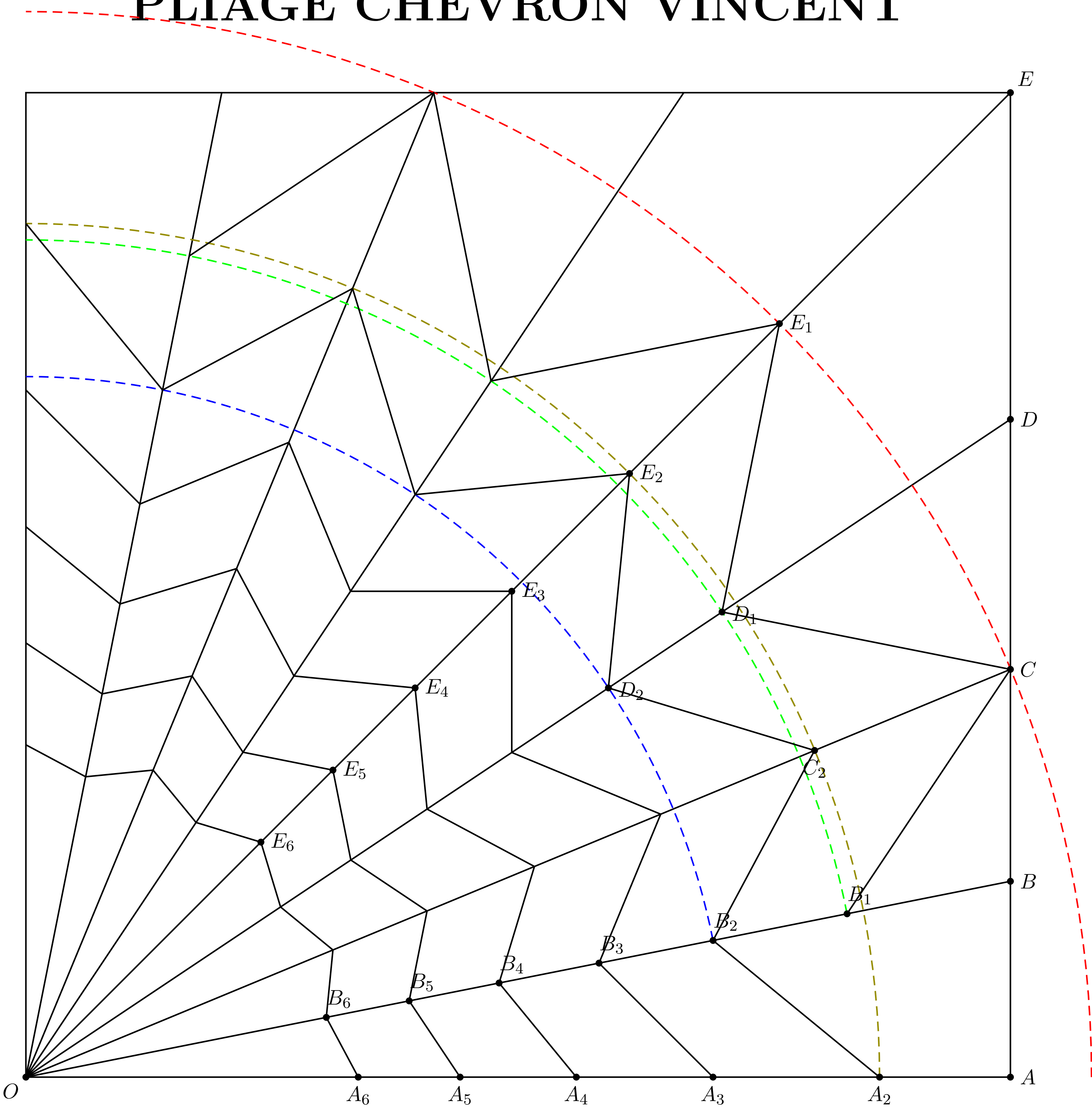
PLIAGE CHEVRON VINCENT



Dimension.

On prend $OA = 1$.

$\cos 22,5^\circ = \dfrac{OA}{OC}$. Donc $OC = \dfrac{OA}{\cos 22,5^\circ} = \dfrac{1}{\frac{1}{2}\sqrt{2+\sqrt{2}}} = \dfrac{2}{\sqrt{2+\sqrt{2}}} = \sqrt{4-2\sqrt{2}} \simeq 1,08239$

$\cos 11,25^\circ = \dfrac{OA}{OB}$. Donc $OB = \dfrac{OA}{\cos 11,25^\circ} = \dfrac{1}{\cos 11,25^\circ} \simeq 1,01959$

1 Pli 1.

$\widehat{OCA} = 90^\circ - 22,5^\circ = 67,5^\circ$
$B_1\widehat{D}A = \dfrac{67,5^\circ}{2} = 33,75^\circ$

Dans le triangle OCB_1, on a : $C\widehat{O}B_1 = 11,25^\circ$, $O\widehat{C}B_1 = 33,75^\circ$ et $O\widehat{B_1}C = 135^\circ$.

$\dfrac{OB_1}{\sin C} = \dfrac{OC}{\sin B_1}$
Donc $OB_1 = OC \times \dfrac{\sin C}{\sin B_1} = OC \times \dfrac{\sin 33,75^\circ}{\sin 135^\circ} = \sqrt{4-2\sqrt{2}} \times \dfrac{\sin 33,75^\circ}{\frac{\sqrt{2}}{2}} = \sqrt{2}\sqrt{4-2\sqrt{2}}\,\sin 33,75^\circ$

$OB_1 = 2\sqrt{2-\sqrt{2}}\,\sin 33,75^\circ \simeq 0,85043$

$y_{B_1} = OB_1 \sin 11,25^\circ = 2\sqrt{2-\sqrt{2}}\,\sin 33,75^\circ \sin 11,25^\circ$
$OD_1 = OB_1$

2 Pli 2.

$OA_2 = OC_2 = OE_2$
$OB_2 = OD_2$

$B_1B = B_1A_2 = B_1C_2$
Soit B_1' la projection de B_1 sur la droite (OA).
Dans le triangle rectangle $B_1B_1'A_2$, on a : $B_1'A_2^2 = B_1A_2^2 - B_1B_1'^2 = B_1B^2 - B_1B_1'^2 = (OB - OB_1)^2 - y_{B_1}^2$

$B_1'A_2^2 = \left(\dfrac{1}{\cos 11,25^\circ} - 2\sqrt{2-\sqrt{2}}\,\sin 33,75^\circ\right)^2 - \left(2\sqrt{2-\sqrt{2}}\,\sin 33,75^\circ \sin 11,25^\circ\right)^2$
D'où : $B_1'A_2 = \sqrt{\left(\dfrac{1}{\cos 11,25^\circ} - 2\sqrt{2-\sqrt{2}}\,\sin 33,75^\circ\right)^2 - \left(2\sqrt{2-\sqrt{2}}\,\sin 33,75^\circ \sin 11,25^\circ\right)^2}$
On en déduit : $OA_2 = OB_1' + B_1'A2 = x_{B_1} + B_1'A2 = ...!$

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Calculons l'angle $O\widehat{A_2}B_1$.

$\sin O\widehat{A_2}B_1 = \dfrac{B_1B_1'}{B_1A_2} = \dfrac{y_{B_1}}{BB_1}$
$O\widehat{A_2}B_1 = 78,75^\circ$
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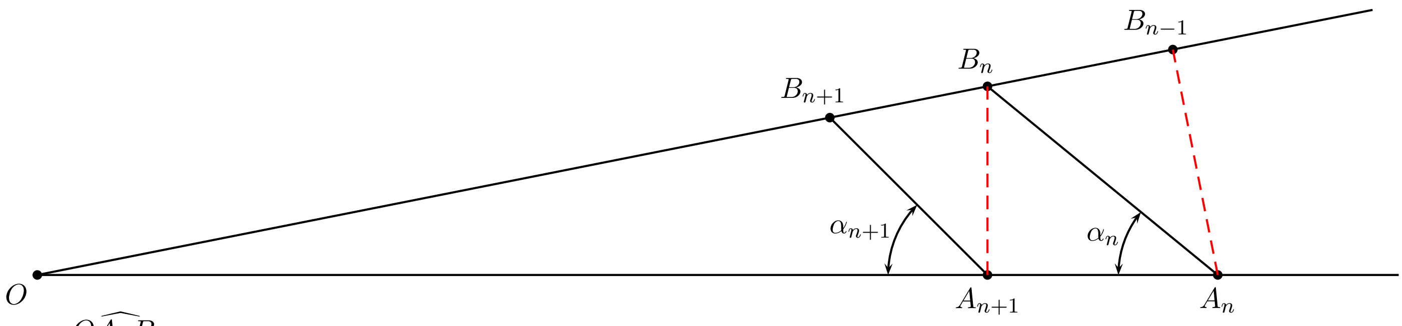
Dans le triangle OA_2B_2, on a : $\widehat{O} = 11,25^\circ$, $\widehat{A}_2 = \dfrac{78,75^\circ}{2} = 39,375^\circ$ et $\widehat{B}_2 = 180^\circ - 11,25^\circ - 39,375^\circ = 129,375^\circ$.

$\dfrac{OB_2}{\sin A_2} = \dfrac{OA_2}{\sin B_2}$
$OB_2 = OA_2 \times \dfrac{\sin A_2}{\sin B_2} = OA_2 \times \dfrac{\sin 39,375^\circ}{\sin 129,375}$

3 Construction des points A_n et B_n.

$A_nB_{n-1}B_nA_{n+1}$ est un cerf-volant.
On a donc : $A_nA_{n+1} = A_nB_{n-1}$ et $B_{n-1}B_n = A_{n+1}B_n$.



$\alpha_n = O\widehat{A_n}B_n$.
Dans le triangle OA_nB_{n-1}, on a : $\widehat{B_{n-1}} = 180^\circ - 11,25^\circ - 2\,\alpha_n$.
Donc $A_n\widehat{A_{n+1}}Bn = 180^\circ - 11,25^\circ - 2\,\alpha_n$.
Autour du point A_{n+1}, on a donc : $2\,\alpha_{n+1} + 180^\circ - 11,25^\circ - 2\,\alpha_n = 180^\circ$.
On en déduit : $\boxed{\alpha_{n+1} = \alpha_n + 5,625^\circ}$. Les angles α forment donc une suite arithmétique.

α_2	$39,375^\circ$
α_3	45°
α_4	$50,625^\circ$
α_5	$56,25^\circ$
α_6	$61,875^\circ$
α_7	$67,5^\circ$

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Calculons OB_n en fonction de OA_n.
Dans le triangle OA_nB_n, on a : $\widehat{O} = 11,25^\circ$, $\widehat{A}_n = \alpha_n$ et $\widehat{B}_n = 180^\circ - 11,25^\circ - \alpha_n$.

$$\frac{OB_n}{\sin A_n} = \frac{OA_n}{\sin B_n}$$

Donc $OB_n = OA_n \times \dfrac{\sin A_n}{\sin B_n} = OA_n \times \dfrac{\sin \alpha_n}{\sin(180^\circ - 11,25^\circ - \alpha_n)} = OA_n \times \dfrac{\sin \alpha_n}{\sin(\alpha_n + 11,25^\circ)}$

$$\boxed{OB_n = OA_n \times \frac{\sin \alpha_n}{\sin(\alpha_n + 11,25^\circ)}}$$

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Calculons OA_n en fonction de ? ? ?.

$OA_{n+1} = OA_n - A_nB_{n-1}$
Avec Pythagore !
$A_nB_{n-1}^2 = (x_{A_n} - x_{B_{n-1}})^2 + (y_{A_n} - y_{B_{n-1}})^2 = (OA_n - x_{B_{n-1}})^2 + y_{B_{n-1}}^2$
$= OA_n{}^2 + x_{B_{n-1}}^2 - 2\,OA_n\,x_{B_{n-1}} + y_{B_{n-1}}^2 = OA_n{}^2 + OB_{n-1}^2 - 2\,OA_n\,x_{B_{n-1}}$