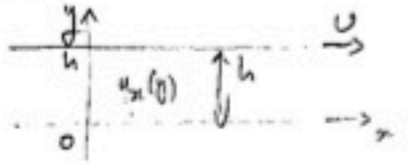
Examen Méca Flu

Planche de mer

Partie I : Fluide visqueux

-2)



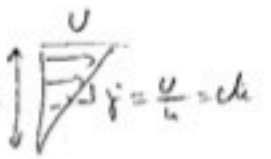
$\vec{u}\begin{vmatrix} u_x(y) \\ 0 \end{vmatrix}$ solution d'écoul^t //

Ecoulement de Couette plan

éq. NS $\begin{cases} 0 = \nu \dfrac{\partial^2 u_x}{\partial y^2} \ (1) \quad \text{car } \dfrac{\partial p}{\partial x} = 0 \\ 0 = \dfrac{\partial p}{\partial y} \ (+\rho g) \end{cases}$

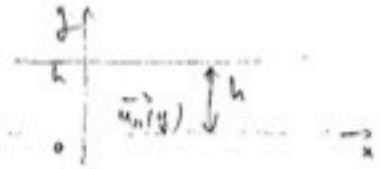
$(1) \Rightarrow \dfrac{\partial^2 u_x}{\partial y^2} = 0 \Rightarrow \dfrac{\partial u_x}{\partial y} = A \qquad u_x(y) = Ay + B.$

2CL : $\left.\begin{array}{l} u_x(y=0) = 0 \Rightarrow B = 0 \\ u_x(y=h) = U \Rightarrow A = \dfrac{U}{h} \end{array}\right\} \Rightarrow u_x(y) = \dfrac{U}{h} y$



$\sigma_{xy} = \nu \dfrac{\partial u_x}{\partial y} = \nu A = \nu \dfrac{U}{h}$ indépendant de y

-1)



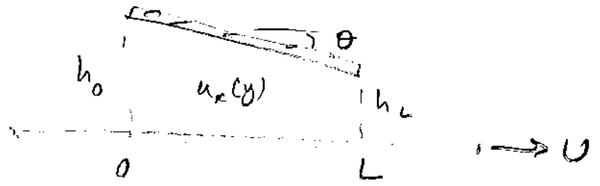
$\vec{u}\begin{vmatrix} u_x(y) \\ 0 \end{vmatrix}$ solution d'écoul^t //

Ecoul^t de Poiseuille plan

éq. NS $\begin{cases} 0 = -\dfrac{\partial p}{\partial x} + \nu \dfrac{\partial^2 u_x}{\partial y^2} \ (1) \\ 0 = \dfrac{\partial p}{\partial y} \ (+\rho g) \end{cases}$

$(1) \Rightarrow \dfrac{\partial^2 u_x}{\partial y^2} = \dfrac{1}{\nu}\dfrac{\partial p}{\partial x} \Rightarrow \dfrac{\partial u_x}{\partial y} = \dfrac{1}{\nu}\dfrac{\partial p}{\partial x} y + A \Rightarrow u_x(y) = \dfrac{1}{2\nu}\dfrac{\partial p}{\partial x} y^2 + Ay + B$

2CL $\left.\begin{array}{l} u_x(y=0) = 0 \Rightarrow B = 0 \\ u_x(y=h) = 0 \Rightarrow A = -\dfrac{1}{2\nu}\dfrac{\partial p}{\partial x} h \end{array}\right\} \quad u_x(y) = -\dfrac{1}{2\nu}\dfrac{\partial p}{\partial x} y(h-y)$



1) $h(x) = h_0 - \theta x$ avec $\theta = \dfrac{h_0 - h_L}{L}$ car $\tan\theta \simeq \theta$ si $\theta \ll 1$

2) $u_x \sim U$ et $u_y \simeq \theta u_x \sim \theta U$

3) Si $Re \ll \dfrac{1}{\theta}$ alors termes inertiels négligeables devant terme visqueux prépondérant

Eq. NS simplifiés $\begin{cases} 0 = -\dfrac{\partial p}{\partial x} + \eta \dfrac{\partial^2 u_x}{\partial y^2} & (1) \\ 0 = \dfrac{\partial p}{\partial y} & (2) \end{cases}$

(2) $\Rightarrow$ p indépendant de y car $u_y \simeq 0$ car $u_y \ll u_x$.

$\dfrac{\partial p}{\partial x}$ car u_x dépend de x car non //.

(1) $\Rightarrow$ $\dfrac{\partial^2 u_x}{\partial y^2} = \dfrac{1}{\eta}\dfrac{\partial p}{\partial x}$ $\Rightarrow$ $u_x(y) = \dfrac{1}{2\eta}\dfrac{\partial p}{\partial x} y^2 + Ay + B$

2CL $u_x(y=0) = U$ $\Rightarrow$ $B = U$
$u_x(y=h) = 0$ $\Rightarrow$ $A = -\dfrac{1}{2\eta}\dfrac{\partial p}{\partial x} h - \dfrac{U}{h}$

$$u_x(y) = \underbrace{-\frac{1}{2\eta}\frac{\partial p}{\partial x} y(h-y)}_{\text{Poiseuille}} + \underbrace{\frac{U}{h}(h-y)}_{\text{Couette}}$$

Superposition car éq. NS linéaire sans les termes non-linéaires.

4) $$Q = \int_0^h u_x(y)\, \ell\, dy = \ell\left[-\frac{1}{2\eta}\frac{\partial p}{\partial x}\left(\frac{hy^2}{2} - \frac{y^3}{3}\right) + \frac{U}{h}\left(hy - \frac{y^2}{2}\right)\right]_0^h$$
$$= \ell\left[-\frac{1}{2\eta}\frac{\partial p}{\partial x}\underbrace{\left(\frac{h^3}{2} - \frac{h^3}{3}\right)}_{\frac{h^3}{6}} + \frac{U}{h}\underbrace{\left(h^2 - \frac{h^2}{2}\right)}_{\frac{h^2}{2}}\right] = \ell\left[\frac{Uh}{2} - \frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right]$$

Q ne dépend pas de x car écoul^t incompressible stationnaire.

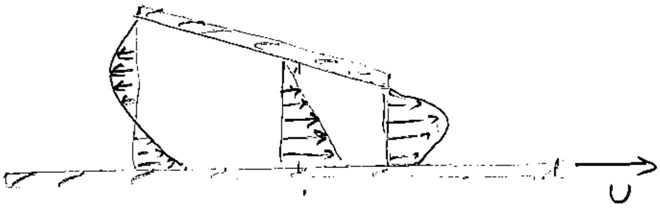
5) $\dfrac{\partial p}{\partial x} = \dfrac{12\eta}{h^3}\left[\dfrac{Uh}{2} - \dfrac{Q}{\ell}\right]$ or $\dfrac{\partial p}{\partial h} = \dfrac{\partial p}{\partial x}\dfrac{\partial x}{\partial h}$ avec $\dfrac{\partial x}{\partial h} = -\dfrac{1}{\theta}$

$\Rightarrow$ $\dfrac{\partial p}{\partial h} = -\dfrac{1}{\theta}\dfrac{\partial p}{\partial x} = \dfrac{12\eta}{\theta h^3}\left[\dfrac{Q}{\ell} - \dfrac{Uh}{2}\right]$

6°/ 2CL $\begin{cases} p = p_{at} \text{ en } h = h_0 \\ p = p_{at} \text{ en } h = h_L \end{cases}$ $\Rightarrow$ $Q =$

$$p(h) = p_{at} + \frac{6\eta U}{\theta}\frac{(h_0 - h)(h - h_L)}{h^2(h_0 + h_L)}$$

7)



$u_x(y)$ ne dépend pas de la viscosité en régime stationnaire ⟹ condus limites établies

8) $$F_N = \int_0^x (p - p_{at})\, l\, dx = \frac{-1}{\theta} \int_{h_0}^{h_L} \frac{6\eta U l}{\theta} \frac{(h_0 - h)(h - h_L)}{h^2 (h_0 + h_L)}\, dh$$

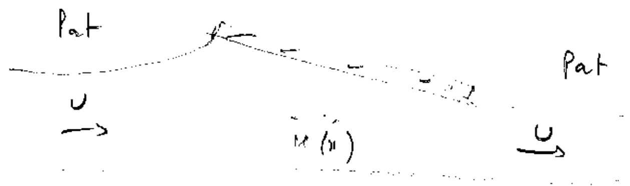
$$F_N \sim \frac{6\eta U l}{\theta^2} \int_{h_L}^{h_0} \frac{(h_0 - h)(h - h_L)}{h^2 (h_0 + h_L)}\, dh \quad \text{avec } \int = \frac{1}{h_0 + h_L} \left[\frac{-h_0 h_L}{h} \right.$$

9) $$F_T = \int_0^L \eta \left.\frac{\partial u_x}{\partial y}\right|_{y=0} l\, dx \quad \text{avec} \quad \left.\frac{\partial u_x}{\partial y}\right|_{y=0} = -\frac{1}{2\eta} \frac{\partial p}{\partial x} h \overset{U/h}{=} \frac{h}{2\eta\theta} \frac{\partial p}{\partial h} - \frac{U}{h} =$$

$$F_T \sim \frac{6\eta U l}{\theta} \qquad \frac{F_N}{F_T} \sim \frac{1}{\theta} \gg 1 \implies F_N \gg F_T$$

10) $$Re = \frac{\rho U h}{\eta}$$

Partie II : Fluide parfait



1) Fluide parfait : viscosité nulle
2) Gliss^t total pour un fluide parfait à la paroi => pas d'entraînement possible
=> la vitesse éventuelle du sol ne change rien au problème

3) Cons. du débit : $u(x)\,h(x) = U h_L \Rightarrow u(x) = U \frac{h_L}{h(x)}$

4) Bernoulli (cons. de l'énergie) : $\frac{1}{2}\rho u^2(x) + p(x) = \frac{1}{2}\rho U^2 + p_{at}$

sur ligne de courant à z = cte entre x et L

$$p(x) = p_{at} + \frac{1}{2}\rho\left(U^2 - u^2(x)\right) = p_{at} + \frac{1}{2}\rho U^2\left(1 - \left(\frac{u(x)}{U}\right)^2\right)$$

$$p(x) = p_{at} + \frac{1}{2}\rho U^2\left[1 - \frac{h_L^2}{h^2(x)}\right]$$

5) $$F_p = \cos\theta \int_0^x [p(x) - p_{at}]\,\ell\,dx = -\frac{\ell}{\theta}\int_{h_0}^{h_L} \frac{1}{2}\rho U^2\left(1 - \frac{h_L^2}{h^2}\right)dh$$

$$F_p = \frac{\rho U^2}{2\theta}\ell \int_{h_L}^{h_0}\left(1 - \frac{h_L^2}{h^2}\right)dh = \frac{\rho U^2 \ell}{2\theta}\left[h + \frac{h_L^2}{h}\right]_{h_L}^{h_0}$$

$$= \frac{\rho U^2 \ell}{2\theta}\left[h_0 - h_L + \frac{h_L^2}{h_0} - h_L\right]$$

or $\underline{h_0 - h_L = \theta L}$

$$= \frac{\rho U^2 \ell}{2\theta}(h_0 - h_L)\left[1 - \frac{h_L}{h_0}\right]$$

$$F_p = \frac{\rho U^2 L \ell}{2}\left(1 - \frac{h_L}{h_0}\right)$$

$F_p = C_p \frac{1}{2}\rho U^2 L \ell$ avec $C_p = 1 - \frac{h_L}{h_0} = \frac{\theta L}{h_0}$

6) $$F_T = \sin\theta \int_0^x [p(x) - p_{at}]\,\ell\,dx = \theta F_p$$

$$\frac{F_\ell}{F_N} = \frac{1}{\theta} \gg 1$$

7)

III Analyse dimensionnelle

1°/ $\cancel{\eta}$, ρ, U, L, ℓ, h_1, h_2 + F_p

$[F] = kg.m.s^{-2}$
$[\rho] = kg.m^{-3}$
$[U] = m.s^{-1}$
$[\ell] = m$
$[h_1] = m$
$[h_2] = m$
$[L] = m$

$7-3 = 4$ paramètres sans dimensions.

$\Pi_1 = F \rho^{\alpha} U^{\beta} h_1^{\gamma}$ $\qquad \Pi_1 = \frac{F}{\rho U^2 h_1^2}$

$\Pi_2 = \frac{L}{h_1}$ $\quad \Pi_3 = \frac{h_2}{h_1}$ $\quad \Pi_4 = \frac{\ell}{h_1}$

$F = \rho U^2 h_1^2 f\left(\frac{L}{h_1}, \frac{\ell}{h_1}, \frac{h_2}{h_1}\right)$ $\qquad$ si $F \propto L$ et $F \propto \ell$

$$\boxed{F = \rho U^2 L \ell\, f\Big(\underbrace{\frac{h_2}{h_1}}_{1+\theta}\Big)}$$

$\frac{h_2}{h_1} - 1 = \theta$

2°/ η, $\cancel{\rho}$, U, L, ℓ, h_1, h_2 + F_p

$\Pi_1 = F \eta^{\alpha} U^{\beta} h_1^{\gamma}$ $\qquad \Pi_1 = \frac{F}{\eta U h_1}$ $\qquad \Pi_2 = \frac{L}{h_1}$ $\quad \Pi_3 = \frac{h_2}{h_1}$ $\quad \Pi_4 = \frac{\ell}{h_1}$

$F = \eta U h_1 f\left(\frac{L}{h_1}, \frac{\ell}{h_1}, \frac{h_2}{h_1}\right)$ $\qquad$ si $F \propto \ell$

$F = \eta U \ell\, f\left(\frac{L}{h_1}, \frac{h_2}{h_1}\right)$

3°/ η, ρ, U, L, ℓ, h_1, h_2 + F_p.

5 paramètres sans dimensions

$$F = \underbrace{\eta U h_1}_{\text{ou } \rho U^2 h_1^2} f\left(\frac{L}{h_1}, \frac{\ell}{h_1}, \frac{h_2}{h_1}, Re\right)$$

avec $Re = \frac{\rho U h_1}{\eta}$

$L = 1\,m \quad \ell = 0{,}5\,m \quad U = 10\,m/s \quad h_1 = 1\,cm \quad h_2 = 2\,cm$

$$\theta = \frac{10^{-2}}{1} = 10^{-2}\,rad$$

$$\frac{F_p}{F_T} = 10^2$$

$$F_{p_v} = \frac{6\eta U \ell}{\theta^2} = \frac{6 \times 10^{-3} \times 10 \times 0{,}5}{10^{-4}} = 300\,N = 30\,kgf$$

$$F_{p_i} = \frac{1}{2}\rho U^2 L \ell = \frac{10^3 \times 10^2 \times 1 \times 0{,}5}{2} \simeq 3.10^4$$

$$\frac{F_{p_v}}{F_{p_i}} = \frac{1}{100}$$

$$Re = \frac{\rho U h}{\eta} = \frac{10^3 \times 10 \times 10^{-2}}{10^{-3}} = 10^5 >> 1$$

$$\frac{F_{p_i}}{F_{p_v}} = \frac{\rho U L}{\eta \theta^2} = \frac{\rho U h}{\eta} \frac{L}{h} \frac{1}{\theta^2}$$

$L = 1\,m \quad \ell = 0{,}5\,m \quad U = 1\,m/s \quad h_1 = 1\,mm \quad h_2 = 2\,mm$

$$\theta = \frac{10^{-3}}{1} = 10^{-3}\,rad$$

$$\frac{F_p}{F_T} = 10^3$$

$$F_{p_v} = \frac{6\eta U \ell}{\theta^2} = \frac{6 \times 10^{-3} \times 1 \times 0{,}5}{10^{-6}} = 3.10^3 = 300\,kgf$$

$$F_{p_i} = \frac{1}{2}\rho U^2 L \ell = \frac{10^3 \times 1 \times 1 \times 0{,}5}{2} = 3.10^2$$

$$\frac{F_{p_v}}{F_{p_i}} = 10$$

$$Re = \frac{\rho U h}{\eta} = \frac{10^3 \times 1 \times 10^{-3}}{10^{-3}} = 10^3 >> 1 \quad \text{mais}$$

$$Re = \frac{1}{\theta}$$

5. $\frac{d}{dt}\iiint_{V} \rho \vec{u}\, d\tau = \iiint_{V} \underbrace{\frac{\partial(\rho\vec{u})}{\partial t}}_{=0} d\tau + \oiint_S \rho \vec{u}(\vec{u}\cdot d\vec{S}) = \sum \vec{F}$

$$\oiint \rho\vec{u}(\vec{u}\cdot d\vec{S}) = -\rho U_A^2 h_A l + \rho U^2 h_1 l = \rho U$$

$$\sum \vec{F} = p_A h_A l - p_{at} h_1 l + F_T$$

$$F_T + p_A h_A l - p_{at} h_1 l = \rho U^2 h_1 l - \rho U_A^2 h_A l$$

or par Bernoulli $\frac{1}{2}\rho U_A^2 + p_A = \frac{1}{2}\rho U^2 + p_{at}$

$$F_T = l(\rho U^2 h_1 - \rho U_A^2 h_A - p_A h_A + p_{at} h_1)$$

or $U h_1 = U_A h_A$

$U^2 h_1^2 = U_A^2 h_A^2$

$$F_T = l\left(\rho U^2 h_1 - \rho U^2 h_1 \frac{h_1}{h_A} - p_A h_A + p_{at} h_1\right) \qquad h_A = h_1 + \theta L$$

$$= l\left[\rho U^2 h_1 \left(1 - \frac{h_1}{h_A}\right) - p_{at} h_A + \frac{1}{2}\rho U^2\left(h_A - \frac{h_1^2}{h_A}\right) + p_{at} h_1\right]$$

$$= l\left[\rho U^2 h_1 \left(1 - \frac{h_1}{h_A}\right) - p_{at}\theta L - \frac{1}{2}\rho U^2 h_A + \frac{1}{2}\rho U^2 \frac{h_1^2}{h_A}\right)$$

$$= l\left[\rho U^2 h_1 - \frac{1}{2}\rho U^2 h_A - p_{at}\theta L\right]$$

$$= l\rho U^2 (h_1 - h_A) \qquad \text{or } h_1 - h_A = \theta L$$

$$= \theta \rho U^2 L l$$