are equal to $2n_a$ and $2n_b$.Their related field $\vec{D}_a$and $\vec{D}_b$ are parallel to these two axes.

Linear Wave Eq. in an anisotropic material

Case of an uniaxial crystal

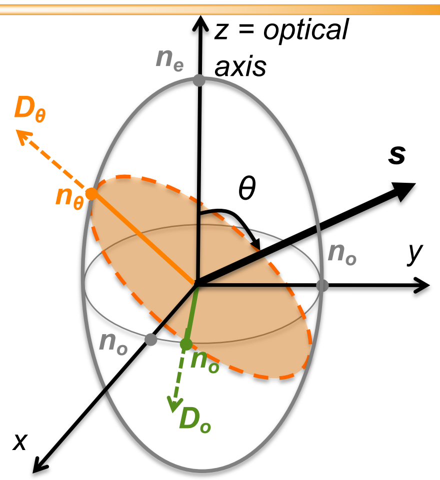
It exhibits two identical principal refractive indices : $n_x = n_y = n_o$ and $n_z = n_e \neq n_o$

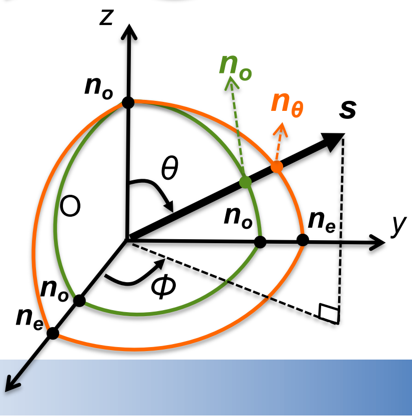
$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

n_o - - Ordinary wave with polarization state
$$e_o = \begin{vmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{vmatrix}$$

n_ϑ - - Extraordinary wave with polarization state
$$e_\theta = \begin{vmatrix} -\cos\theta\cos\phi \\ -\cos\theta\sin\phi \\ \sin\theta \end{vmatrix}$$

$$\left(\frac{1}{n_\theta}\right)^2 = \left(\frac{\cos\theta}{n_o}\right)^2 + \left(\frac{\sin\theta}{n_e}\right)^2$$





Stationary Nonlinear Wave Equation

Nonlinear wave equation at steady state

Nonlinear dielectric material

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \vec{P}_L + \vec{\mathcal{P}}_{\mathrm{NL}}$$ Nonlinear polarization

$$\vec{P}_L(\omega) = \varepsilon_0 \underline{\underline{\chi}}^{(1)}(\omega)\vec{E}(\omega)$$

Wave equation in frequency domain including a nonlinear polarization:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r},\omega) = \frac{\omega^2}{c^2}\underline{\underline{\epsilon}}(\boldsymbol{r},\omega)\boldsymbol{E}(\boldsymbol{r},\omega) + \omega^2\mu_0 \boldsymbol{P}^{(NL)}(\boldsymbol{r},\omega)$$

Steady-state solution of the form: $\boldsymbol{E}(\boldsymbol{r},\omega) = A(\boldsymbol{r})e^{\imath \boldsymbol{k}\cdot\boldsymbol{r}}\boldsymbol{e}$

$$-\left[\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{e}) + \frac{\omega^2}{c^2}\underline{\underline{\epsilon}}(\omega)\boldsymbol{e}\right]A(\boldsymbol{r})$$

$$+\imath\left[\nabla A\times(\boldsymbol{k}\times\boldsymbol{e}) + \boldsymbol{k}\times(\nabla A\times\boldsymbol{e})\right] + \nabla\times(\nabla A\times\boldsymbol{e}) \quad = \quad -\omega^2\mu_0 \boldsymbol{P}_{NL}(z,\omega)e^{-\imath\boldsymbol{k}\cdot\boldsymbol{r}}$$

Nonlinear Wave Equation

$$-\left[\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{e}) + \frac{\omega^2}{c^2}\underline{\underline{\epsilon}}(\omega)\boldsymbol{e}\right]A(\boldsymbol{r})$$

$$+\imath\left[\nabla A\times(\boldsymbol{k}\times\boldsymbol{e}) + \boldsymbol{k}\times(\nabla A\times\boldsymbol{e})\right] + \nabla\times(\nabla A\times\boldsymbol{e}) \quad = \quad -\omega^2\mu_0 \boldsymbol{P}_{NL}(z,\omega)e^{-\imath\boldsymbol{k}\cdot\boldsymbol{r}}$$

- Simplification : $\vec{E} = \vec{E}_o + \vec{E}_\theta = \vec{E}_{ordinary} + \vec{E}_{extraordinary}$

Eigen polarization states of Fresnel Eq. :

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{e}) + \frac{\omega^2}{c^2}\underline{\underline{\epsilon}}(\omega)\boldsymbol{e} = 0$$

- **Slowly-varying amplitude approximation :**

$$\left|k\frac{\partial A}{\partial x_i}\right| \gg \left|\frac{\partial^2 A}{\partial x_i \partial y_i}\right|$$

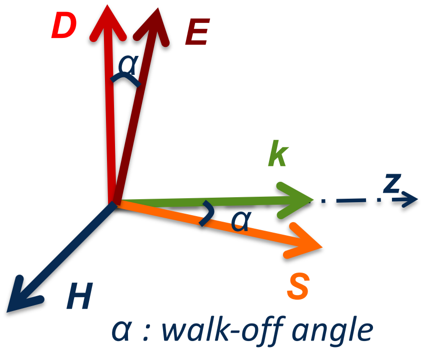
Slow variation of the field amplitude on a typical length of the order of λ

Nonlinear Wave Equation

$$2\imath\nabla A\cdot[(\boldsymbol{k}\times\boldsymbol{e})\times\boldsymbol{e}]=\omega^2\mu_0\boldsymbol{e}\cdot\boldsymbol{\Pi}_{NL}(z,\omega)e^{\imath\Delta\boldsymbol{k}\cdot\boldsymbol{r}}$$

where $\boldsymbol{P}_{NL}(z,\omega)=\boldsymbol{\Pi}_{NL}(z,\omega)\,e^{\imath\boldsymbol{k}_p(\omega)\cdot\boldsymbol{r}}$

- Now, vector $\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{e})$ is parallel to Poynting vector



Wave equation in the coordinate system $(\vec{D},\vec{H},\vec{k})$

$$-\tan\alpha\frac{\partial A}{\partial x}+\frac{\partial A}{\partial z}=\frac{\imath\omega^2\mu_0}{2k\cos^2\alpha}\boldsymbol{e}\cdot\boldsymbol{\Pi}_{NL}(z,\omega)e^{\imath\Delta\boldsymbol{k}\cdot\boldsymbol{r}}$$

Wavevector missmatch

α : walk-off angle

Nonlinear Wave Equation

- **Neglecting the walk-off angle, form for wave equation :**

$$\frac{\partial A(z)}{\partial z}=\frac{\imath\omega}{2\epsilon_0 nc}\boldsymbol{e}\cdot\boldsymbol{P}_{NL}(z,\omega)e^{-\imath kz}$$

$$\boldsymbol{P}_{NL}(z,\omega)=\boldsymbol{\Pi}_{NL}(z,\omega)\,e^{\imath\boldsymbol{k}_p(\omega)\cdot\boldsymbol{r}}$$

$$\frac{\partial A(z)}{\partial z}=\frac{\imath\omega}{2\epsilon_0 nc}\boldsymbol{e}\cdot\boldsymbol{\Pi}_{NL}(z,\omega)e^{-\imath\Delta kz}$$

$A(\omega)$	Slowly varying electric field amplitude of wave ω
$\vec{P}_{NL}(\omega)$	Complex amplitude of the nonlinear polarization at ω
$\vec{e}$	Polarization unit vector of the electric field
n	Refractive index at frequency ω
ω	Frequency (in vacuum)
$\Delta\vec{k}=\vec{k}_s-\vec{k}$	Wavevector missmatch : difference between the wavevectors of the NL polarization and of the electric field

CONCLUSIONS

• **Nonlinear propagation of waves in an <u>anisotropic</u> material can be treated as the propagation in an <u>isotropic</u> material by priorly decompose the incoming field other the eigen polarization modes, the ordinary and the extraordinary fields.**

• Energy transfer condition :

- phase matching condition ***Δk=0***
- Non-zero projection between the electric field and the NL polarization $\vec{e}\,.\,\vec{P}_{NL} \neq 0$

Lecture 4 - Content

- **2nd ORDER NONLINEARITIES**
  - The Manley-Rowe Relations
  - Three wave mixing in $\chi^{(2)}$ materials
  - **Second Harmonic Generation**
    - Phase matching consideration
    - Reminder : propagation in a linear anisotropic material
    - **Phase matching condition in birefringent materials**
  - Frequency generation - Parametric processes
    - optical parametric fluoresence and amplification
    - optical parametric oscillation : OPO
  - Quasi-phase matched materials

Phase-matching condition in birefringent materials

Example : Second Harmonic Generation

The fields at ω and 2ω are decomposed into the sum of ordinary and extraordinary vibration modes :

$$\boldsymbol{E}(\omega) = \boldsymbol{E}_o(\omega) + \boldsymbol{E}_\theta(\omega)$$

$$\left| \begin{array}{l} \boldsymbol{E}_o(\omega) = A_o(\omega)\boldsymbol{e}_o \exp i(\boldsymbol{k}_o(\omega).\boldsymbol{s}) \\ \boldsymbol{E}_\theta(\omega) = A_\theta(\omega)\boldsymbol{e}_\theta \exp i(\boldsymbol{k}_\theta(\omega).\boldsymbol{s}) \end{array} \right.$$

$$\boldsymbol{E}(2\omega) = \boldsymbol{E}_o(2\omega) + \boldsymbol{E}_\theta(2\omega)$$

$$\left| \begin{array}{l} \boldsymbol{E}_o(2\omega) = A_o(2\omega)\boldsymbol{e}_o \exp i(\boldsymbol{k}_o(2\omega).\boldsymbol{s}) \\ \boldsymbol{E}_\theta(2\omega) = A_\theta(2\omega)\boldsymbol{e}_\theta \exp i(\boldsymbol{k}_\theta(2\omega).\boldsymbol{s}) \end{array} \right.$$

Nonlinear wave equations :

$$\begin{cases} \dfrac{\partial A_o(2\omega)}{\partial s} = \dfrac{i(2\omega)}{2\epsilon_0 n_o(2\omega)c}\boldsymbol{e}_o.\boldsymbol{\mathcal{P}}_{NL}(2\omega)e^{-i\boldsymbol{k}_o(2\omega).\boldsymbol{s}} \\ \dfrac{\partial A_\theta(2\omega)}{\partial s} = \dfrac{i(2\omega)}{2\epsilon_0 n_\theta(2\omega)c}\boldsymbol{e}_\theta.\boldsymbol{\mathcal{P}}_{NL}(2\omega)e^{-i\boldsymbol{k}_\theta(2\omega).\boldsymbol{s}} \end{cases}$$

With :

$$\boldsymbol{\mathcal{P}}_{NL}(2\omega) = \boldsymbol{P}_{NL}(2\omega)e^{i\boldsymbol{k}_P(2\omega).\boldsymbol{s}}$$

Phase matching condition :

$$\boldsymbol{k}_P(2\omega) = \boldsymbol{k}_o(2\omega) \quad \text{OR} \quad \boldsymbol{k}_P(2\omega) = \boldsymbol{k}_\theta(2\omega)$$

(vectorial relation)

Phase-matching condition in birefringent materials

Example : Second Harmonic Generation

$$\begin{aligned} \boldsymbol{\mathcal{P}}_{NL}(2\omega) &= \epsilon_0 \underline{\underline{\chi}}^{(2)}(\omega,\omega)\boldsymbol{E}(\omega)\boldsymbol{E}(\omega), \\ &= \epsilon_0 \underline{\underline{\chi}}^{(2)}(\omega,\omega)\left[\boldsymbol{E}_o(\omega)\boldsymbol{E}_o(\omega) + \boldsymbol{E}_\theta(\omega)\boldsymbol{E}_\theta(\omega) + \boldsymbol{E}_o(\omega)\boldsymbol{E}_\theta(\omega) + \boldsymbol{E}_\theta(\omega)\boldsymbol{E}_o(\omega)\right] \end{aligned}$$

Type I :

$$k_p(2\omega) = \boldsymbol{k}_\theta(\omega) + \boldsymbol{k}_\theta(\omega) = \boldsymbol{k}_o(2\omega) \Rightarrow n_\theta(\omega) = n_o(2\omega)$$

$$k_p(2\omega) = \boldsymbol{k}_o(\omega) + \boldsymbol{k}_o(\omega) = \boldsymbol{k}_\theta(2\omega) \Rightarrow n_o(\omega) = n_\theta(2\omega)$$

Type II :

$$k_p(2\omega) = \boldsymbol{k}_\theta(\omega) + \boldsymbol{k}_o(\omega) = \boldsymbol{k}_o(2\omega) \Rightarrow \frac{n_\theta(\omega) + n_o(\omega)}{2} = n_o(2\omega)$$

$$k_p(2\omega) = \boldsymbol{k}_\theta(\omega) + \boldsymbol{k}_o(\omega) = \boldsymbol{k}_\theta(2\omega) \Rightarrow \frac{n_\theta(\omega) + n_o(\omega)}{2} = n_\theta(2\omega)$$

Collinear interactions

Phase-matching condition in birefringent materials

CONCLUSION

➜ Use of birefringence properties of crystals to satisfy the phase matching condition.

- Phase matching condition can be fulfilled by playing with the birefringent properties of the materials
- This condition to be satisfied sets the polarization states of the interacted fields.
- Note that in some cases, it might be impossible to find a proper phase matching configuration

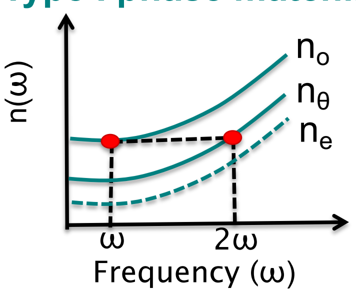
Phase-matching condition in birefringent materials

CONCLUSION

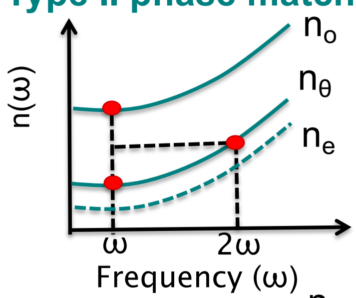
➜ Case of SHG in uniaxial crystals

- In a negative (positive) uniaxial crystal : the 2ω wave is necessarily polarized along the extraordinary (ordinary) direction

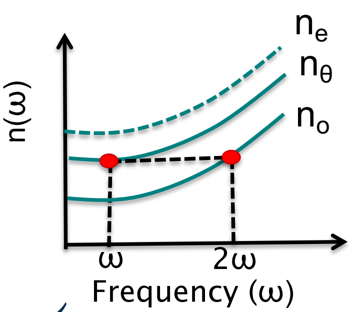
Type I phase matching



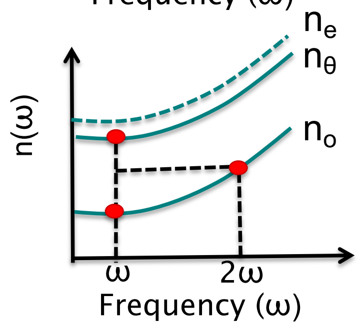
Type II phase matching



Negative uniaxial crystal



Positive uniaxial crystal



Reminder : $n_e = n_{\theta=\pi/2}$