MATHS FOR DATA SCIENCE

CLASS 4 - EXERCISES

EXERCISE 1

LET (λ_i, v_i), $i = 1, ..., m$ BE PAIRS OF EIGENVALUES AND EIGENVECTORS OF A $m \times m$ MATRIX A, WITH $\{v_1, ..., v_m\}$ LINEARLY INDEPENDENT.

IDENTIFY EIGENVALUES AND EIGENVECTORS OF $A^2, \dots, A^k$ $k \in \mathbb{N}$.
ARE THEY DIAGONALIZABLE?

EXERCISE 2

LET A BE A $m \times m$ MATRIX.
1) SHOW THAT $\mathrm{Ker}(A^T A) = \mathrm{Ker}(A)$.
2) USING 1), SHOW THAT $\mathrm{rank}(A^T A) = \mathrm{rank}(A)$.

EXERCISE 3

1) SHOW THAT IF A IS SYMMETRIC POSITIVE SEMIDEFINITE, THEN ALL THE EIGENVALUES OF A ARE NON NEGATIVE (≥ 0).

2) SHOW THAT IF A IS SYMMETRIC POSITIVE DEFINITE, THEN ALL THE EIGENVALUES OF A ARE STRICTLY POSITIVE (> 0).