Hints for Homework (tutorial) 6

Exercise 1:

1. Find the principal plane. How are $\mathbf{d}'$ and $\mathbf{d}''$ related to the principal plane? Draw C, the cross-section of the index ellipsoid in the principal plane, add $\mathbf{u}$, the wave vector direction and $\mathbf{OM}''$, in the direction of $\mathbf{d}''$, with M'' on C. Find the equation for the ellipse C. Find the coordinates of M'' in terms of n'' and the angle $\mathbf{u}$ makes with the optic axis. Use the above to solve for n''.

Answers: $\mathbf{d}'$ is into the page, $\mathbf{d}''$ is along the entrance face. $n'' = 1.57$

2. Recall: **D,B,k** form an orthogonal triad, as do **E,B,** and **S.** Recall that **E** is normal to the index ellipsoid at M'' and **S** is tangential.

Exercise 2:

1. and 2. Find the wave normal surfaces in the incidence plane either using geometrical arguments or the normal surface equation with x=0. What is n'' equal to? Use Snell's law to find the angles of refraction.

Answers: r_0=27.256°; r_e=27.085°

3. Find the wave normal surfaces again using geometric arguments or the normal surface equation, this time with z=0. Finding n'' is more difficult this time. Let OP'' be in the direction of the refracted extraordinary ray, with the point P'' on the elliptically-shaped wave normal surface cross-section. Write an expression for n'' in terms of the coordinates of P''. Find an expression for the x coordinate of P'' (i.e., in the direction of the interface) from Snell's law. Use these two expressions plus the equation for the elliptically-shaped wave normal surface cross-section to find n''. The angle of refraction is found from Snell's law.

Answers: ordinary ray—as above; $n'' = 1.5511$; r_e=27.121°

Exercise 3

1. Use the same arguments as Exercise 1 to find the polarization directions. Is there refraction for normal incidence?
2. The direction of a *ray* is the direction of *energy propagation.* What can you say right away about the ordinary ray? To find the direction of the extraordinary ray

there are several possibilities; you can (1) use the fact that the Poynting vector **S** is perpendicular to the wave normal surface in the direction of **u**; (2) or that **S** is tangent to the cross section of the index ellipsoid in the principle plane at the point M'' (see Exercise 1 for the definition of M''); (3) or that the electric field of the extraordinary wave is normal to the index ellipsoid at the point M'', and **S, E,** and **B** form an orthogonal triad. Draw careful diagrams. Suggestion (2) is the closest to what was done in class.

Answers: angle between ordinary and extraordinary rays $\rho = 45° - \tan^{-1}\frac{n_e^2}{n_o^2} = 6.2°$.

Distance separating the rays: $\delta = w\tan\rho = 5.5\ \mathrm{mm}$