MQ PARTIEL OCT 2021

$\boxed{\text{OP en MQ}}$

1. $P_u = |u\rangle\langle u| = \frac{1}{2}\begin{pmatrix} 1 \\ -i \end{pmatrix}^{(1\ i)} = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

2. $P_u^+ = P_u$ OK

3. $\begin{vmatrix} \frac{1}{2}-\lambda & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2}-\lambda \end{vmatrix} = \left(\frac{1}{2}-\lambda\right)^2 - \frac{1}{4} = -\lambda(1-\lambda)$ $\to \lambda = 0$, $\to \lambda = 1$

$\lambda = 1 \to |u\rangle$

$\lambda = 0 \to |v\rangle \perp |u\rangle = \cancel{\frac{1-i}{2}} = \frac{|1\rangle - i|2\rangle}{\sqrt{2}}$

par ex.

$\boxed{\text{La Mesure en MQ}}$

1. a et b

2. $P(a) = |\langle a|\psi\rangle|^2 = |\alpha|^2$

$P(b) = |\langle b,1|\psi\rangle|^2 + |\langle b,2|\psi\rangle|^2 = |\beta_1|^2 + |\beta_2|^2$

3 si $a \to |a\rangle$

si $b \to \beta_1|b,1\rangle + \beta_2|b,2\rangle$ + renormalisation

$\hookrightarrow \dfrac{\beta_1|b,1\rangle + \beta_2|b,2\rangle}{\sqrt{|\beta_1|^2 + |\beta_2|^2}}$

4) $\langle A\rangle = \langle\psi|A|\psi\rangle = |\alpha|^2 a + \left(|\beta_1|^2 + |\beta_2|^2\right) b$

DOUBLE PUITS QUANTIQUE

Dble puits sym.

1. $H = H_0 + W = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$

2. $\begin{vmatrix} E_0 - E & -A \\ -A & E_0 - E \end{vmatrix} = (E_0 - E)^2 - A^2$

$= (E_0 - A - E)(E_0 + A - E)$

$\Rightarrow E_\pm = E_0 \pm A$

3. $E_+ \to |+\rangle = \dfrac{|\psi_g\rangle - |\psi_d\rangle}{\sqrt{2}}$; $E_- \to |-\rangle = \dfrac{|\psi_g\rangle + |\psi_d\rangle}{\sqrt{2}}$

4) $|\psi(0)\rangle = |\psi_g\rangle = \dfrac{|+\rangle + |-\rangle}{\sqrt{2}}$

$\hookrightarrow |\psi(t)\rangle = \dfrac{e^{-\frac{iE_+ t}{\hbar}} |+\rangle + e^{-\frac{iE_- t}{\hbar}} |-\rangle}{\sqrt{2}}$

$= e^{-\frac{iE_0 t}{\hbar}} \dfrac{e^{-\frac{iAt}{\hbar}} |+\rangle + e^{+\frac{iAt}{\hbar}} |-\rangle}{\sqrt{2}}$

5) $P_d = |\langle \psi_d | \psi(t)\rangle|^2$

$= \frac{1}{2} \left| e^{-\frac{iAt}{\hbar}} \langle \psi_d | + \rangle + e^{\frac{iAt}{\hbar}} \langle \psi_d | - \rangle \right|^2$

avec $\langle \psi_d | + \rangle = -1/\sqrt{2}$ et $\langle \psi_d | - \rangle = \frac{1}{\sqrt{2}}$:

$P_d = \frac{1}{4} \left| -e^{-iAt/\hbar} + e^{iAt/\hbar} \right|^2$

$P_d = \sin^2\left(\frac{At}{\hbar}\right)$

si $\frac{At}{\hbar} = \frac{\pi}{2} (\pi)$ on a $P_d = 1$

Dble puits asymétrique

6 - $H' = H_0' + W = \begin{pmatrix} E_g & -A \\ -A & E_d \end{pmatrix}$

7 - $\begin{vmatrix} E_g - E & -A \\ -A & E_d - E \end{vmatrix} = (E_g - E)(E_d - E) - A^2$

$$= E_g E_d - (E_g + E_d)E + E^2 - A^2$$

$$= E^2 - (E_g + E_d)E + E_g E_d - A^2$$

$$\hookrightarrow E'_\pm = \frac{1}{2}\left(E_g + E_d \pm \sqrt{(E_g - E_d)^2 + 4A^2} \right)$$

8 - $E'_\pm = \frac{1}{2}\left(E_g + E_d \pm |E_g - E_d| \sqrt{1 + \frac{4A^2}{(E_g - E_d)^2}} \right)$

$$\simeq \frac{1}{2}\left(E_g + E_d \pm |E_g - E_d| + \frac{2A^2}{|E_g - E_d|} \right)$$

$E'_\pm$ en A^2 $E_\pm$ en A !

si A petit, puits asym - sensible au couplage tunnel que le puits sym.

9) $|\psi(t=0)\rangle = |\psi_g\rangle = \cos(\frac{\theta}{2})|+'\rangle - \sin(\frac{\theta}{2})|-'\rangle$

$$\hookrightarrow |\psi(t)\rangle = \cos(\frac{\theta}{2})\, e^{-\frac{iE_+ t}{\hbar}} |+'\rangle - \sin(\frac{\theta}{2})\, e^{-\frac{iE_- t}{\hbar}} |-'\rangle$$

10) $P_d = |\langle \psi_d | \psi(t) \rangle|^2$

$$= \left| \cos(\frac{\theta}{2}) e^{-\frac{iE_+ t}{\hbar}} \langle \psi_d | +' \rangle - \sin(\frac{\theta}{2}) e^{-\frac{iE_- t}{\hbar}} \langle \psi_d | -' \rangle \right|^2$$

avec $\langle \psi_d | +' \rangle = \sin\frac{\theta}{2}$ et $\langle \psi_d | -' \rangle = \cos\frac{\theta}{2}$

$$\Rightarrow P_d = \left| \sin\frac{\theta}{2} \cos\frac{\theta}{2} \right|^2 \left| e^{-\frac{iE_+ t}{\hbar}} - e^{-\frac{iE_- t}{\hbar}} \right|^2$$

$$= \frac{1}{4} \sin^2 \theta \times 4 \sin^2 ((E_+ - E_-) t / 2\hbar)$$

$$= \sin^2 \theta \, \sin^2 \frac{(E_+ - E_-) t}{2\hbar}$$

$$\sin^2\theta = \frac{\tan^2\theta}{1+\tan^2\theta} = \frac{4A^2/\Delta E^2}{1+\frac{4A^2}{\Delta E^2}} = \frac{4A^2}{4A^2+\Delta E^2}$$

et $E_+ - E_- = \sqrt{\Delta E^2 + 4A^2}$

$$\Rightarrow P_d(t) = \frac{4A^2}{4A^2+\Delta E^2}\sin^2\frac{\sqrt{\Delta E^2+4A^2}\,t}{2\hbar}$$

en g^al $P_{d_{MAX}} = \frac{4A^2}{4A^2+\Delta E^2} < 1$ donc jamais 100%

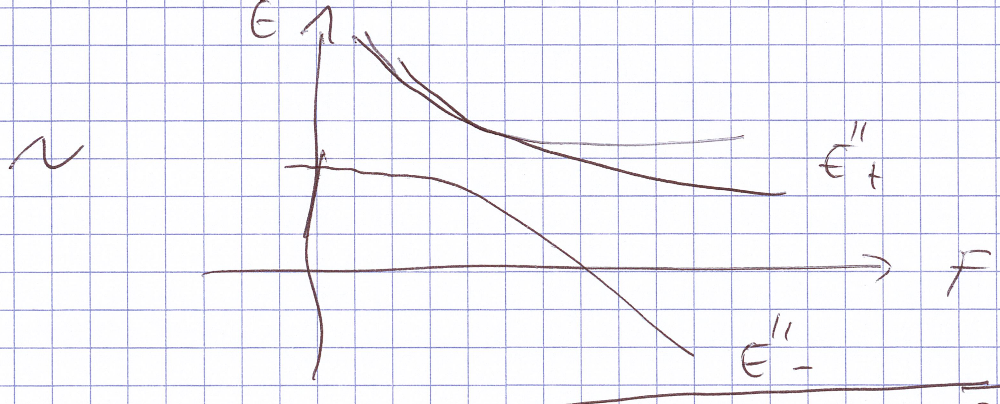
sûr de trouver l'e⁻ ds l'état $|\Psi d\rangle$

11) $$H'' = \begin{pmatrix} E_g+\alpha_g F & -A \\ -A & E_d+\alpha_d F \end{pmatrix}$$

chg^t $E_g \to E_g + \alpha_g F$ et $E_d \to E_d + \alpha_d F$

d'où $E''_\pm = \frac{1}{2}\left(E_g+\alpha_g F + E_d + \alpha_d F \pm \sqrt{(E_g+\alpha_g F - E_d - \alpha_d F)^2 + 4A^2}\right)$

12) 2 branches d'hyperboles



13) $\delta = E''_+ - E''_- = \sqrt{\left(E_g - E_d + (\alpha_g - \alpha_d)F\right)^2 + 4A^2}$

$\sqrt{x^2+\alpha^2}$ min si $x = 0$

↳ δ min si $\left(E_g - E_d + (\alpha_g - \alpha_d)F\right)^2 = 0$

i.e. si $E_g + \alpha_g F = E_d + \alpha_d F$

~~14)~~ δ. et alors $\delta_{min} = 2A$

14) "Freq de Bohr" (ou plutôt pulsation)

$$\omega_{Bohr} = \frac{E_+ - E_-}{\hbar} = \frac{\delta}{\hbar} = \frac{2A}{\hbar} \text{ à résonance}$$

15) Dipôle oscillant -> rayonnement

$$\lambda = \frac{c}{\nu} = \frac{2\pi c}{\omega} = \frac{2\pi \hbar c}{2A}$$

$$\Rightarrow A = \pi \frac{\hbar c}{\lambda}$$

AN $A \simeq 3 \times \frac{200 \text{ MeV fm}}{10^{-4} \cancel{m}} = 3 \times \frac{200\ 10^6 \text{ eV fm}}{10^{-11} \text{ fm}}$

$$A \simeq 6 \times 10^{-3} \text{ eV}$$

Question subsidiaire :

énergies liaisons typiques ds conducteurs

ou 1/2 conducteurs $\gtrsim$ eV

↳ couplage faible.