MATHS FOR DATA SCIENCE

CLASS 1 - EXERCISES

EXERCISE 1

FOR $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $w = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

DESCRIBE GRAPHICALLY ALL POINTS cv WITH

a) c BEING AN INTEGER, THAT IS, $c \in \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2 \ldots\}$

b) $c \in \mathbb{R}$, WITH $c \geq 0$.

DESCRIBE $cv + dw$ WHERE $d \in \mathbb{R}$ AND c IS LIKE IN a) OR b).

EXERCISE 2

IS $z = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ IN THE SPAN OF $x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ AND $y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$?

IF SO, FIND $\alpha, \beta \in \mathbb{R}$ SUCH THAT $z = \alpha x + \beta y$.

EXERCISE 3

1) PROVE THAT $u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $u_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, $u_3 = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ ARE LINEARLY INDEPENDENT.

2) IS $\{u_1, u_2, u_3\}$ A BASIS OF $\mathbb{R}^3$?

EXERCISE 4

CONSIDER THE FOLLOWING TRANSFORMATIONS

$L_1 : \mathbb{R}^2 \to \mathbb{R}^2, \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y \\ x \end{pmatrix},$

$L_2 : \mathbb{R}^2 \to \mathbb{R}^2, \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ y \end{pmatrix},$

$L_3 : \mathbb{R}^3 \to \mathbb{R}^3, \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} z+y \\ z-y \\ 0 \end{pmatrix}$

ARE THEY LINEAR?
JUSTIFY YOUR ANSWERS

EXERCISE 5

- PROVE THAT IF $T : U \to V$ IS A LINEAR TRANSFORMATION, THEN $T(0_U) = 0_V$.

- PROVE THAT A LINEAR TRANSFORMATION $T : U \to V$ IS INJECTIVE IF AND ONLY IF $\ker(T) = \{0_U\}$.