Correction effet Doppler

1°) $(K_0) = (\Lambda)(K)$ A 1D on peut écrire simplement

$$\begin{pmatrix} \frac{\omega_0}{c} \\ k_{0x} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \omega/c \\ k_x \end{pmatrix}$$

$\rightarrow \omega_0 = \gamma(\omega - \beta c k_x)$ et $k_x = k\cos\theta$ avec $k = \|\vec{k}\|$

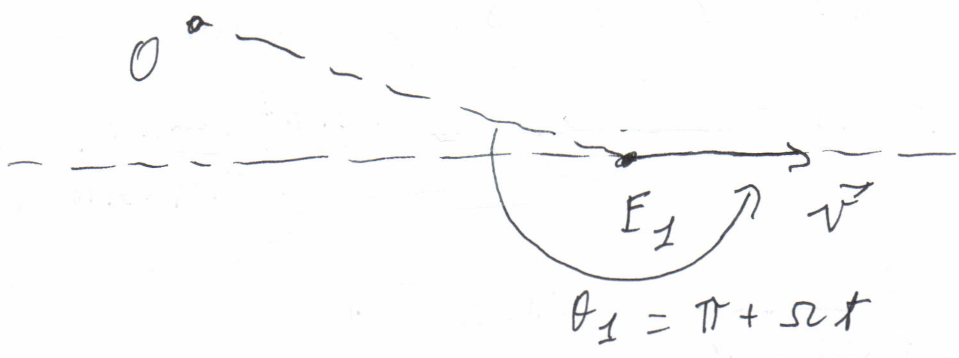
Dans le vide $\omega = kc$

$\rightarrow \omega_0 = \gamma\omega(1 - \beta\cos\theta) \rightarrow \omega = \dfrac{\omega_0}{\gamma(1-\beta\cos\theta)}$

et donc $\boxed{\nu = \dfrac{\nu_0}{\gamma(1-\beta\cos\theta)}}$ et $\boxed{\lambda = \gamma\lambda_0(1-\beta\cos\theta)}$

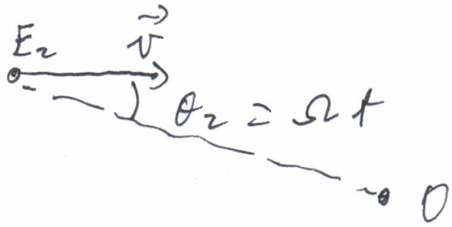
avec $\omega = 2\pi\nu = \dfrac{2\pi c}{\lambda}$

2°) E_1 s'éloigne de la Terre. On a donc la figure ci-dessous



$\rightarrow \lambda_1 = \gamma\lambda_0(1 - \beta\cos(\pi + \Omega t)) = \gamma\lambda_0(1 + \beta\cos\Omega t)$

Pour E_2 on a la situation suivante



$$\lambda_2 = \gamma \lambda_0 (1 - \beta \cos \Omega t)$$

$$\Delta \lambda (t) = \gamma \lambda_0 \, 2\beta \cos \Omega t = 2\gamma\lambda_0 \frac{\Omega r}{c} \cos \Omega t$$

3°/a) Lorsque les 2 étoiles sont alignées sur l'axe d'observation $\Omega t = \frac{\pi}{2}$, la vitesse des étoiles est perpendiculaire à l'axe d'observation et $\Delta\lambda = 0$. On observe la raie H_α affectée du facteur γ : $\lambda = \gamma \lambda_0$. La période de révolution est :

$$T = 2 \times (1{,}886 - 0{,}061) = 3{,}65 \text{ jours}$$

b) Lorsque les étoiles E_1 et E_2 se trouvent sur l'axe $\perp$ à l'axe d'observation, l'effet Doppler est maximal et $\Delta\lambda = \pm 2\gamma\lambda_0\beta$. Dans un cas on a $\lambda_1 < \lambda_2$ dans l'autre on a $\lambda_2 < \lambda_1$.

c) Lorsque $\Omega t = \frac{\pi}{2}$ on a $\lambda = \gamma\lambda_0 \simeq 6545$ Å

Lorsque $\Omega t = 0$ (ou π) $\Delta\lambda = 2\gamma\lambda_0\beta = 2\gamma\lambda_0 \frac{\Omega r}{c} \simeq 2$ Å

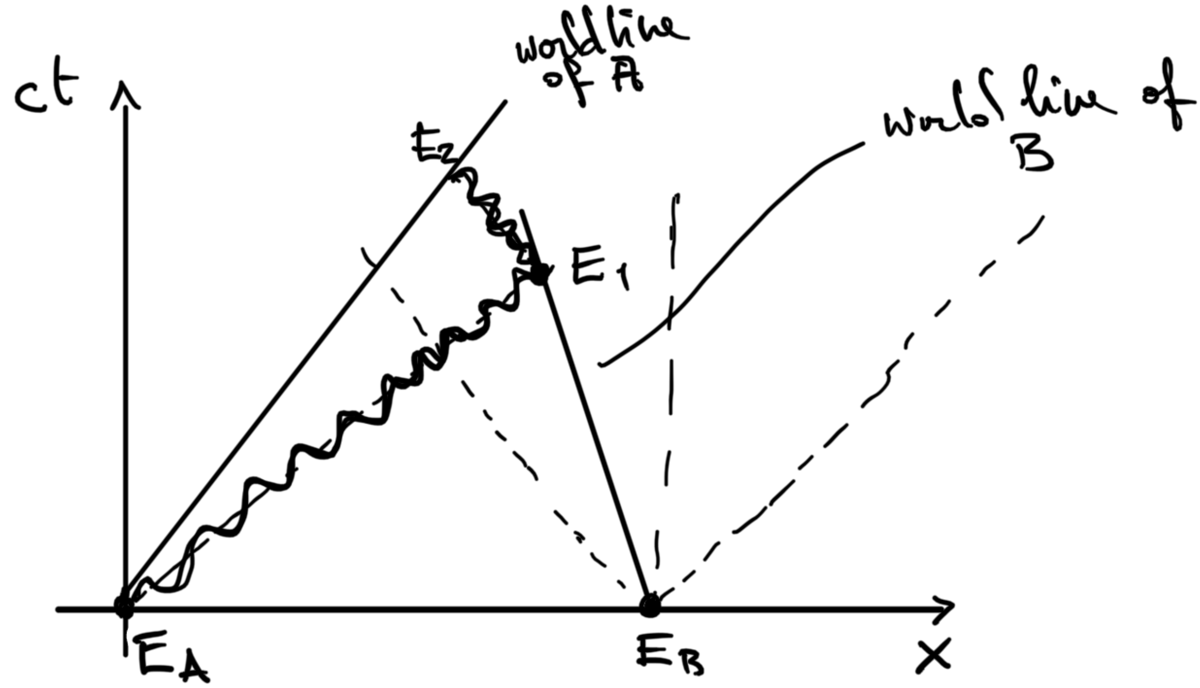
$$\rightarrow \frac{\Delta\lambda}{\lambda} = \frac{2\Omega r}{c} \rightarrow r = \frac{c}{2\Omega}\frac{\Delta\lambda}{\lambda} = \frac{3.10^5 \text{ (km/s)}}{2\Omega} \cdot \frac{2}{6545}$$

$$\text{avec } \Omega = \frac{2\pi}{3{,}65 \times 24 \times 3600} = 1{,}99.10^{-5} \text{ rd/s}$$

soit $r = 2{,}3 . 10^6$ km

Correction Space-time signal

(1) $E_A = (0,0)$ $E_B = (0, L)$



(3) Worldline of A

$$x_A = v_A t$$

Worldline of B

$$x_B = L - v_B t$$

Worldline of the first photon

$$x_{\nu_1} = ct$$

We determine E_1:

At E_1:

$$x_B = x_{\nu_1}$$

$$L - v_B t_{E_1} = c t_{E_1} \Rightarrow c t_{E_1} = \frac{L}{1+\beta_B}$$

$$E_1 = (ct_{E_1}, ct_{E_1}) = \frac{L}{1+\beta_B}(1,1)$$

The 2^{nd} photon has worldline

$$x_{\nu_2} = ct_{E_1} - c(t - t_{E_1})$$

We can determine E_2

$$x_{\nu_2}(t_{E_2}) = v_A t_{E_2} = x_A(t_{E_2})$$

$$v_A t_{E_2} = c[2t_{E_1} - t_{E_2}]$$

$$ct_{E_2} = 2ct_{E_1} = \frac{2L}{\quad}$$

$$\frac{}{1+\beta_A} \qquad (1+\beta_A)(1+\beta_B)$$

At that time we have

$$x_{E_2} = v_A t_{E_2} = \frac{2\beta_A L}{(1+\beta_A)(1+\beta_B)}$$

$$E_2 = \frac{2L}{(1+\beta_A)(1+\beta_B)} \left(\overset{1}{\cancel{\beta_A}} , \overset{\beta_A}{\cancel{1}} \right)$$

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We need now to Lorentz transform to R'

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma_A \begin{pmatrix} 1 & -\beta_A \\ -\beta_A & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$E_1' = \gamma_A (1-\beta_A)\, c t_{E_1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \gamma_A (1-\beta_A) \frac{L}{1+\beta_B} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$E_1' = \gamma_A \frac{(1-\beta_A)}{1+\beta_B} \begin{pmatrix} 1 \\ 1 \end{pmatrix} L$$

$$E_2' = \frac{2L\gamma_A}{(1+\beta_A)(1+\beta_B)} \begin{pmatrix} 1-\beta_A^2 \\ 0 \end{pmatrix}$$

$$E_2' = \frac{2L\gamma_A}{(1+\beta_A)(1+\beta_B)} \begin{pmatrix} 1-\beta_A^2 \\ 0 \end{pmatrix} = 2L\gamma_A\left(\frac{1-\beta_A}{1+\beta_B}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

5. $E_1' = (c\tau', c\tau')$ with

$$c\tau' = \gamma_A \frac{(1-\beta_A)}{1+\beta_B} L$$

If a photon travel a length ℓ in a time $c\tau = \ell$ in R it will travel a length

$c\tau' = \ell' = \gamma_A(1-\beta_A)\ell$ in a time $c\tau'$ in R'.

Since $\ell = \frac{L}{1+\beta_B}$ we find

$$c\tau' = \frac{\gamma_A(1-\beta_A)L}{1+\beta_B}$$

- τ' is the flight time of the photon in R'

6. $|E_1| = |E_1'| = 0$ light

(7) $x_B = L - v_B t$

$$\begin{pmatrix} ct' \\ x'_B \end{pmatrix} = \gamma_A \begin{pmatrix} 1 & -\beta_A \\ -\beta_A & 1 \end{pmatrix} \begin{pmatrix} ct \\ L - v_B t \end{pmatrix}$$

$$\begin{cases} ct' = \gamma_A (ct - \beta_A L + \beta_A \beta_B ct) \\ x'_B = \gamma_A (L - (\beta_A + \beta_B) ct) \end{cases}$$

$$\frac{ct'}{\gamma_A} + \beta_A L = (1 + \beta_A \beta_B) ct$$

$$\Rightarrow ct = \frac{\beta_A L}{1+\beta_A\beta_B} + \frac{ct'}{\gamma_A(1+\beta_A\beta_B)}$$

$$x'_B = \gamma_A \left[L - \frac{\beta_A(\beta_A+\beta_B)L}{1+\beta_A\beta_B} \right] - \frac{(\beta_A+\beta_B)}{1+\beta_A\beta_B} ct'$$

$$= \gamma_A L \frac{1-\beta_A^2}{1+\beta_A\beta_B} - \frac{\beta_A+\beta_B}{1+\beta_A\beta_B} ct$$

(8)

$$E'_A = (0,0)$$

$$E'_B = \gamma_A \begin{pmatrix} 1 & -\beta_A \\ -\beta_A & 1 \end{pmatrix} \begin{pmatrix} 0 \\ L \end{pmatrix}$$

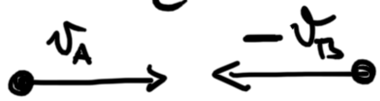
$$E'_B = \gamma_A (-\beta_A L, L)$$

not simultaneous.

(9) $$\begin{pmatrix} c\,dt' \\ dx' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} c\,dt \\ dx \end{pmatrix} = \gamma \begin{pmatrix} c\,dt - \beta dx \\ dx - \beta c\,dt \end{pmatrix}$$

$$\frac{dx'}{c\,dt'} = \frac{dx - \beta c\,dt}{c\,dt - \beta dx} = \frac{v_x/c - \beta}{1 - \beta \frac{v_x}{c}}$$

$$\frac{v'_x}{c} = \frac{v_x/c - \beta}{1 - \beta v_x/c}$$



(10)

We use this transformation with

$v_x \longrightarrow -v_B$

$\beta \longrightarrow +\beta_A$

$$-v'_B = -\frac{v_B + v_A}{1 + \beta_A b c}$$

$$x'_B = \gamma_A L - \frac{v_B + v_A}{1+\beta_A\beta_B}\left(t + \frac{v_A}{c^2} L \gamma_A\right)$$

$$= \gamma_A L \left[\frac{1 + \cancel{\beta_A\beta_B} - \cancel{\beta_A\beta_B} - \beta_A^2}{1+\beta_A\beta_B}\right] - \frac{v_B+v_A}{1+\beta_A\beta_B}\, t$$

which coincides with the previous expression.

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