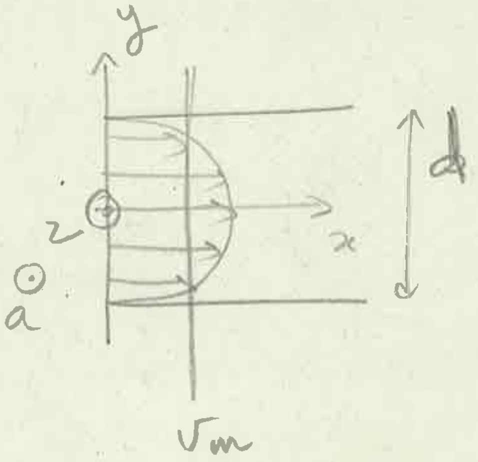
ex2 suite

3 — débit volum

$$Q = \iint \vec{v}\, \underset{\substack{\downarrow \\ dy\,dz}}{dS}\, \vec{n} \qquad \vec{n} \to \vec{u_x}$$



$$Q = \iint v_0 \left(1 - 4\frac{y^2}{d^2}\right) dy\,dz$$

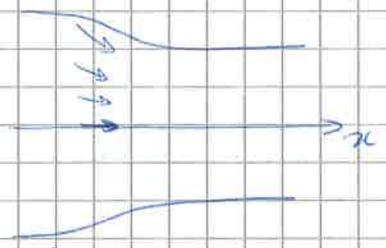
$$= a\, v_0 \int_{-\frac{d}{2}}^{\frac{d}{2}} \left[1 - 4\frac{y^2}{d^2}\right] dy$$

$$\underbrace{d - \frac{4}{d^2}\underbrace{\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{\frac{d}{2}}}_{\frac{4}{3d^2}\times\left(\underbrace{\frac{d^3}{8}+\frac{d^3}{8}}_{\frac{d^3}{4}}\right) = \frac{d}{3}}}_{2\frac{d}{3}}$$

$$Q = \underbrace{a d \times \underbrace{\frac{2}{3} v_0}_{}}_{\bar{v}}$$

4 — $\frac{d\vec{v}}{dt} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{0} + (\vec{v}\cdot\vec{\nabla})\vec{v}$ $\quad v_y = v_z = 0$ $\quad \frac{dv_x}{dt} = v_x \frac{\partial v_x}{\partial x} = 0 \quad \Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \vec{0}$

Ex ~~1~~ 3



$\vec{v} = v_z \vec{e}_z - \alpha r \vec{e}_r$

1 – Pas de dépdce en t ⇒ stationnaire

2 – $\|\vec{v}\| \nearrow$ qd $r \nearrow$ à x fixé et $v_r < 0$ ⇒ voir schéma ci-dessus

3 – $\operatorname{div} \vec{v} = 0$

$$\operatorname{div} \vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \underbrace{\frac{\partial v_z}{\partial z}}_{\text{~~ici x et non z~~}}$$

$$\frac{\partial v_z}{\partial z} + \frac{1}{r}\underbrace{\frac{\partial(-\alpha r^2)}{\partial r}}_{-2\alpha r} = 0$$

$$\frac{\partial v_z}{\partial z} = 2\alpha$$

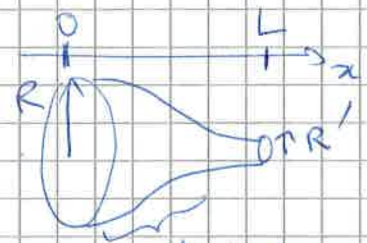
$$v_z = 2\alpha z + B$$

$$v_z(0) = B = v_0$$

$\Rightarrow v_z = 2\alpha z + v_0$ $\quad$ $v \nearrow$ avec z normal car rétrécisse^T

cons du débit et section ↘ ⇒ $v \nearrow$ (⇒ P ↘ cf cours)

4 –



slt ds la partie où le ∅ varie on a le chp de vitesses donné

~~Sinon on suppose~~

$z = 0$ $\quad v_z = v_0$ $\quad$ écoule^T "bouchon"

fluide parfait

$\iint \rho \vec{v} \cdot d\vec{S}\vec{n} = \iint \rho v_z dS$

$Q_m = \rho v_0 \pi R^2$ à l'entrée en $x = 0$

en $x = L$ $\quad Q_m = \rho(2\alpha L + v_0)\pi R'^2$ m̄ débit qu'à l'entrée

5 — $\vec{v} = -\alpha r \vec{u}_r + v_z \vec{e}_z \qquad v_z = 2\alpha z + v_0$

$$(\vec{v}\cdot\vec{\nabla})\vec{v} = \left(-\alpha r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z}\right)\left(-\alpha r \vec{u}_r + v_z \vec{u}_z\right)$$

$$= -\alpha r \frac{\partial}{\partial r}(-\alpha r)\,\vec{u}_r + v_z \frac{\partial v_z}{\partial z}\,\vec{u}_z$$

$$(\vec{v}\cdot\vec{\nabla})\vec{v} = \alpha^2 r\,\vec{u}_r + 2\alpha(2\alpha z + v_0)\,\vec{u}_z$$

$$\frac{\partial \vec{v}}{\partial t} = 0$$

$$\frac{d\vec{v}}{dt} = \alpha^2 r\,\vec{u}_r + 2\alpha(2\alpha z + v_0)\,\vec{u}_z$$