Hints for Homework (tutorial) 7

Exercise I:

1.

a) Find the principal plane. How are $\mathbf{d}'$ and $\mathbf{d}''$ related to the principal plane?

b) Recall: x and y components propagate at different velocities.

c) For what types of wave plates will an initial linear polarization remain linear? (There are *two* types, one of which was not (or hardly) discussed in class but is “obvious”). Once the appropriate phase change is found, use the answer to b) to find *d.*

d) For the type of wave plate *not* discussed in class, there is no change in the orientation of the linear polarization.

Answers: a) $\mathbf{d}'$ is into the page, $\mathbf{d}''$ is along the entrance face. b) $\varphi = \frac{2\pi}{\lambda} \; n_e - n_o \; d$

c) half wave and *full* wave plates d) half wave plate 2α rotation.

2.

a) See I.1.d)

b) What is the effect of reflection on the polarization? The wave plate “doesn’t care” about the propagation direction, it adds the same extra phase difference between the components of **D** for each passage. What is the total phase change after passing twice through a half wave plate?

3.

a) What are the conditions on the amplitudes and phase difference of the components of **D** for circularly polarized light?

b) c) Recall that the handedness of light is defined *with respect to the direction of propagation.*

d) See 2.b)

e) What is the orientation of the polarization of the reflected and transmitted light as compared to the polarizer?

Exercise II:

1. Jones vectors and matrices

  a) See class notes.
  b) Recall the Jones matrix for an ideal polarizer (y-axis transmission=0); for this partial polarizer, the y-axis *intensity* transmission is ε. What is the y-axis *field* transmission? Where does this value go in the Jones matrix for a partial polarizer?
  c) Let n' correspond to the x-axis. What can you say about the relative values of n' and n" ?
  d) Rotate! (See lecture notes).
  e) Let ϕ=90° in answer for d). f) Let ϕ=180° in answer for d). g) Careful with order. Recall order of matrices in matrix optics.

Answers: b) $P = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\varepsilon} \end{pmatrix}$ c) $\mathbf{M} = \begin{pmatrix} e^{-i\frac{\varphi}{2}} & 0 \\ 0 & e^{i\frac{\varphi}{2}} \end{pmatrix}$;

d) $\mathbf{M}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\frac{\varphi}{2}} & 0 \\ 0 & e^{i\frac{\varphi}{2}} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

e) $\mathbf{M}_\theta(\varphi = \frac{\pi}{2}) = e^{-\frac{\pi}{4}} \begin{pmatrix} \cos^2\theta + i\sin^2\theta & 1-i\ \sin\theta\cos\theta \\ 1-i\ \sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$

f) $\mathbf{M}_\theta(\varphi = \pi) = -i \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ g) $M_{tot}=M_2M_1$.

2. Cavity

  a) Recall: wave plates “don’t care” about propagation direction.
  b) Find eigenvectors and eigenvalues (λ_i) of matrix $\mathbf{M}_{\mathbf{cavity}}$. For resonance, accumulated phase after a round trip is a multiple of 2π, i.e. $kL_{cav} + \arg\ \lambda_i\ = 2m\pi$
  c) Align incident polarization along an eigenvector!
  d) and e) Use the appropriate matrix elements.
  f) To find field: add the Jones vectors for fields in both directions (left-to-right and right-to-left).
  g) Think about the different phase delays.

Answers:

a) $\mathbf{M}_{cavity} = \mathbf{M}(\varphi = \frac{\pi}{2}, \theta = \frac{\pi}{4})\mathbf{M}(\varphi = \pi, \theta = \rho)\mathbf{M}(\varphi = \frac{\pi}{2}, \theta = \frac{\pi}{4})$;

$\mathbf{M}(\varphi = \frac{\pi}{2}, \theta = \frac{\pi}{4}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$ (Use this version for mathematical simplicity).

$\mathbf{M}_{cavity} = -i\begin{pmatrix} e^{-2i\rho} & 0 \\ 0 & -e^{2i\rho} \end{pmatrix}$ (Note that versions with an extra phase factor are also correct).

b) $\mathbf{u}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{u}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; $\lambda_x = e^{-\left(2i\rho + \frac{\pi}{2}\right)}$; $\lambda_y = e^{+\left(2i\rho + \frac{\pi}{2}\right)}$; (absolute phase unknown)

$\nu_{x,m} = \left[m + \frac{1}{4} + \frac{\rho}{\pi}\right]\frac{c}{L_{cav}}$; $\nu_{y,m} = \left[m - \frac{1}{4} - \frac{\rho}{\pi}\right]\frac{c}{L_{cav}}$; d), e) both right circular in the usual convention.

f) $\mathbf{E} = 2\sqrt{2}\begin{bmatrix} \cos kz \\ \sin kz \end{bmatrix}\cos \omega t$ g) Linear polarization with 45° angle to wave plate axes.