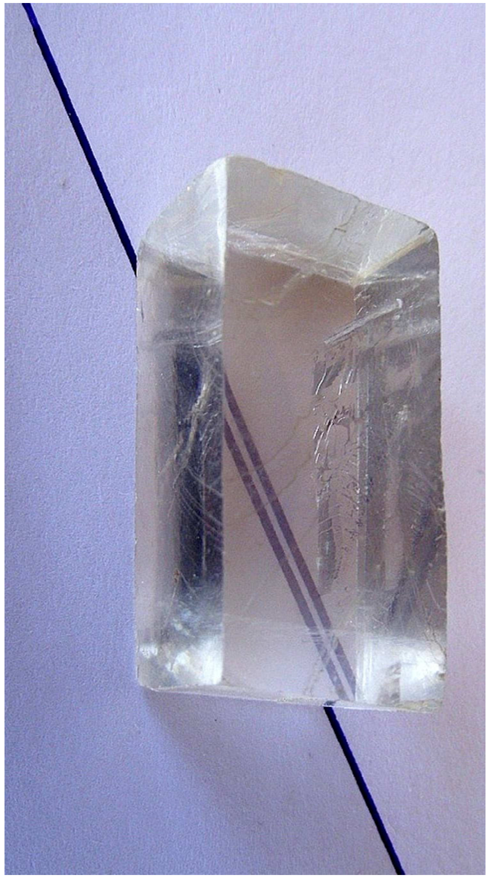
Calcite

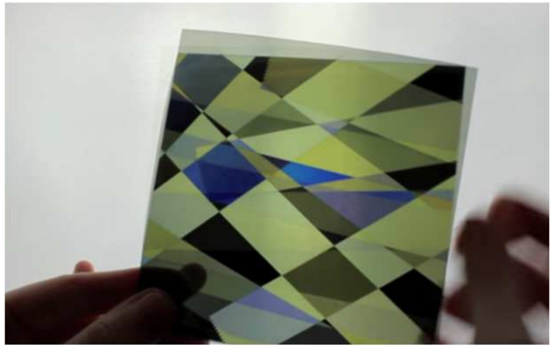


- Experiment:
  - Draw a black dot on a piece of paper
  - Place calcite crystal on top
  - Rotate crystal

  => <u>observations</u>
  - Look through polarizered sunglasses at crystal
  - Rotate crystal or sunglasses

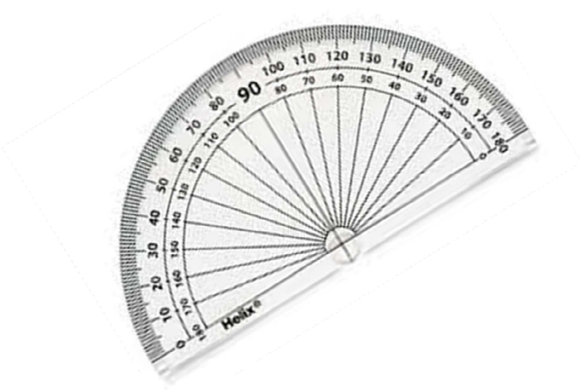
  => <u>observations</u>

Experiments



- Experiments:
  - Place Scotch tape between crossed polarizers
  - Rotate Scotch tape

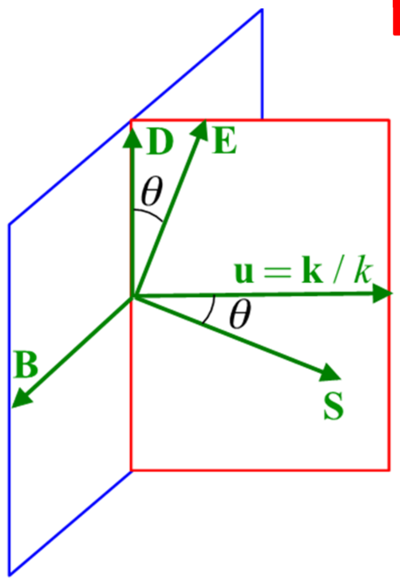
  ⇒ <u>observations</u>



- Place a clear plastic object (e.g. protractor) between crossed polarizers

=> <u>observations</u>

Recall lecture 6



$\vec{D} = \epsilon \vec{E}$

In an anisotropic medium:

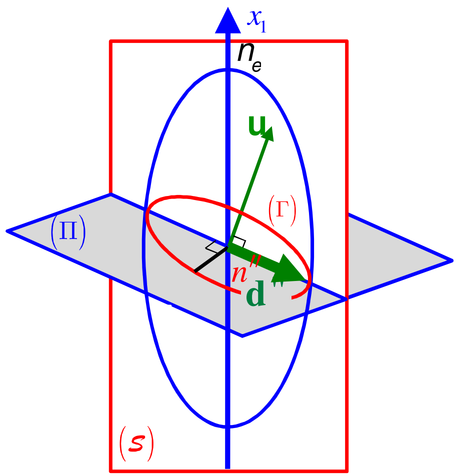
$$\boxed{\mathbf{D} = \varepsilon_0 \quad \mathbf{E}}$$

Tensor!

- Wave vector **k** is (in general) **NOT** in the same direction as the **propagation of energy** $\vec{S} = \vec{E} \times \vec{E}$

- **Two waves**! unless…

- Graphical method for finding indices and polarizations



$\hat{u} = \dfrac{\vec{k}}{|\vec{k}|}$

$\boxed{\hat{u} \perp \vec{D}}$

$n', n'', \vec{d}', \vec{d}''$

→ 1 seul axe optique

Propagation in a uniaxial crystal: arbitrary **u**

$x_i = n \dfrac{\vec{D}}{|\vec{D}|}$

(S) principal plane: formed by **u** and the optic axis

$n_2 = n_3 \equiv n_o$ "ordinary" index

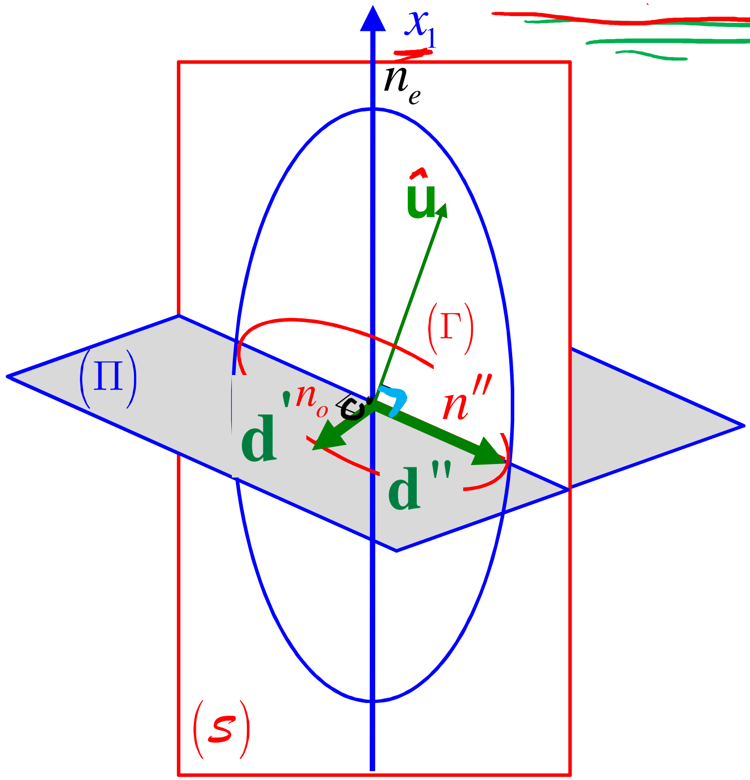
$n_1 \equiv n_e$ "extraordinary" index

x_1: optic axis

$$1 = \frac{x_1^2}{n_e^2} + \frac{x_2^2}{n_o^2} + \frac{x_3^2}{n_o^2}$$

ellipsoïde des indices

- The index ellipsoid is *symmetrical* about the principal plane
- Plane Π: normal to **u**
- Resulting elliptical cross-section Γ: *symmetrical* about (S)
- Semi-axes of Γ:
  - One *perpendicular* to (S): n_o, $\mathbf{d}'$
  - One *parallel* to (S): n'', $\mathbf{d}''$



Propagation in a uniaxial crystal

- Semi-axes of Γ:
  - One *perpendicular* to (S) : n_o, $\mathbf{d}'$
  - One *parallel* to (S) : n'', $\mathbf{d}''$

$n_2 = n_3 \equiv n_o$ "ordinary" index
$n_1 \equiv n_e$ "extraordinary" index

x_1: optic axis

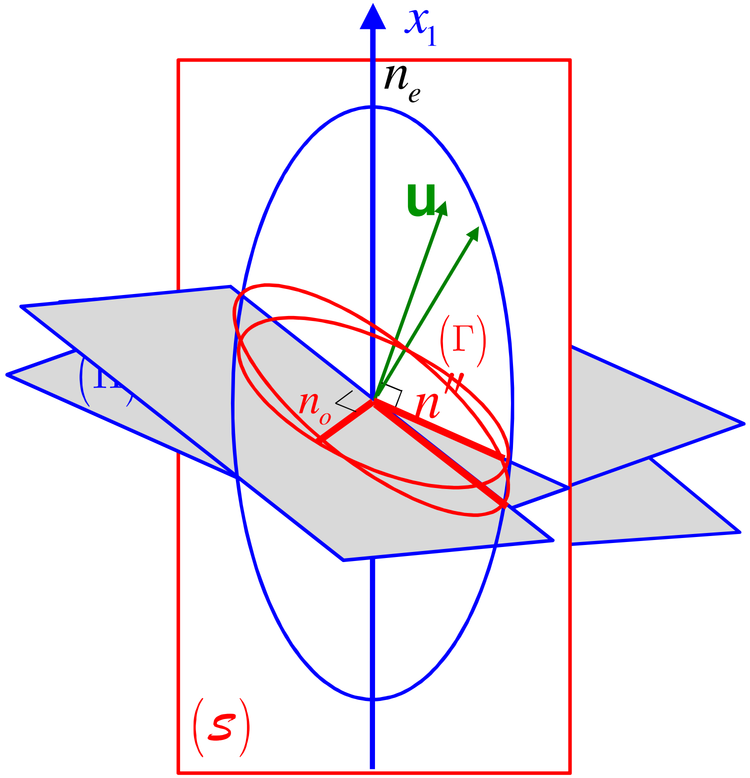
$$1 = \frac{x_1^2}{n_e^2} + \frac{x_2^2}{n_o^2} + \frac{x_3^2}{n_o^2}$$

- No matter how **u** is tilted, *perpendicular* semi-axis always has the *same length and direction*

Ordinary wave

- The length and direction of the *parallel* semi-axis *depends on* **u**

Extraordinary wave



$$\frac{x_1^2}{n_e^2} + \frac{x_2^2}{n_o^2} + \frac{x_3^2}{n_o^2} = 1$$

Propagation in a uniaxial crystal

determinés par cristal

$n_2 = n_3 \equiv n_o$ "ordinary" index

$n_1 \equiv n_e$ "extraordinary" index

x_1: optic axis

Ordinary wave

- **d'** is perpendicular to (S)
- $n' = n_o$ no matter the orientation of **u**
- **d'** is in the (x_2, x_3) plane, and $\varepsilon_{r2} = \varepsilon_{r3} = n_o^2$

$$\Rightarrow \quad D_2 = \varepsilon_0 n_o^2 E_2; \quad D_3 = \varepsilon_0 n_o^2 E_3$$

⇓

D and E are parallel!

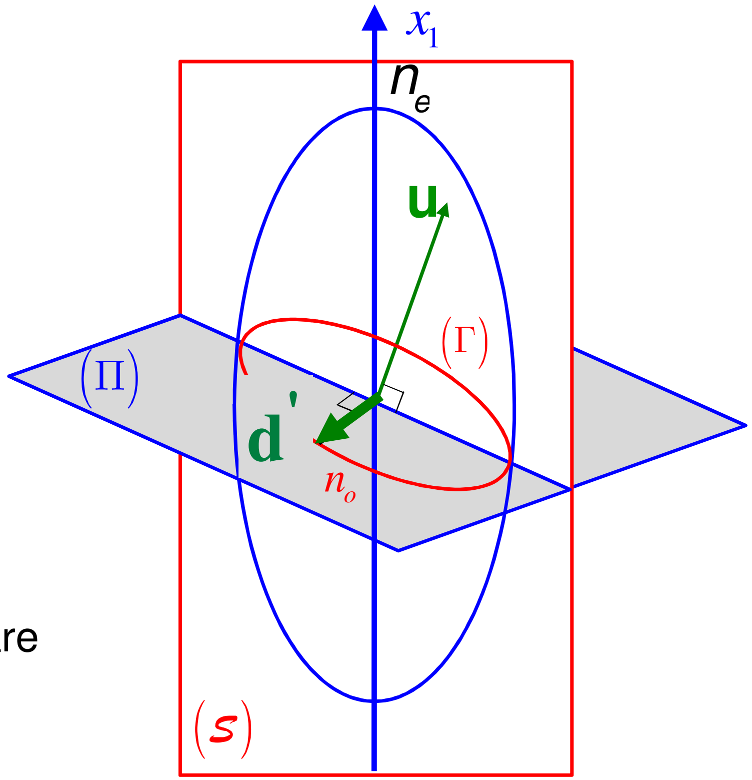
⇓

$\vec{k}$

Poynting vector **S** and the wave normal **u** are also parallel

⇓

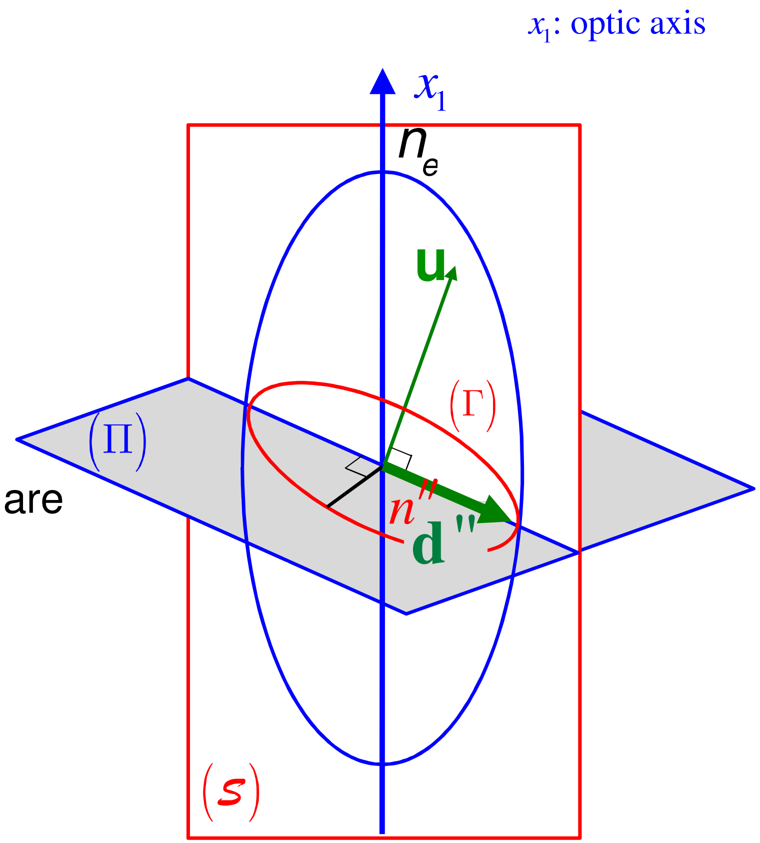
Wave behaves as if it is in an isotropic medium!



Propagation in a uniaxial crystal

Extraordinary wave

- **d"** is in the plane $(\mathcal{S})$
- $n"$ <u>depends</u> on the orientation of **u**
- **D** and **E** are **NOT** parallel
- Poynting vector **S** and the wave normal **u** are **NOT** parallel
- Wave does NOT behave as if it is in an isotropic medium!



Propagation in a uniaxial crystal: determining n''

$n' = n_o$

- *C:* cross-section of the index ellipsoid in the principal plane
- n''*:* length of the semi-axis of the ellipse (Γ)

(Γ) is in the plane perpendicular to **u**

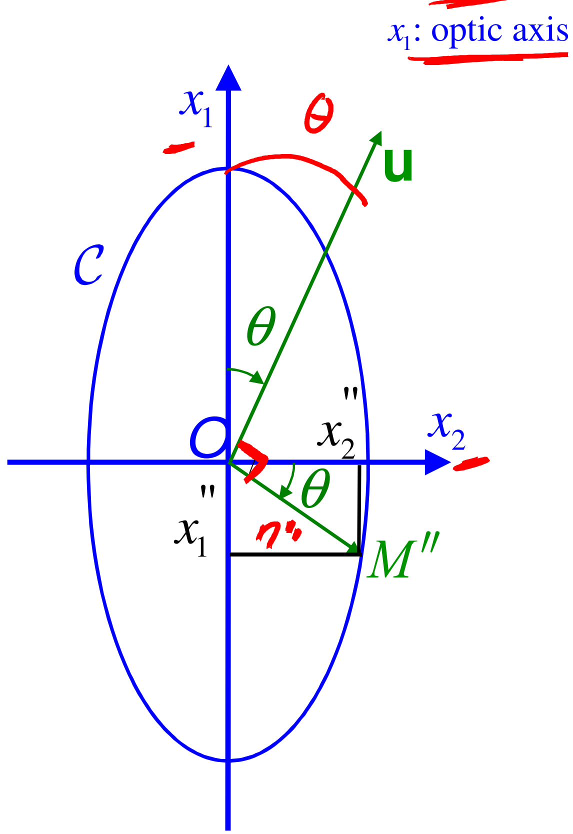
⇓

length of the vector **OM''**

coordinates of **OM''**: $\boxed{x_2'' = n''\cos\theta\ ;\ x_1'' = n''\sin\theta}$ (A)

equation of ellipse *C:* $\boxed{1 = \dfrac{x_1^2}{n_e^2} + \dfrac{x_2^2}{n_o^2}}$ (B)

Plug (A) into (B) and solve for n''

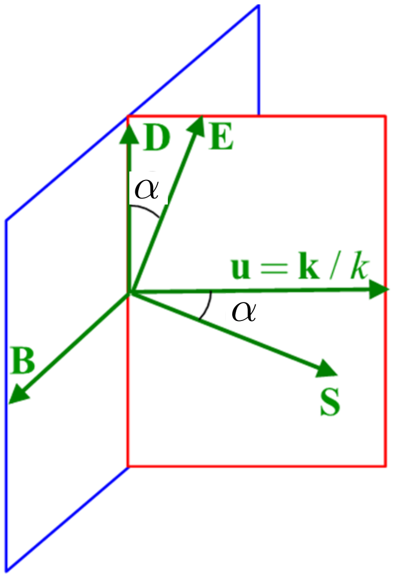


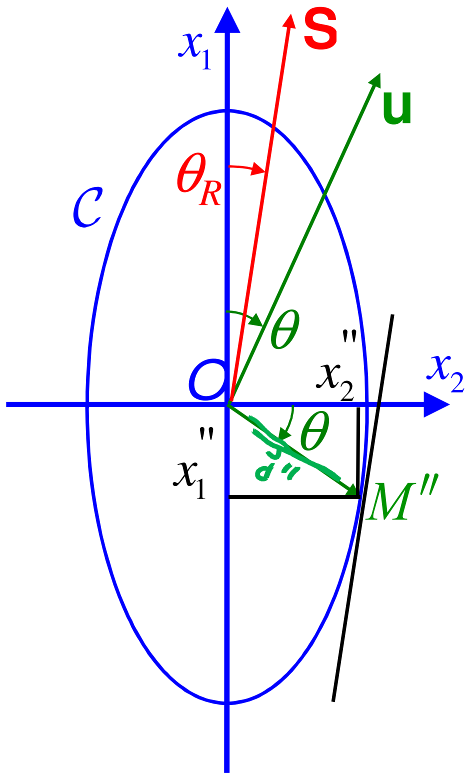
$$\boxed{n'' = 1\Big/\sqrt{\dfrac{\sin^2\theta}{n_e^2} + \dfrac{\cos^2\theta}{n_o^2}}}$$

Propagation in a uniaxial crystal: determining the direction of **S,** the Poynting vector

- Recall: **S** in the same plane as **u, E, D**, perpendicular to **B**
- Recall: **S** is tangential to ellipsoid at M''

Want to find θ_R





Propagation in a uniaxial crystal: determining the direction of **S**, the Poynting vector

- Recall: **S** in the same plane as **u, E, D**, perpendicular to **B**
- Recall: **S** is tangential to ellipsoid at M''

⬇

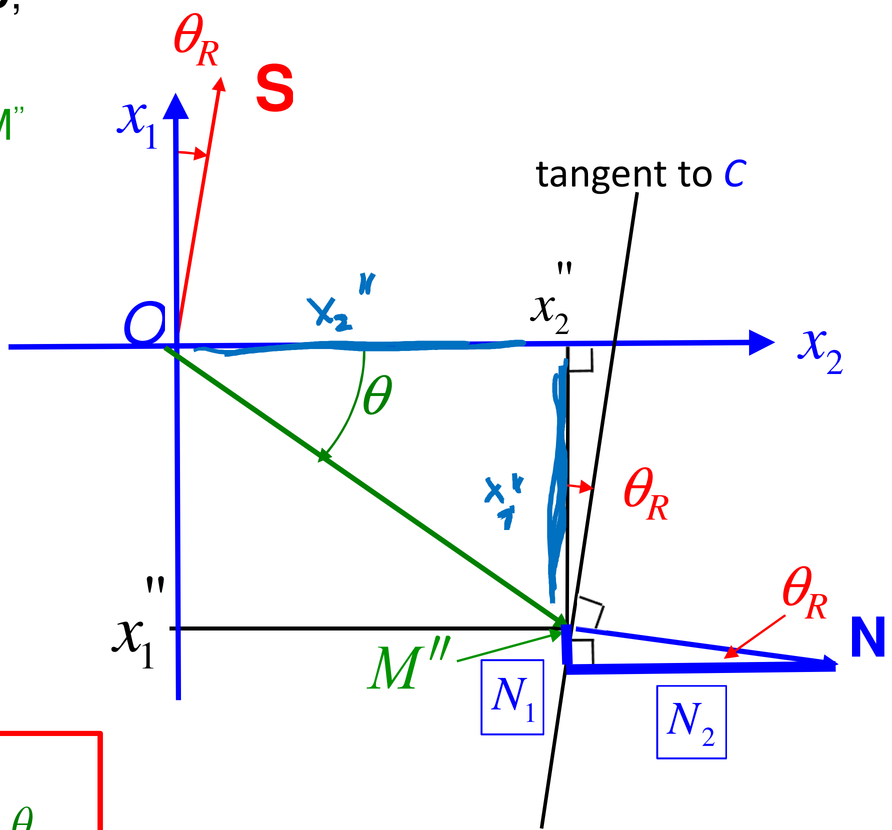
Want to find θ_R

- Find first the <u>normal</u> to *C* at M''

$$g(x_1, x_2) = \frac{x_1^2}{n_e^2} + \frac{x_2^2}{n_o^2} - 1$$

$$\mathbf{N} = \begin{pmatrix} \partial g / \partial x_1 \\ \partial g / \partial x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 / n_e^2 \\ 2x_2 / n_o^2 \end{pmatrix} \equiv \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

$$\boxed{\tan \theta_R = \frac{N_1}{N_2} = \frac{x_1'' / n_e^2}{x_2'' / n_o^2} = \frac{n_o^2}{n_e^2} \tan \theta}$$



Index ellipsoid and wave normal surfaces

surfaces des indices

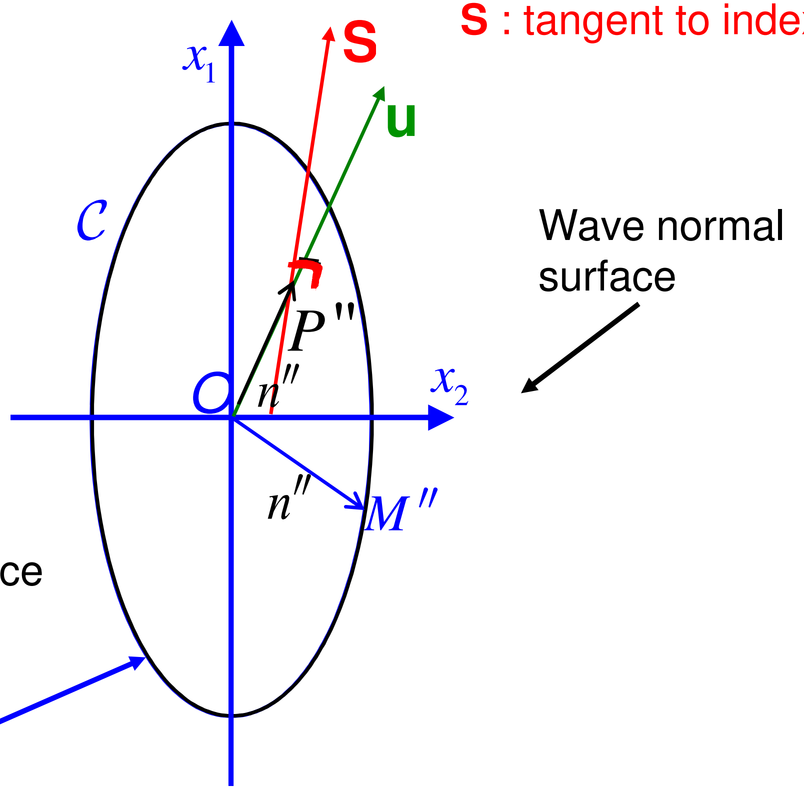
Recall: wave-normal surface:

- For each **u,** associate two vectors whose lengths are proportional to the two corresponding indices of refraction *n'* and *n"*

P'' is on the normal surface

S is perpendicular to wave normal surface

S : tangent to index ellipsoid

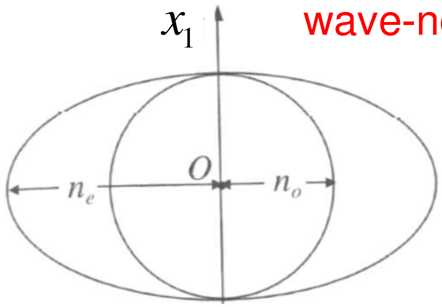


Index ellipsoid

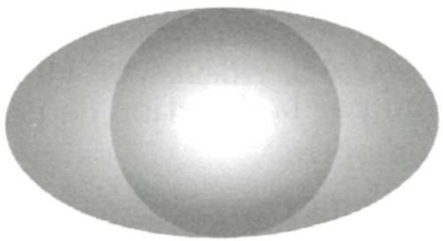
Wave normal surface: the set of all points N such that **ON**=*n***u**

Index ellipsoid:

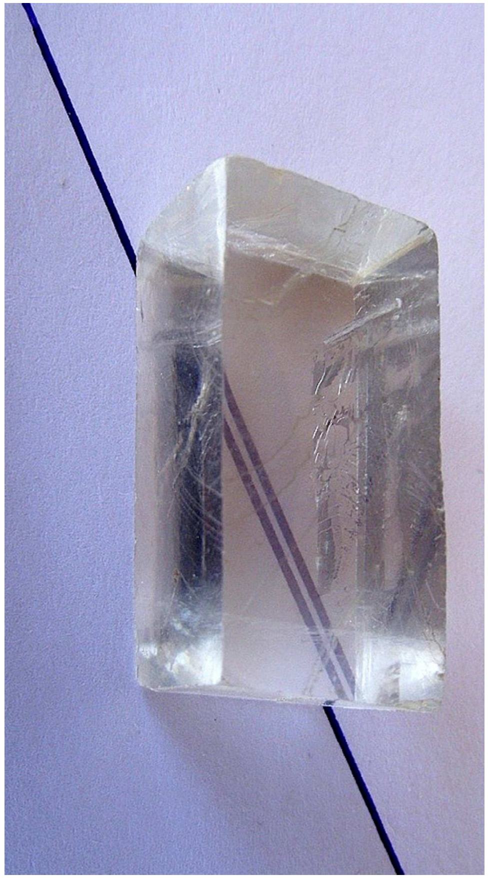
$$x_i \equiv n \frac{D_i}{\|\mathbf{D}\|}$$



wave-normal surface



Experiment!



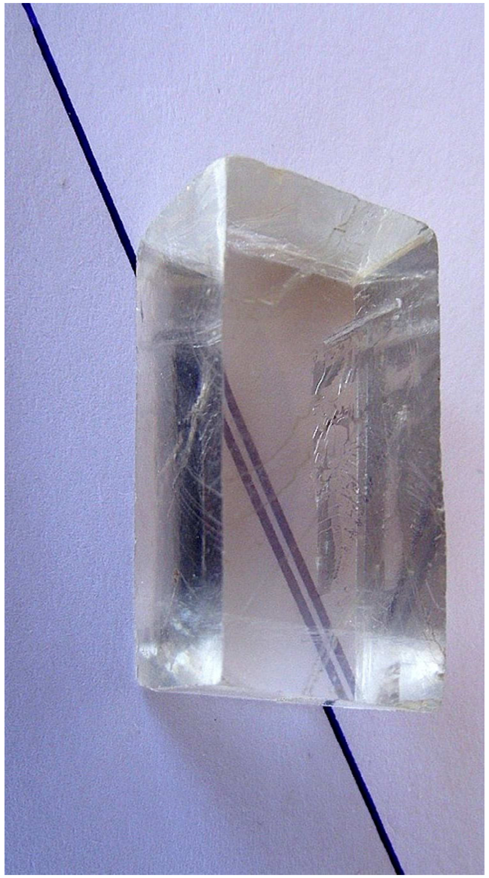
- Experiment:
  - Draw a black dot on a piece of paper
  - Place calcite crystal on top
  - Rotate crystal

There are two dots! One is stationary, the other rotates as the crystal rotates.

Stationary dot: ordinary wave
Rotating dot: extraordinary wave

Extraordinary wave Poynting vector changes direction as the optic axis direction is changed!

Experiment



- Experiment:
  - Look through a polarizer at crystal
  - Rotate polarizer

One or the other of the black dots is visible.

Polarization directions of the ordinary and extraordinary waves are perpendicular!!!

Snell’s (Descartes’?) laws for anisotropic media

According to Dijksterhuis,[12] "In *De natura lucis et proprietate* (1662) Isaac Vossius said that *Descartes had seen Snell's paper and concocted his own proof*. We now know this charge to be undeserved but it has been adopted many times since." *Both Fermat and Huygens repeated this accusation that Descartes had copied Snell.* In French, Snell's Law is called "la loi de Descartes" or "loi de Snell-Descartes."

Wikipedia *(italics and* underlining mine)

Snell's laws

- Basic premise: the tangential component of the wave vector **k** must be continuous across an interface

=> conservation de quantité de mouvement
=> conditions aux limites Maxwell

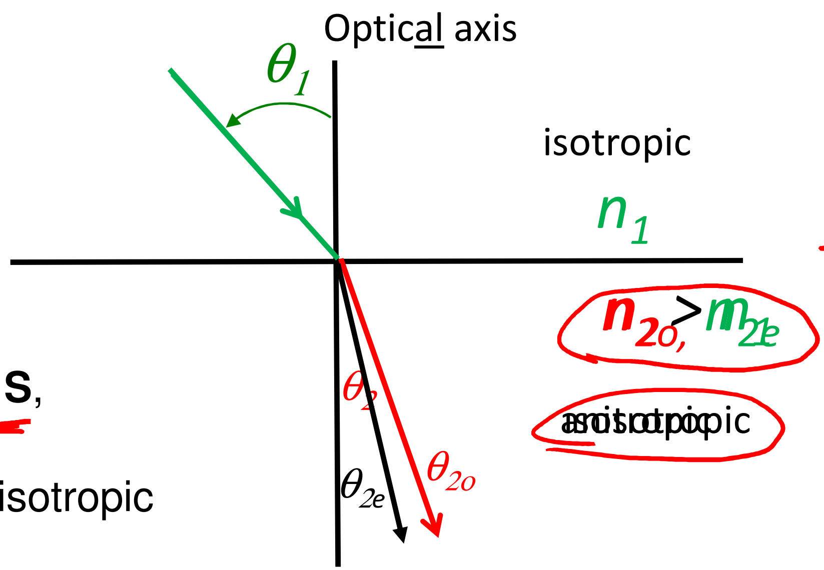
Recall Snell's laws for isotropic media:

- The wave normals (**u**) of the *incident*, *refracted* waves and the normal to the interface are all in the same plane (the plane of incidence)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Second medium anisotropic (uniaxial):

$$n_1 \sin\theta_1 = n_{2o} \sin\theta_{2o} = n_{2e} \sin\theta_{2e}$$



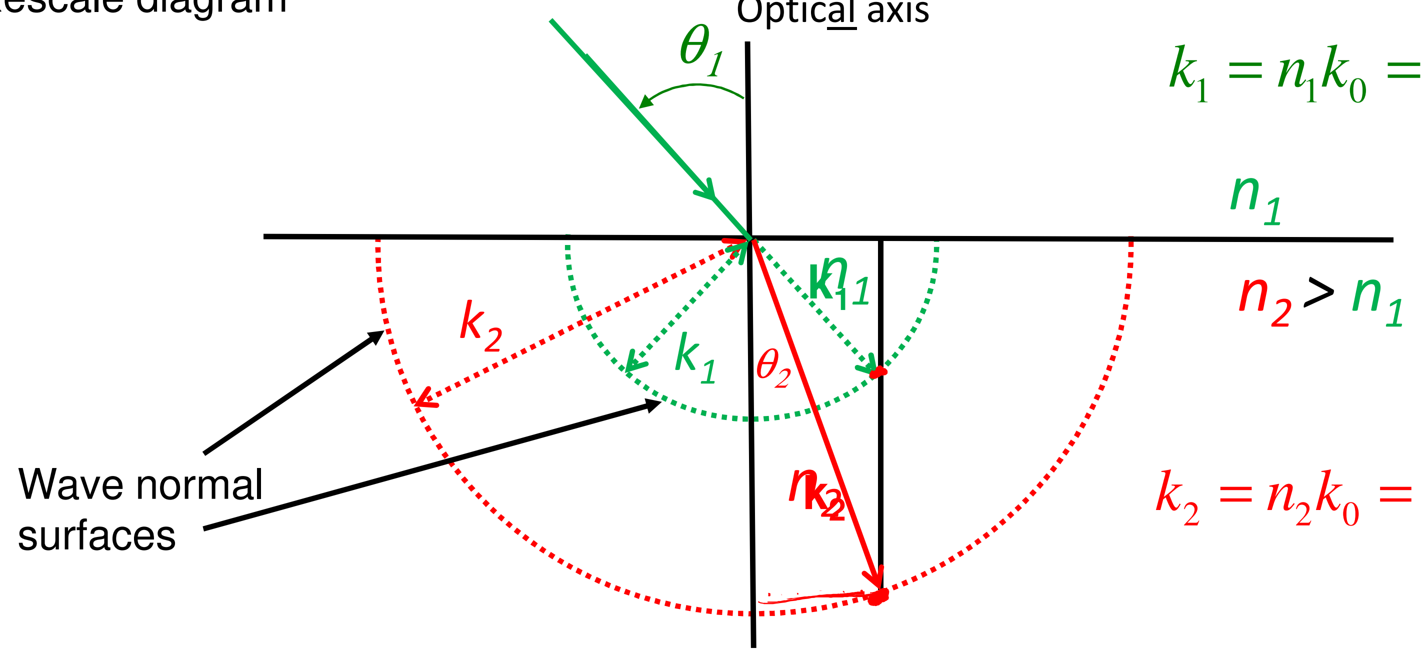
Note:
- Snell's laws apply to **k** and NOT to **S**, the Poynting vector
- n" changes with direction in the anisotropic medium!

Graphical method for applying Snell's laws: two *isotropic* media

Method:

- Draw half circle with radius k_1 ⇒ <u>the tangential components are equal</u>
- Draw half circle with radius k_2

Rescale diagram



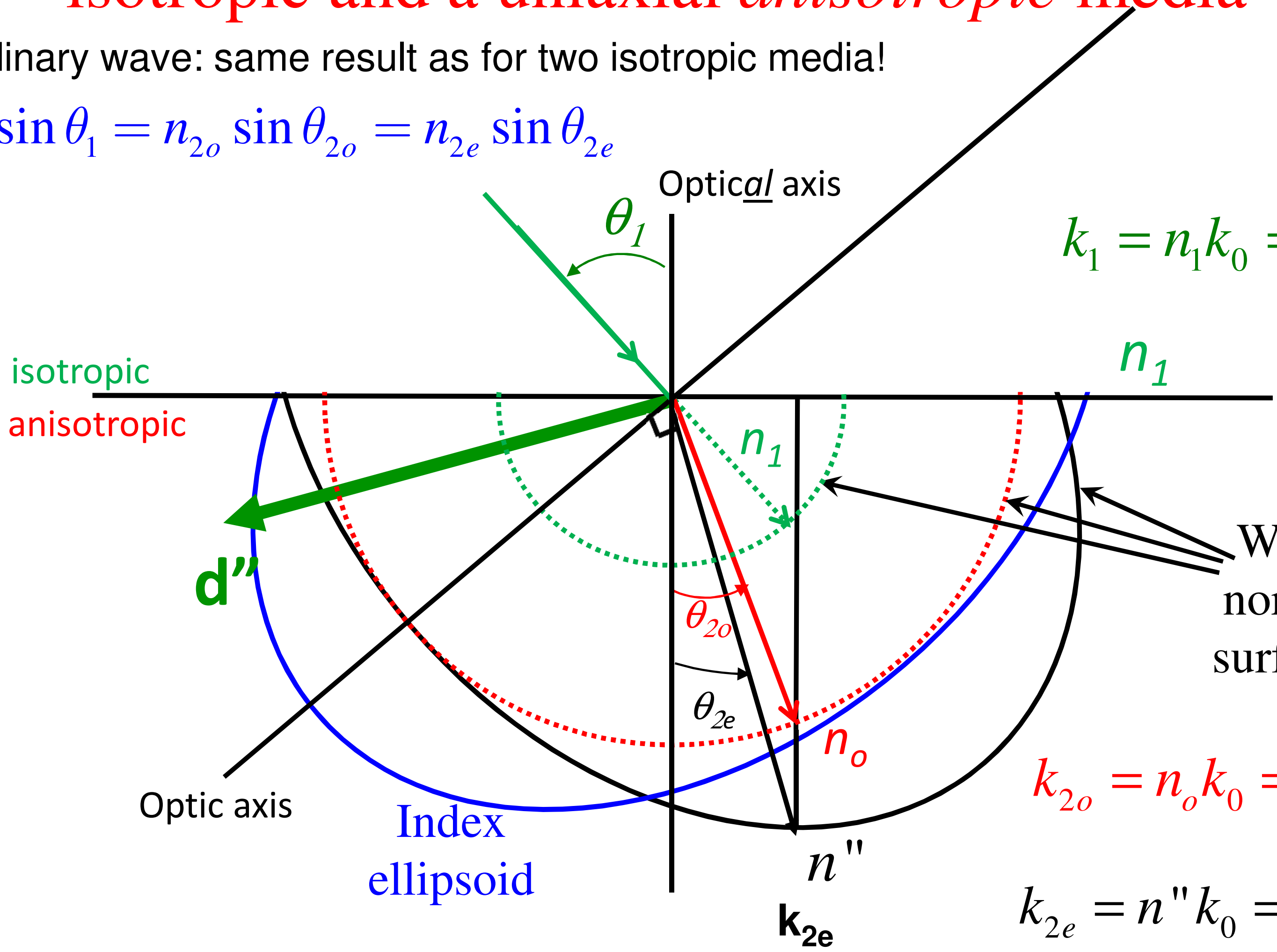
$$k_1 = n_1 k_0 = n_1 \frac{\omega}{c}$$

$$k_2 = n_2 k_0 = n_2 \frac{\omega}{c}$$

Graphical method for Snell's laws: an isotropic and a uniaxial *anisotropic* media

Ordinary wave: same result as for two isotropic media!

$$n_1 \sin\theta_1 = n_{2o} \sin\theta_{2o} = n_{2e} \sin\theta_{2e}$$

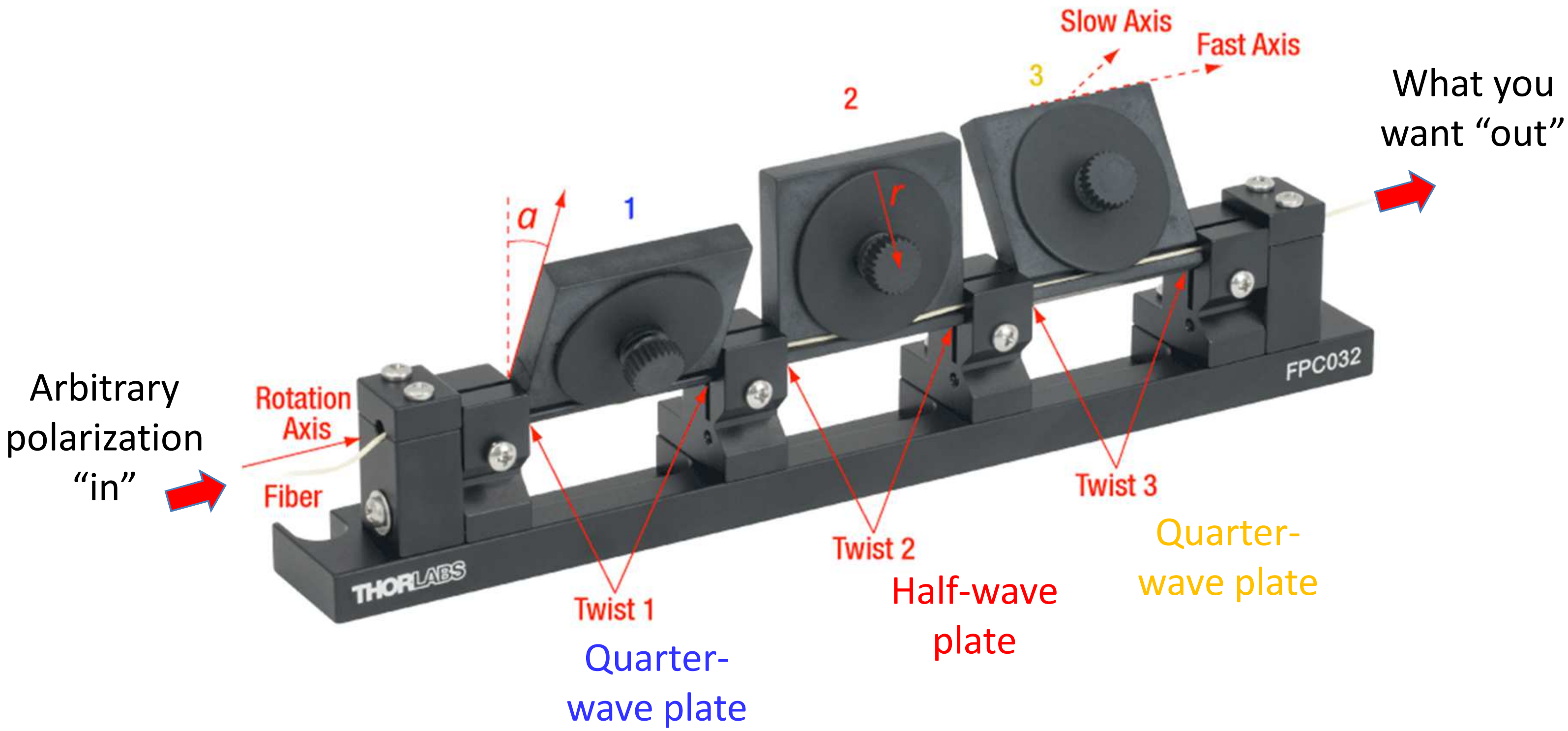


$$k_1 = n_1 k_0 = n_1 \frac{\omega}{c}$$

$$k_{2o} = n_o k_0 = n_o \frac{\omega}{c}$$

$$k_{2e} = n'' k_0 = n'' \frac{\omega}{c}$$

Controlling the polarization of light

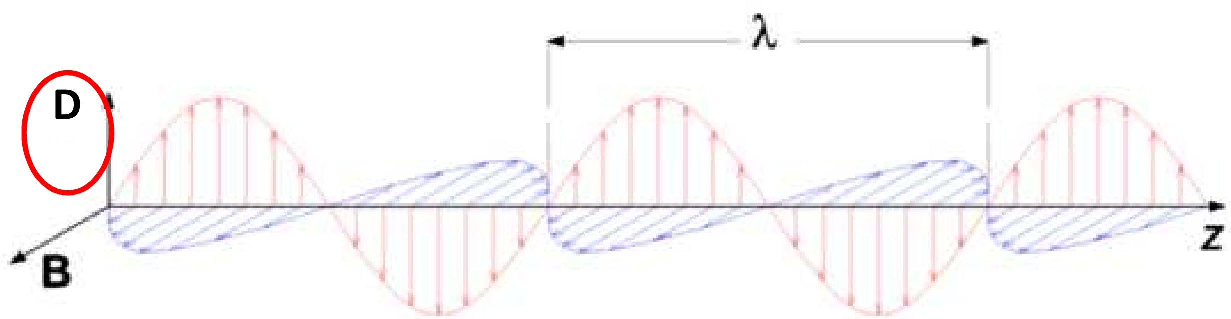


Outline

– Polarization states of light—mathematical description

– Jones vectors

– Manipulation and control of the polarization of light



Polarization

Polarization:

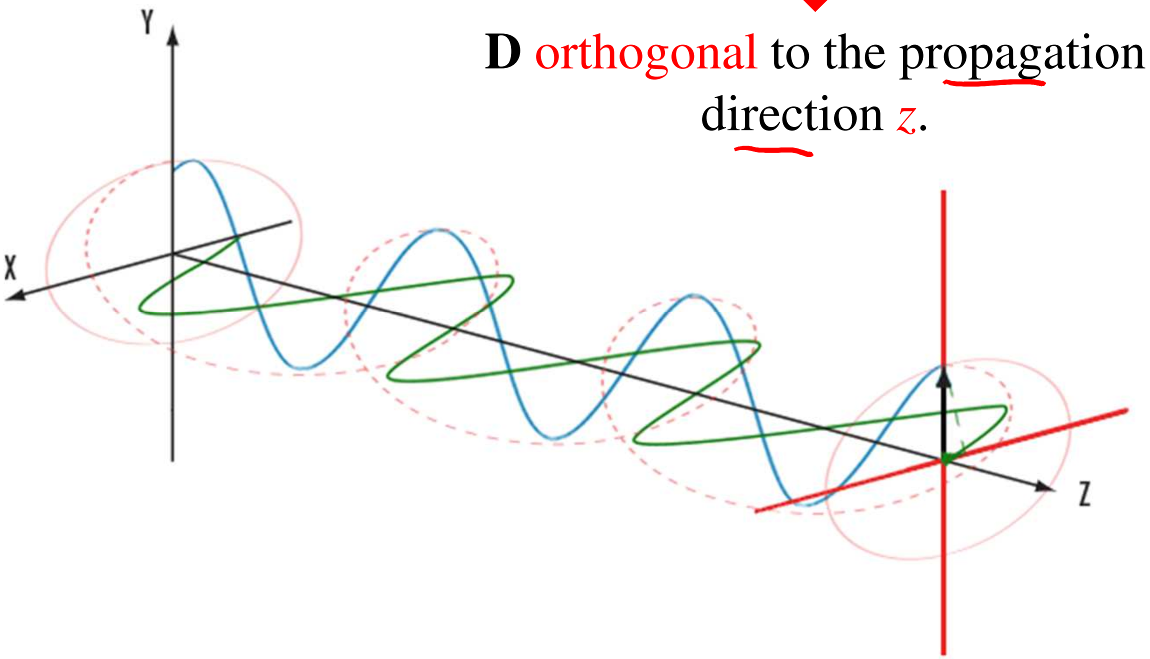
- direction and variation of the electric displacement vector **D** during propagation

- **Monochromatic plane wave** in a transparent medium

⬇

D orthogonal to the propagation direction *z*.

$\vec{k}$ ($\hat{u} = \frac{\vec{k}}{|k|}$)



https://www.edmundoptics.com/resources/application-notes/optics/introduction-to-polarization/

Polarization

- **Monochromatic sinusoidal plane wave** in a transparent medium

⬇

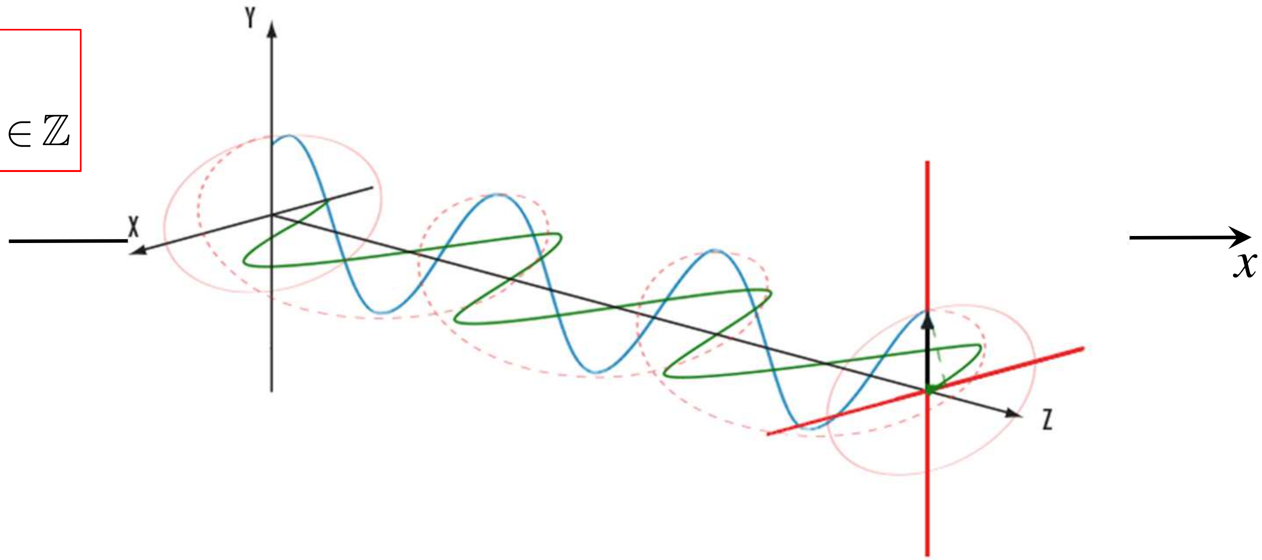
D orthogonal to propagation direction z.

$$\boxed{\begin{aligned} D_x &= D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right) \\ D_y &= D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right) \end{aligned}}$$

$\boxed{D_{0x}, D_{0y} \geq 0}$ Amplitudes

$\boxed{\psi_x, \psi_y}$ Phases, constant

$\boxed{\begin{aligned} &D_{0x} \neq D_{0y} \\ &\psi_y - \psi_x \neq m\pi,\ m \in \mathbb{Z} \end{aligned}}$

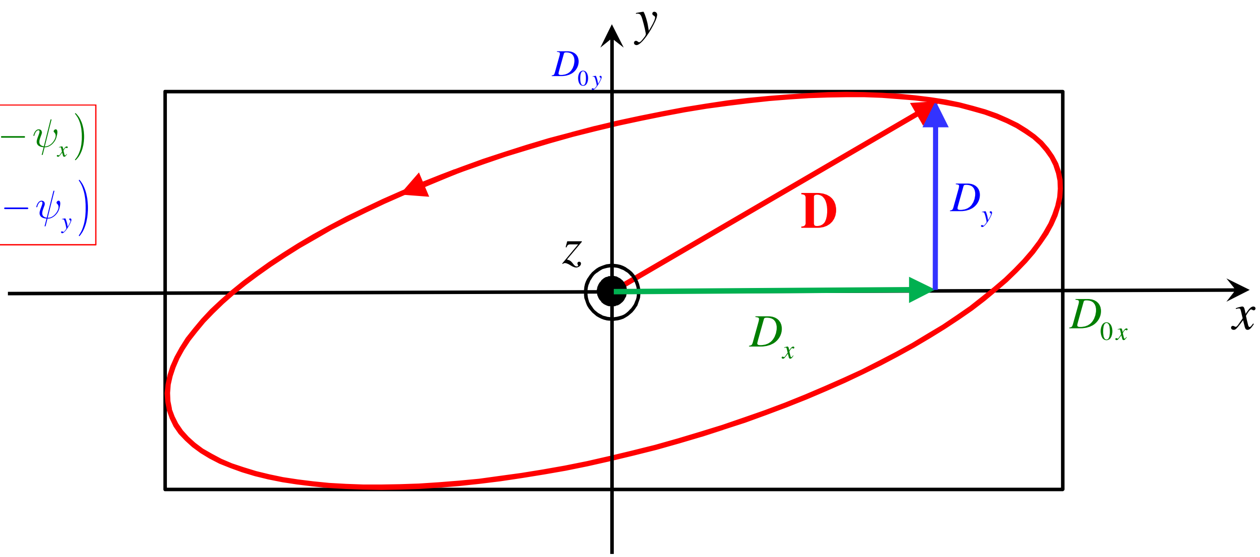


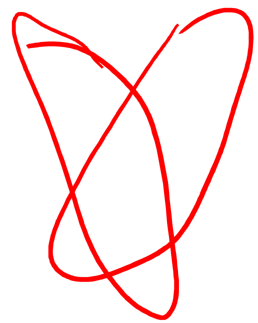
State of polarization

The *state of polarization* is determined by

- $\dfrac{D_{0y}}{D_{0x}}$ OR   • $\dfrac{D_{0y}}{D_{0x}}$ • $\psi_y - \psi_x$
- angle of ellipse axis
- rotation *direction*

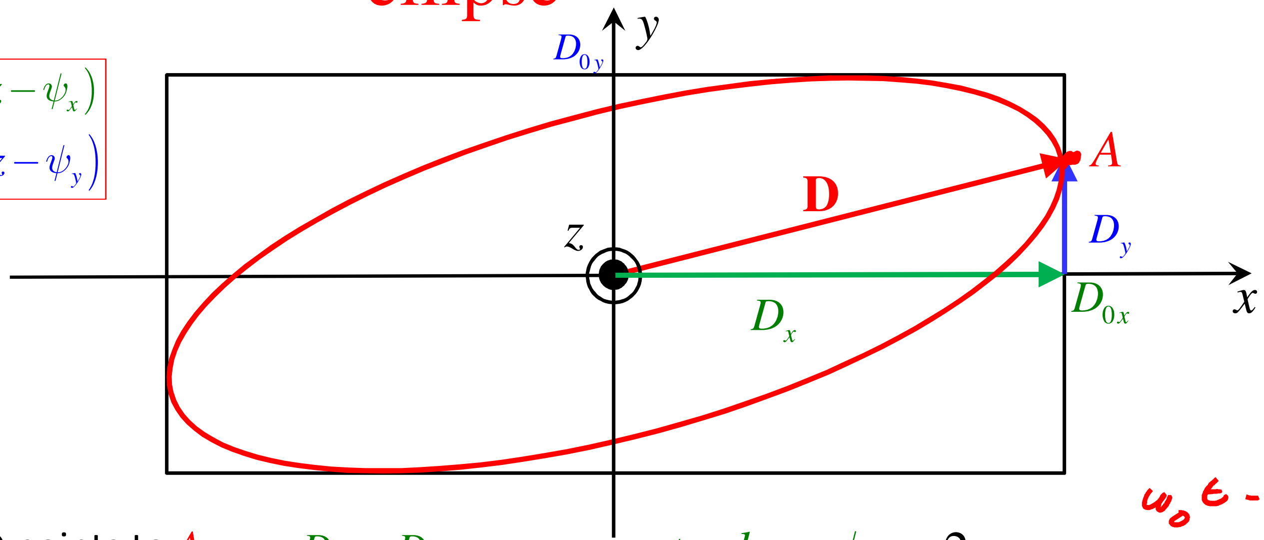
$$\boxed{\begin{aligned} D_x &= D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right) \\ D_y &= D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right) \end{aligned}}$$





Determining the rotation direction of the polarization ellipse

$$D_x = D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right)$$
$$D_y = D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right)$$

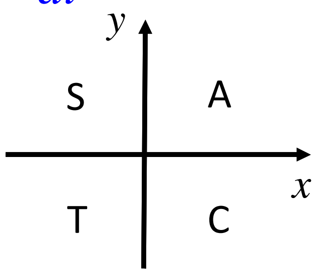


$\omega_0 t - kz = 2m\pi - \psi_x$

- Consider when **D** points to *A*: $D_x = D_{0x}$ $\quad \omega_0 t - kz - \psi_x = 2m\pi$
- In order to know the rotation direction, we need to know the sign of $\dfrac{dD_y}{dt}$

$$\frac{dD_y}{dt} = -\omega_0 D_{0y}\sin\left(\overbrace{\omega_0 t - kz}^{2m\pi - \psi_x} - \psi_y\right) = \omega_0 D_{0y}\sin\left(\psi_y - \psi_x\right)$$

La lumière se propage vers nous

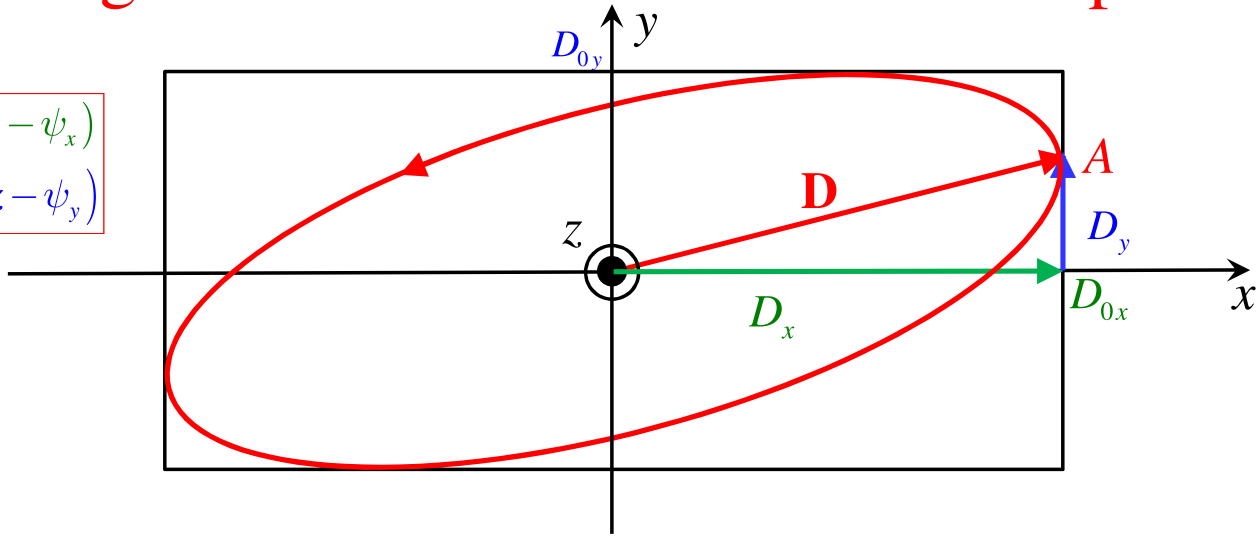


$0 < \psi_y - \psi_x < \pi \Leftrightarrow$ left polarization — counter-clockwise

$-\pi < \psi_y - \psi_x < 0 \Leftrightarrow$ right polariation — clockwise

Determining the rotation direction of the ellipse

$$\boxed{\begin{aligned} D_x &= D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right) \\ D_y &= D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right) \end{aligned}}$$



$$\frac{dD_y}{dt} = -\omega_0 D_{0y}\sin\left(\omega_0 t - kz - \psi_y\right) \quad = \omega_0 D_{0y}\sin\left(\psi_y - \psi_x\right)$$

$0 < \psi_y - \psi_x < \pi \Leftrightarrow$ left elliptically polarized

$-\pi < \psi_y - \psi_x < 0 \Leftrightarrow$ right elliptically polarized

counter-clockwise rotation as a function of time when the wave is travelling ***towards*** the observer

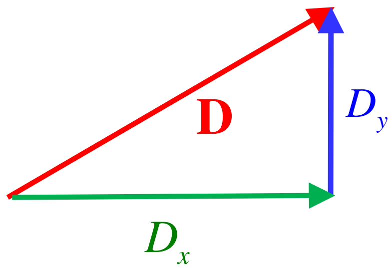
Polarization: special cases

$$\boxed{\begin{aligned} D_x &= D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right) \\ D_y &= D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right) \end{aligned}}$$

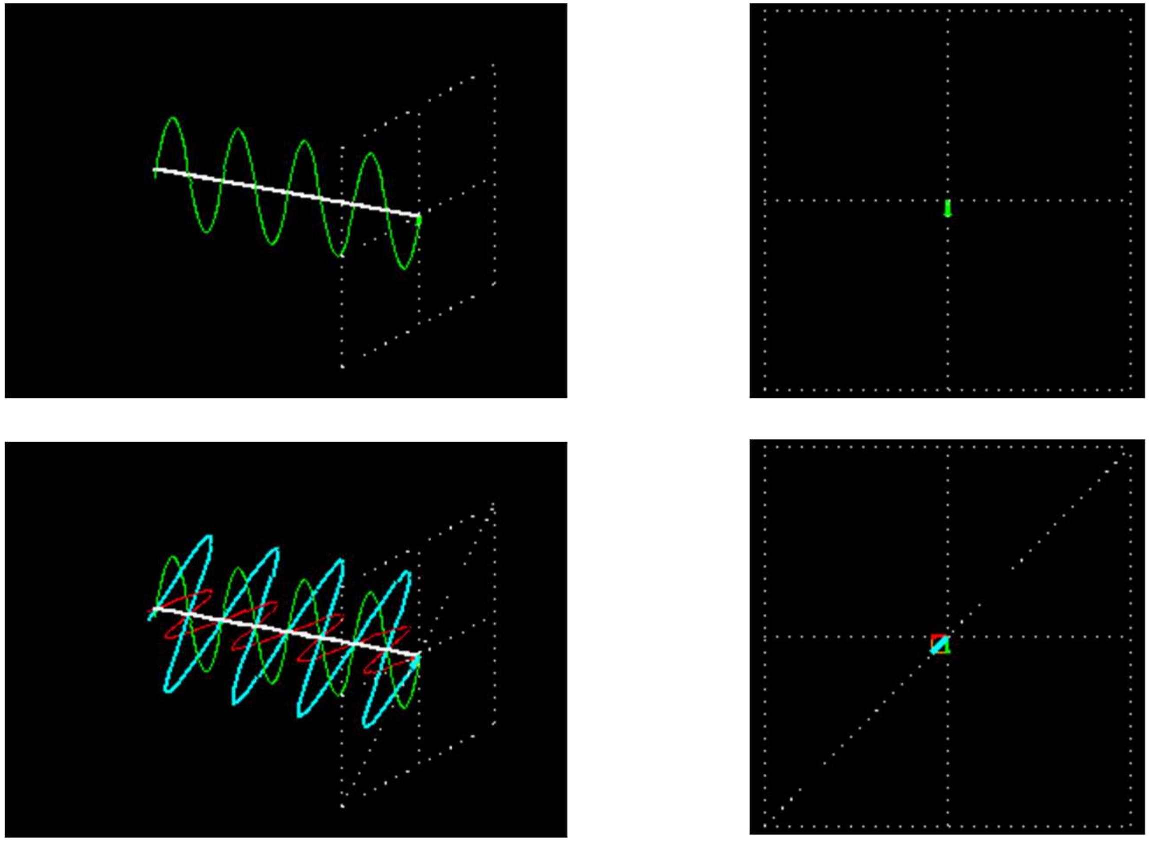
$$\boxed{D_{0x}, D_{0y} \geq 0}$$

If $\psi_y - \psi_x = 0 \text{ or } \pi$

➡ components are *<u>in phase</u>* or *<u>out of phase</u>*



Linear polarization



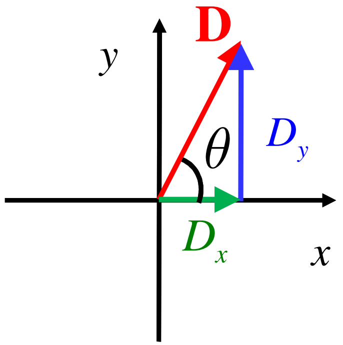
http://cddemo.szialab.org/

Linear polarization

$$\boxed{\begin{aligned} D_x &= D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right) \\ D_y &= D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right) \end{aligned}}$$

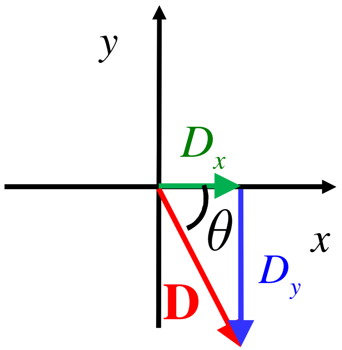
$$\boxed{D_{0x}, D_{0y} \geq 0}$$

If $\psi_y - \psi_x = 0$:



$$\boxed{\begin{aligned} &\text{If } \psi_y - \psi_x = 0 : \tan\theta = \frac{D_y}{D_x} = \frac{D_{0y}}{D_{0x}}. \\ &\text{If } \psi_y - \psi_x = \pi : \tan\theta = \frac{D_y}{D_x} = -\frac{D_{0y}}{D_{0x}}. \end{aligned}}$$

If $\psi_y - \psi_x = \pi$:



D makes an angle $\pm\theta$ with the x axis.

Circular polarization

What if $\psi_y - \psi_x = \pm\, \pi/2$ and $D_{0x} = D_{0y}$?

$$D_x = D_{0x} \cos\left(\omega_0 t - kz - \psi_x\right)$$
$$D_y = D_{0y} \cos\left(\omega_0 t - kz - \psi_y\right)$$

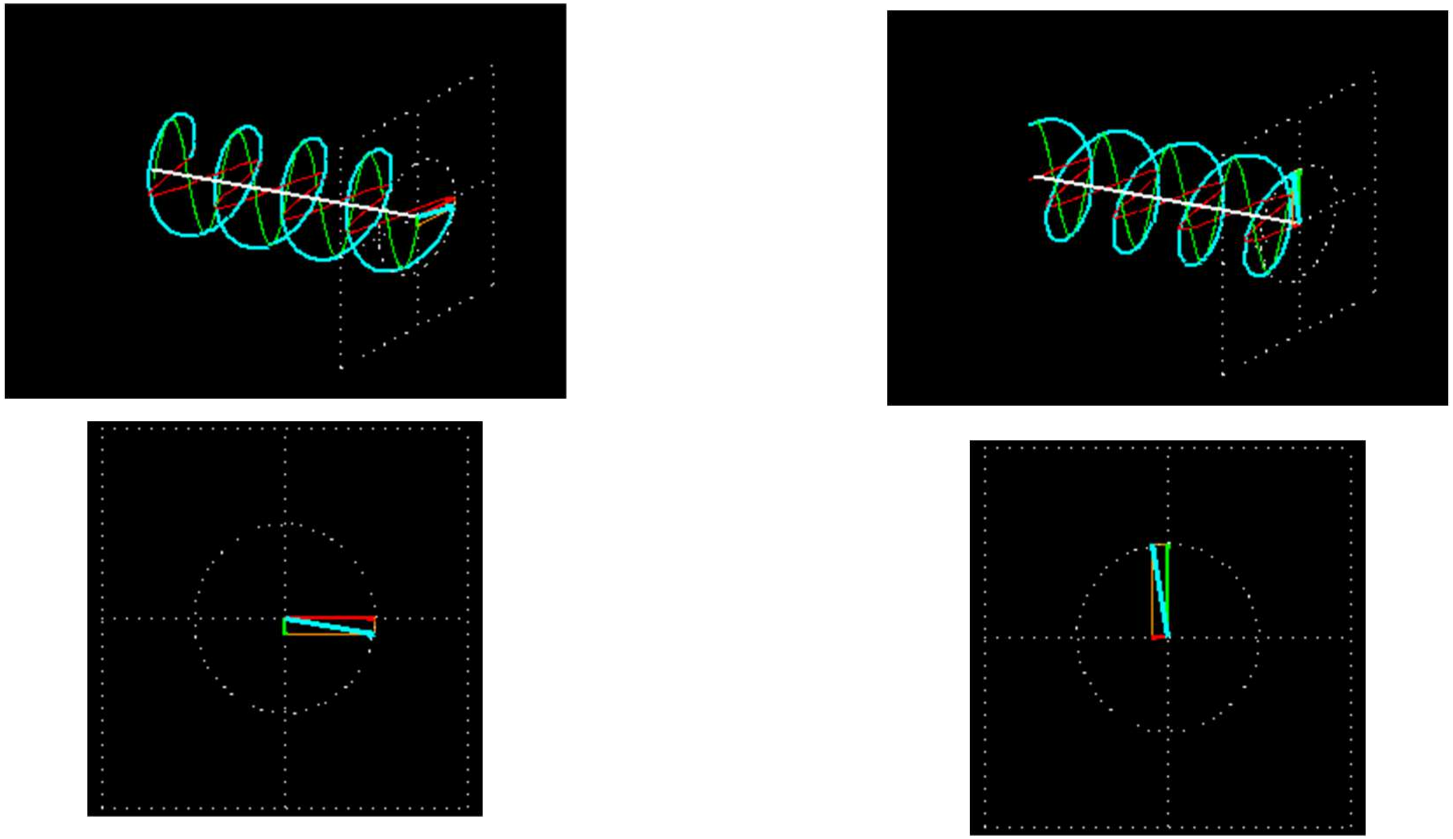
$$\Rightarrow$$

$$D_x = +D_{0x} \cos\left(\omega_0 t - kz - \psi_x\right)$$
$$D_y = \pm D_{0x} \sin\left(\omega_0 t - kz - \psi_x\right)$$

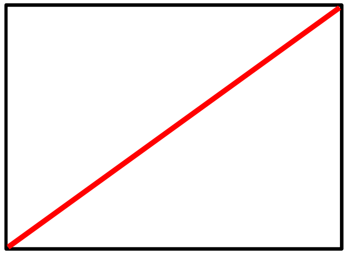
The end of **D** draws a circle of radius D_{0x}.

- If $\psi_y - \psi_x = +\, \pi/2$: left circular polarization
- If $\psi_y - \psi_x = -\, \pi/2$: right circular polarization

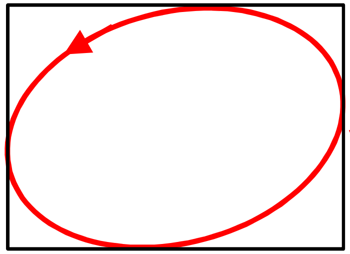
Circular polarization



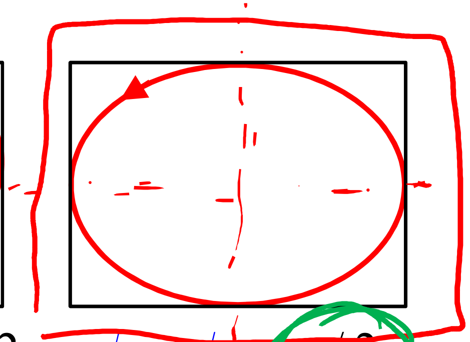
Diverse polarization states



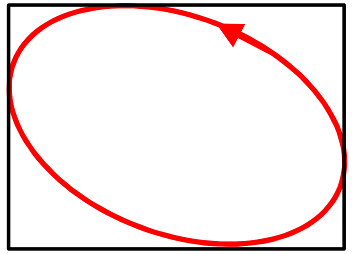
$\psi_y - \psi_x = 0$



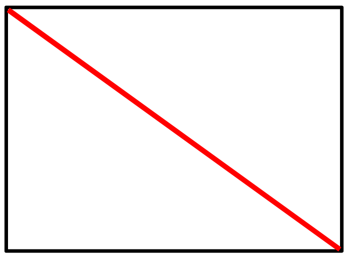
$0 < \psi_y - \psi_x < \pi / 2$



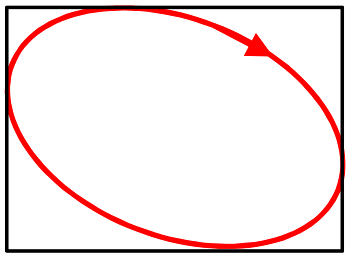
$\psi_y - \psi_x = \pi / 2$



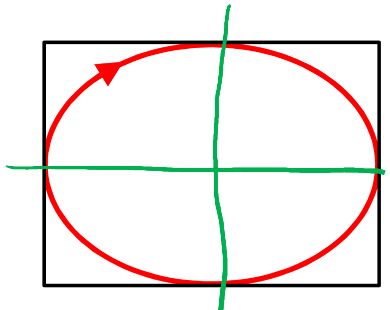
$\pi / 2 < \psi_y - \psi_x < \pi$



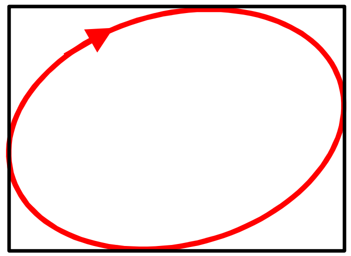
$\psi_y - \psi_x = \pi$



$\pi < \psi_y - \psi_x < 3\pi / 2$



$\psi_y - \psi_x = 3\pi / 2$



$3\pi / 2 < \psi_y - \psi_x < 2\pi$

$$\boxed{\begin{aligned} D_x &= D_{0x} \cos\left(\omega_0 t - kz - \psi_x\right) \\ D_y &= D_{0y} \cos\left(\omega_0 t - kz - \psi_y\right) \end{aligned}}$$

Jones vectors

Handy mathematical formulism for describing and manipulating polarization states

Write
$$D_x = D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right)$$
$$D_y = D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right)$$
in complex notation, i.e.,

$$\mathcal{D}_x = D_{0x}\exp\left[i(kz - \omega t + \psi_x)\right]$$
$$\mathcal{D}_y = D_{0y}\exp\left[i(kz - \omega t + \psi_y)\right]$$

$$\boxed{\begin{aligned}\mathcal{D}_{0x} &\equiv D_{0x}\exp\left[i\psi_x\right] \\ \mathcal{D}_{0y} &\equiv D_{0y}\exp\left[i\psi_y\right]\end{aligned}}$$

Define the Jones vector for the polarization state as:

$$\boxed{\mathbf{u} = \begin{pmatrix}\mathcal{D}_{0x} \\ \mathcal{D}_{0y}\end{pmatrix} = \begin{pmatrix}D_{0x}\exp\left[i\psi_x\right] \\ D_{0y}\exp\left[i\psi_y\right]\end{pmatrix}}$$

Jones vectors

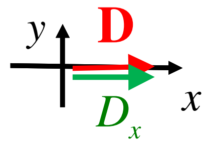
$$\boxed{\mathbf{u} = \begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y} \end{pmatrix} = \begin{pmatrix} D_{0x} \exp[i\psi_x] \\ D_{0y} \exp[i\psi_y] \end{pmatrix}}$$

Jones vectors for linear polarization:

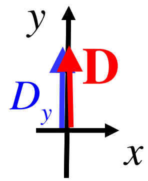
$D_{0y} = 0$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Linear polarization oriented along the Ox axis: $\mathbf{u}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

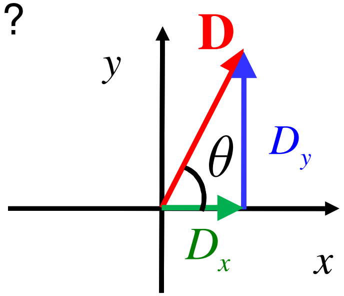


Linear polarization oriented along the Oy axis: $\mathbf{u}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



Orthonormal basis!

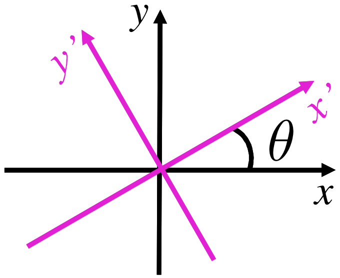
What is the Jones vector for linear polarization at an angle θ with respect to the Ox axis?



$u_\theta = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

Jones vectors

$$\mathbf{u} = \begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y} \end{pmatrix} = \begin{pmatrix} D_{0x} \exp[i\psi_x] \\ D_{0y} \exp[i\psi_y] \end{pmatrix}$$



Recall from math class:

$$R(\theta)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$R(\theta)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

$$\Rightarrow \quad R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Jones vector <u>for linear polarization at an angle θ with respect to the Ox axis</u>:

$$\Rightarrow \quad \mathbf{u}_\theta = R(\theta)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

(normalized) Jones vectors for <u>**circular polarization**</u>**?**

Recall: $\psi_y - \psi_x = \pm \pi/2$ and $D_{0x} = D_{0y}$

$$\mathbf{u} = \begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y} \end{pmatrix} = \begin{pmatrix} D_{0x} \exp[i\psi_x] \\ D_{0x} \exp\left[i\psi_x \pm \frac{\pi}{2}\right] \end{pmatrix} = D_{0x} \exp[i\psi_x] \begin{pmatrix} 1 \\ e^{\pm i\left(\frac{\pi}{2}\right)} \end{pmatrix} \propto \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

<u>**circular polarization**</u>

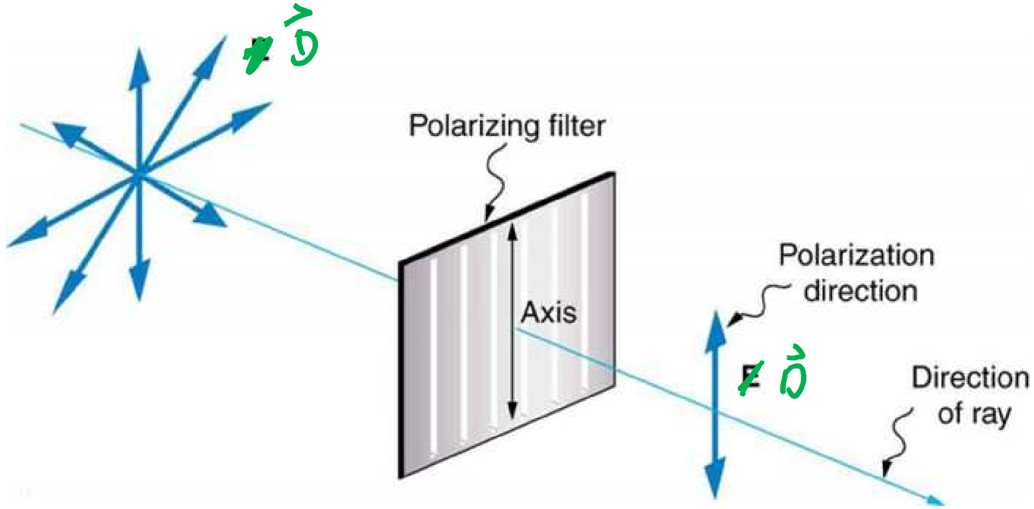
Jones vectors for circularly polarized light

Left circular polarization: $$\mathbf{u}_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix}$$

Right circular polarization: $$\mathbf{u}_R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Orthonormal basis!

Polarizers and the Jones formulism



$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}$$

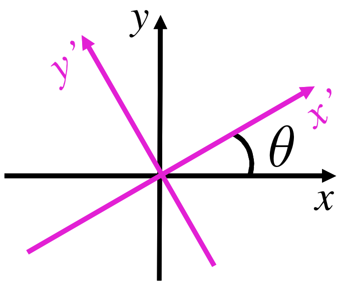
Ideal polarizer whose transmission axis is aligned with Ox:

$$\mathbf{P}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

What about the Jones matrix for an ideal polarizer whose transmission axis is at an angle θ to the Ox axis?

Polarizers in the Jones formulism

What about the Jones matrix for an ideal polarizer whose transmission axis is at an angle θ to the Ox axis?



Recall: $\boxed{R(\theta)=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}}$ gives the coordinates of the new axes $Ox'y'$ in terms of the *old* coordinates Oxy

1. Find *old* axes in terms of the *new* coordinate system

$$R^{-1}(\theta)\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos\theta\\-\sin\theta\end{pmatrix}$$

$$R^{-1}(\theta)\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\sin\theta\\\cos\theta\end{pmatrix}$$

$$R^{-1}(\theta)=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

Thus, the incoming polarization $\begin{pmatrix}D_x\\D_y\end{pmatrix}$ in terms of the coordinate system of the polarizer is

$$R^{-1}(\theta)\begin{pmatrix}D_x\\D_y\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}D_x\\D_y\end{pmatrix}$$

Polarizers in the Jones formulism

2. Next, apply the effect of the polarizer:

$$\mathbf{P}_0 R^{-1}(\theta)\begin{pmatrix} D_x \\ D_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} D_x \\ D_y \end{pmatrix}$$

3. Finally, express in terms of the original coordinate system

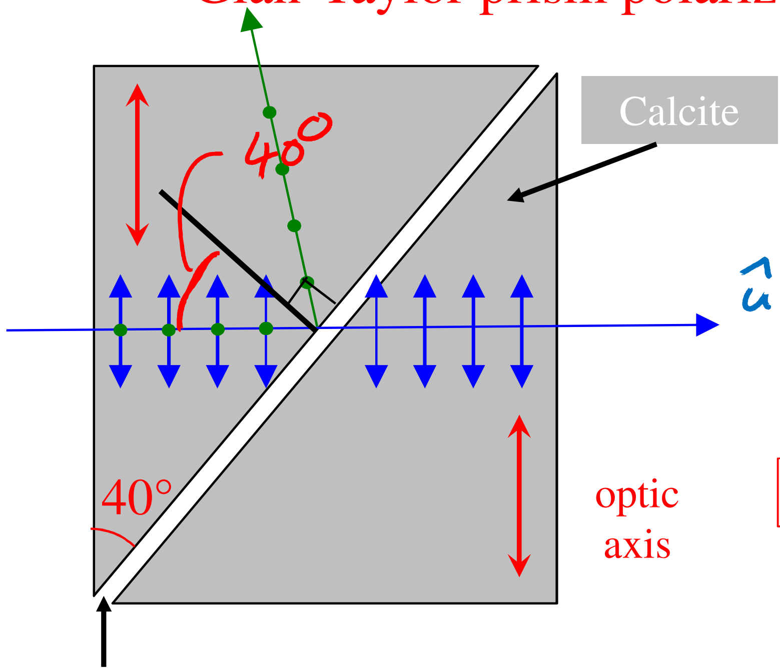
$$\begin{aligned} \mathbf{P}_\theta &= R(\theta)\mathbf{P}_0 R^{-1}(\theta) \\ &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \\ &= \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \end{aligned}$$

Jones matrix for an ideal polarizer whose transmission axis is at an angle θ to the Ox axis

Controling the polarizaton

Goal: light with arbitrary polarization "in", linearly polarized light "out".

Glan-Taylor prism polarizer



Ordinary wave polarization:

out of screen $n_o = 1.658$

Extraordinary wave polarization:

parallel to optic axis $n_e = 1.486$

Use total internal reflection to separate!!!

$$\theta_{crit\text{-}o} = \arcsin(1/n_o) = 37°$$

$$\theta_{crit\text{-}e} = \arcsin(1/n_e) = 42°$$

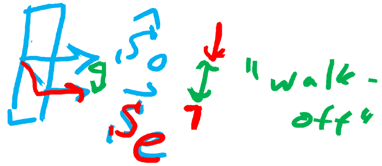
Extraordinary wave continues! Ordinary wave is reflected!

Controling the polarization of light

- Use polarizers

lame d'ondes

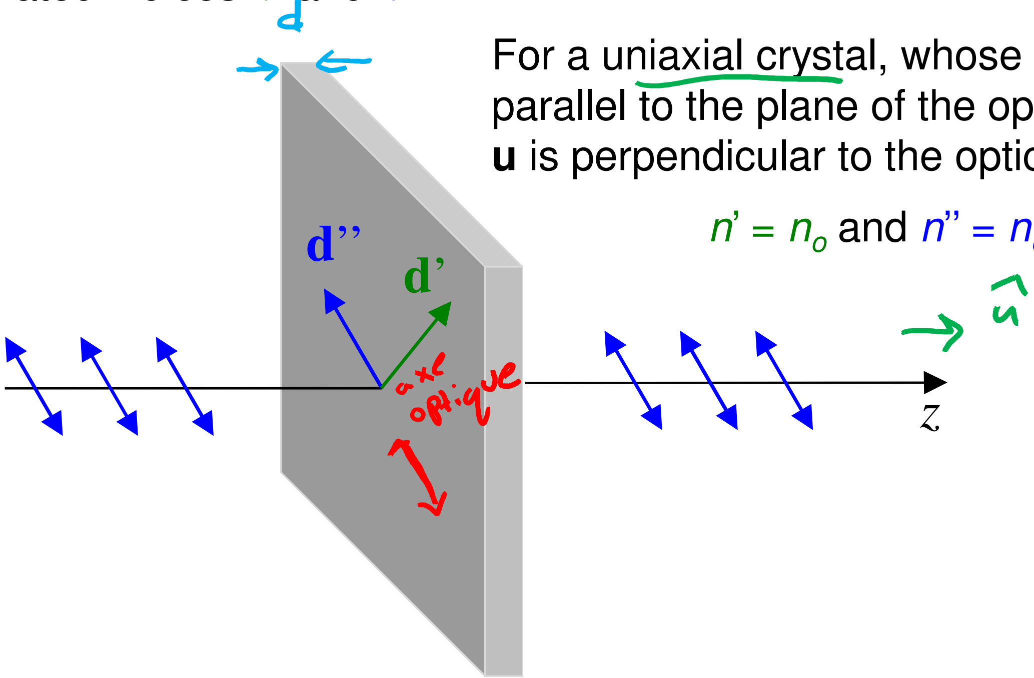
- Use a birefringent optical flat ➡ **"wave plates"** or **"retarders"**
    - Normal incidence
    - Walk-off is negligible



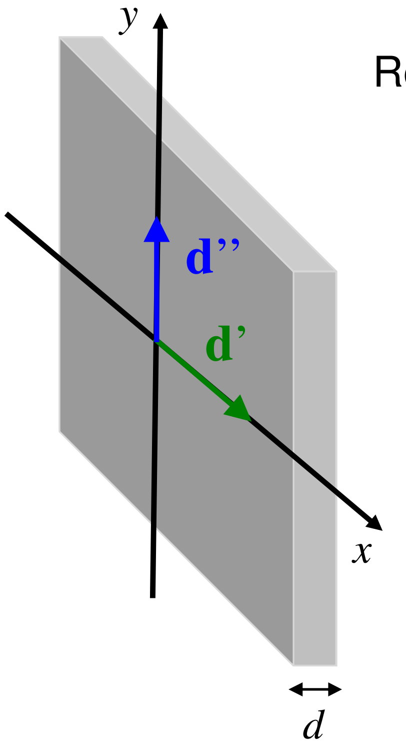
Recall:
- **d'** and **d''**, special polarization directions for which the polarization is maintained as the light propagates
- associated indices n' and n''

For a uniaxial crystal, whose optic axis is parallel to the plane of the optical flat (i.e., **u** is perpendicular to the optic axis)

$n' = n_o$ and $n'' = n_e$



What happens to the polarization as the light propagates through a birefringent crystal?



Choose x and y to be along **d'** and **d''** respectively

"lignes neutres"

Recall: $D_x = D_{0x}\cos(\omega_0 t - kz - \psi_x)$ $n' k_0$

$D_y = D_{0y}\cos(\omega_0 t - kz - \psi_y)$ $n'' k_0$

$k_0 = 2\pi/\lambda = \omega/c$

$$\mathbf{u} = \begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y} \end{pmatrix} = \begin{pmatrix} D_{0x}\exp[i\psi_x] \\ D_{0y}\exp[i\psi_y] \end{pmatrix}$$

Each component travels at a different speed!

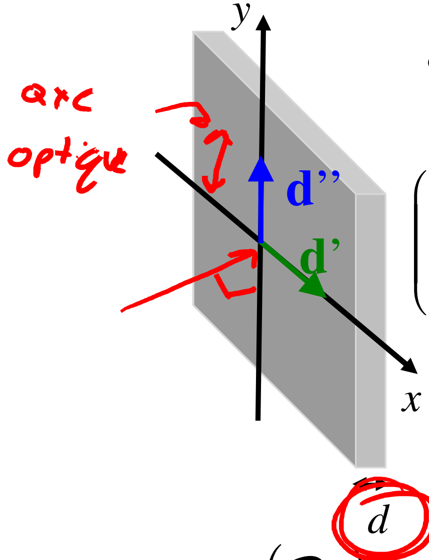
$$\mathcal{D}_x = \mathcal{D}_{0x}\exp(ik'd) = \mathcal{D}_{0x}\exp\left(i\frac{2\pi}{\lambda}n'd\right)$$

$$\mathcal{D}_y = \mathcal{D}_{0y}\exp(ik''d) = \mathcal{D}_{0y}\exp\left(i\frac{2\pi}{\lambda}n''d\right)$$

Note that **if** $n' < n''$, **d'** and **d''** are called the **fast** and **slow** axes respectively

$v = \frac{c}{n}$

What happens to the polarization as the light propagates through a birefringent crystal?



Find Jones matrix corresponding to propagation through a birefringent crystal of thickness d

$$\mathbf{u} = \begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{D}_x \\ \mathcal{D}_y \end{pmatrix} = \begin{pmatrix} \mathcal{D}_{0x} \exp\left(i\frac{2\pi}{\lambda}n'd\right) \\ \mathcal{D}_{0y} \exp\left(i\frac{2\pi}{\lambda}n''d\right) \end{pmatrix} = \exp\left(i\frac{2\pi}{\lambda}n'd\right)\begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y} \exp\left(i\frac{2\pi}{\lambda}(n''-n')d\right) \end{pmatrix}$$

Let $\varphi = \frac{2\pi}{\lambda}(n''-n')d$ Phase delay

$n'' = n_e$

$n' = n_o$

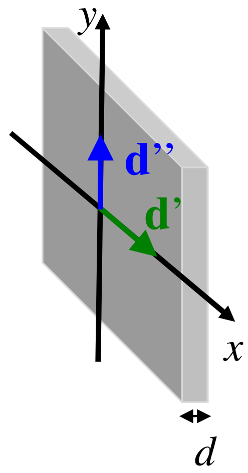
$$\begin{pmatrix} \mathcal{D}_x \\ \mathcal{D}_y \end{pmatrix} = \exp\left(i\frac{2\pi}{\lambda}n'd\right)\begin{pmatrix} \mathcal{D}_{0x} \\ \mathcal{D}_{0y}e^{i\varphi} \end{pmatrix}$$

Jones matrix for a wave plate:

$$\boxed{\mathbf{M} \propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}}$$

$$\boxed{\mathbf{M} \propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}}$$

Quarter wave plate



A phase of 2π corresponds to so a quarter wave plate corresponds to a phase delay of

Choose thickness *d* such that

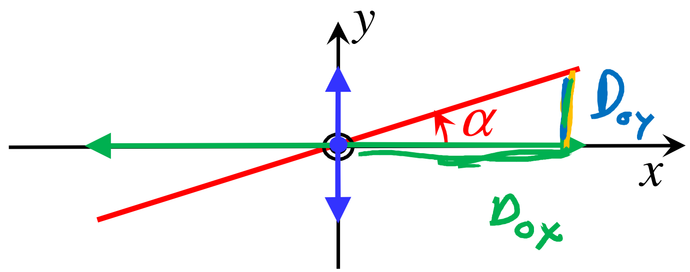
Phase delay: $$\boxed{\varphi = \frac{2\pi}{\lambda}(n_e - n_o)d = \frac{\pi}{2}}$$

Example: quartz at 560 nm: $(n_e - n_o) = 0.0091$ $$\boxed{d = \frac{\lambda}{4(n_e - n_o)} = 15.4\,\mu\text{m}}$$

$$\boxed{\mathbf{M} \propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \propto \begin{pmatrix} \exp(-i\pi/4) & 0 \\ 0 & \exp(i\pi/4) \end{pmatrix}}$$

$$\varphi = \frac{\pi}{2}$$

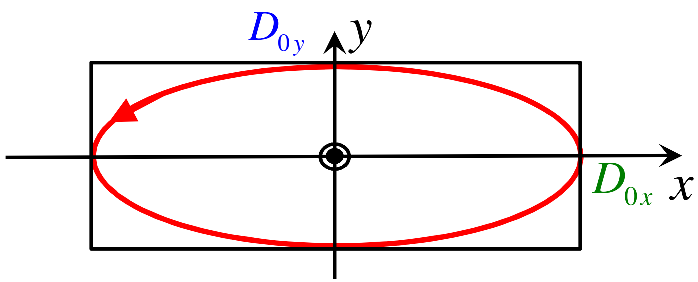
How does a quarter wave plate change the polarization of a linearly polarized wave?



Before

$$\boxed{\varphi = \frac{2\pi}{\lambda}\left(n_e - n_o\right)d = \frac{\pi}{2}}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$$



After

left elliptically polarized light!!!

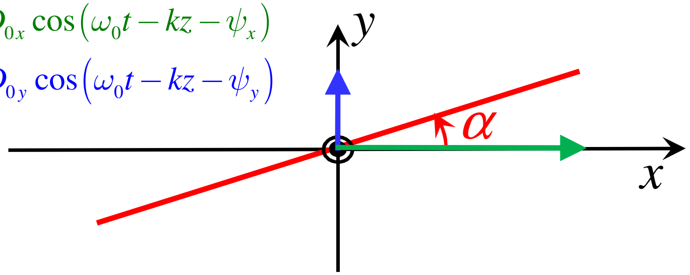
$$\psi_y - \psi_x = \frac{\pi}{2}; \quad \tan\alpha = \frac{D_{0y}}{D_{0x}}$$

How does a quarter wave plate change the polarization of a linearly polarized wave?

$$\boxed{\begin{aligned}\mathcal{D}_{0x} &\equiv D_{0x}\exp\left[i\psi_x\right]\\ \mathcal{D}_{0y} &\equiv D_{0y}\exp\left[i\psi_y\right]\end{aligned}}$$

$D_x = D_{0x}\cos\left(\omega_0 t - kz - \psi_x\right)$

$D_y = D_{0y}\cos\left(\omega_0 t - kz - \psi_y\right)$



$$\boxed{\varphi = \frac{2\pi}{\lambda}\left(n_e - n_o\right)d = \frac{\pi}{2}}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$$

What is the Jones vector for the initial state?

Before

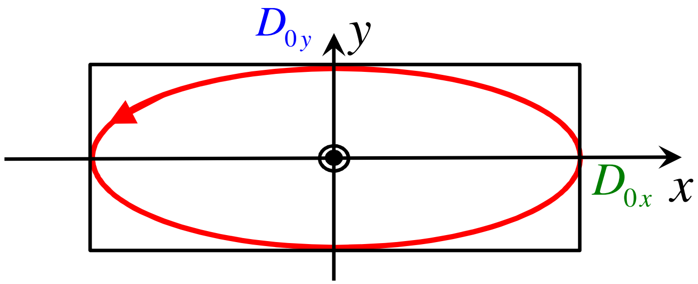
$$\begin{pmatrix}\mathcal{D}_x\\ \mathcal{D}_y\end{pmatrix} = R(\alpha)\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$$

Apply the quarter wave plate:

After

$$\begin{pmatrix}\mathcal{D}_x\\ \mathcal{D}_y\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix} = \begin{pmatrix}\cos\alpha\\ i\sin\alpha\end{pmatrix}$$

$$\Rightarrow \begin{cases}\psi_x = 0\\ \psi_y = \pi/2\end{cases}$$



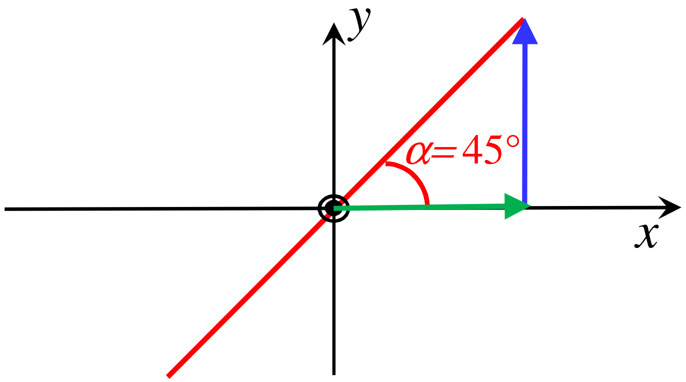
elliptically polarized light

$$D_x = \cos\alpha\cos\left(\omega_0 t - kz - 0\right) = \cos\alpha\cos\left(\omega_0 t - kz\right)$$

$$D_y = \sin\alpha\cos\left(\omega_0 t - kz - \frac{\pi}{2}\right) = \sin\alpha\sin\left(\omega_0 t - kz\right)$$

$$\psi_y - \psi_x = \frac{\pi}{2};\quad \tan\alpha = \frac{D_{0y}}{D_{0x}}$$

Quarter wave plate: special case, α = π/4



$$\cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}}$$

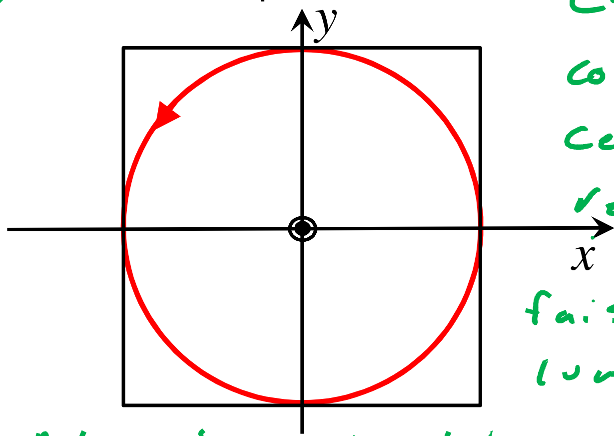
amplitudes are equal!

Recall: quarter wave plate adds π/2 phase delay.

Equal amplitudes + π/2 phase delay gives → Circular polarization!

$$D_x = \cos\alpha\cos(\omega_0 t - kz - 0) = \frac{1}{\sqrt{2}}\cos(\omega_0 t - kz)$$

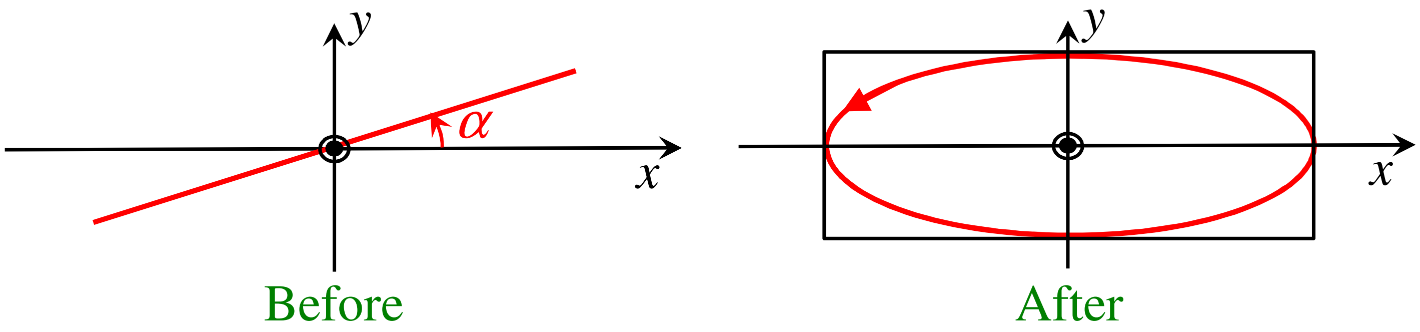
$$D_y = \sin\alpha\cos\left(\omega_0 t - kz - \frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}\sin(\omega_0 t - kz)$$



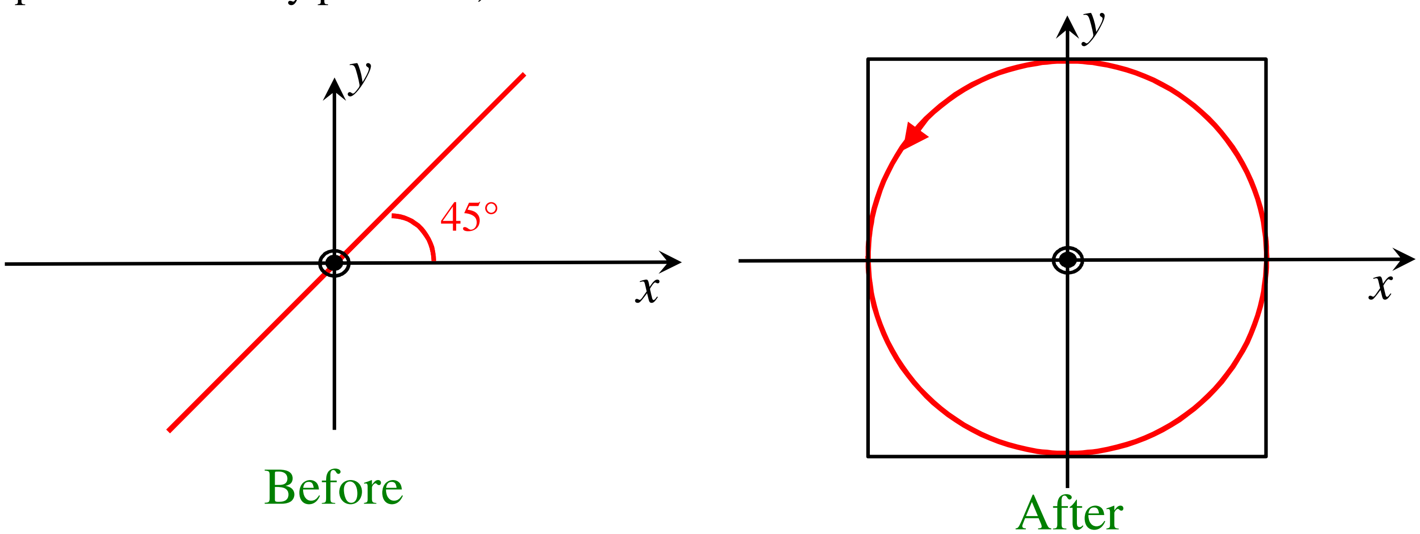
* C'est comme ceci qu'on réalise des faisceaux de lumière polarisée circulairement.

Quarter wave plate

Input wave linearly polarized:



Input wave linearly polarized, $\alpha=\pi/4$:



Half wave plate

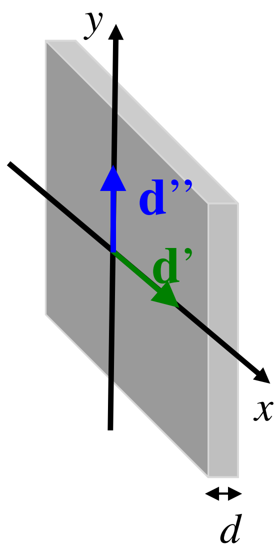
A phase of 2π corresponds to so a half wave plate corresponds to a phase delay of

Choose thickness *d* such that

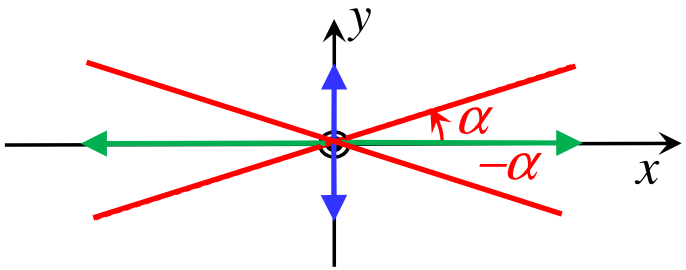
Phase delay: $$\boxed{\varphi = \frac{2\pi}{\lambda}\left(n_e - n_o\right)d = \pi}$$

$$\boxed{\mathbf{M} \propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \propto \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}}$$

$$\varphi = \pi$$

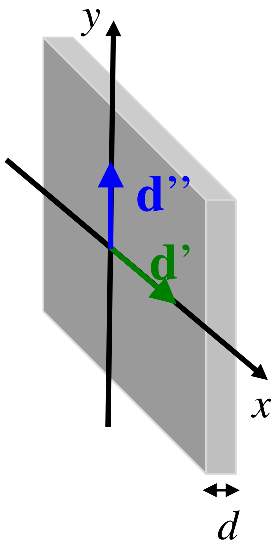


How does a half wave plate change the polarization of a linearly polarized wave?



$$\varphi = \frac{2\pi}{\lambda}\left(n_e - n_o\right)d = \pi$$

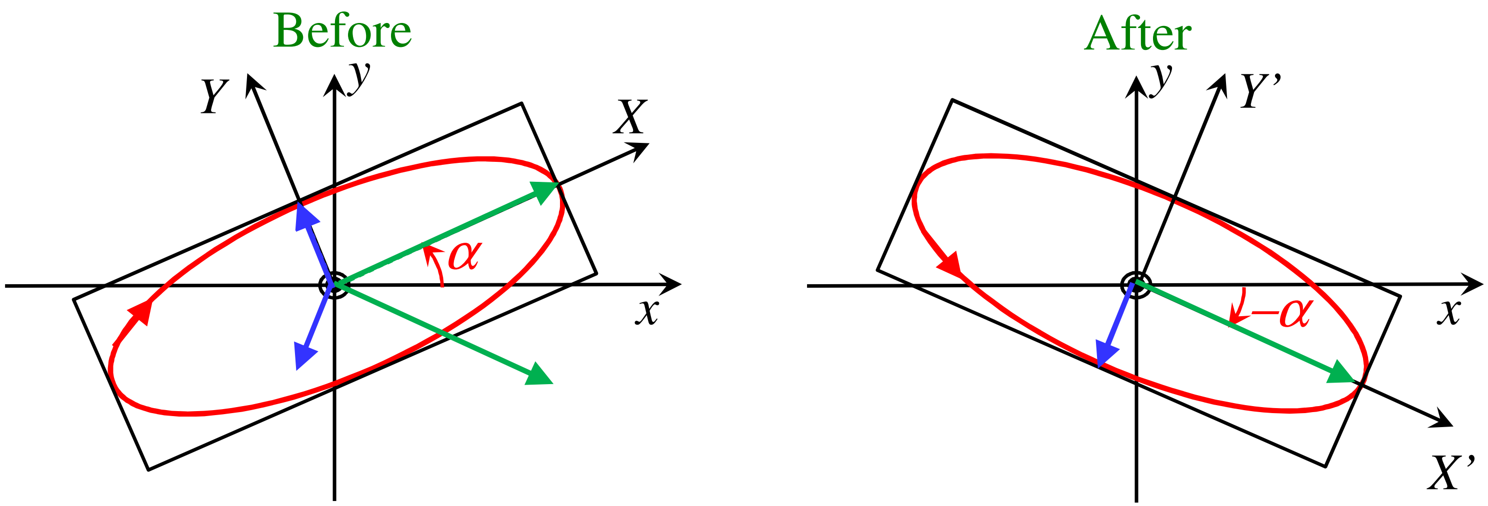
$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix}$$



Get **linearly** polarized light "rotated" by 2α

symmetrical with respect to $\mathbf{d}', \mathbf{d}''$

How does a half wave plate change the polarization of an elliptically polarized wave?

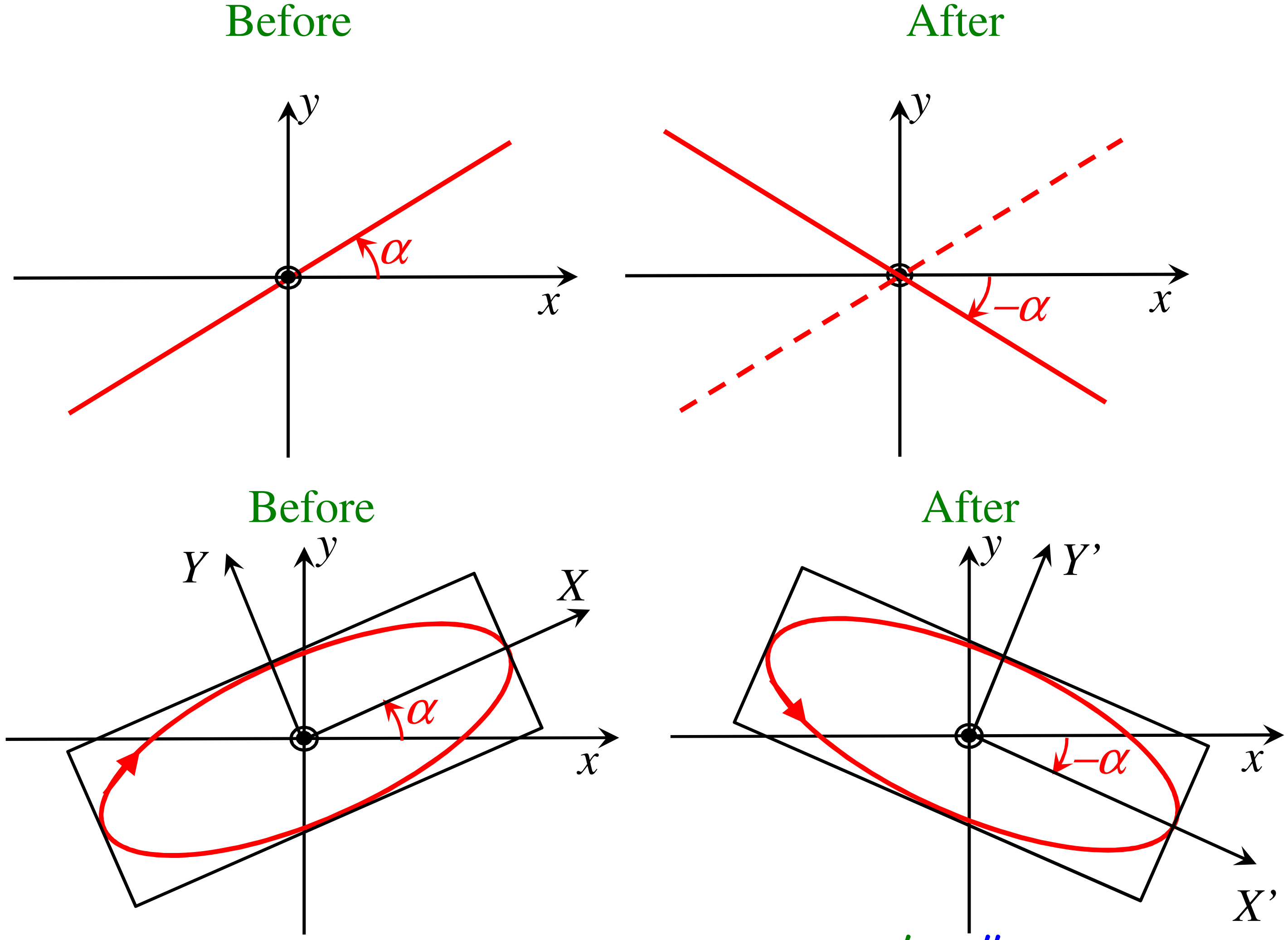


- Consider elliptical polarization as a sum of two linear polarizations
- Submit each y component of each linear polarization to a π delay

- Right polarization becomes left and vice versa!

Half wave plate

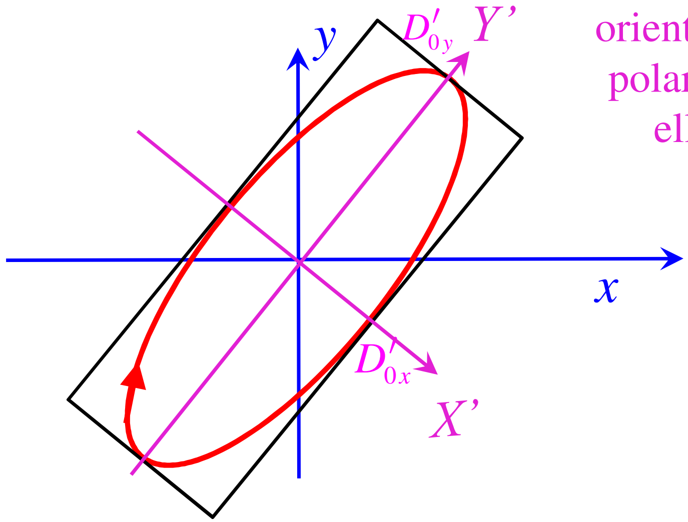
Wave plate axes
= x and y
= $\mathbf{d}'$, $\mathbf{d}''$



Symmetry with respect to $\mathbf{d}'$, $\mathbf{d}''$

Producing the desired state of polarization

Goal: elliptical polarization with a specific orientation and axis ratio

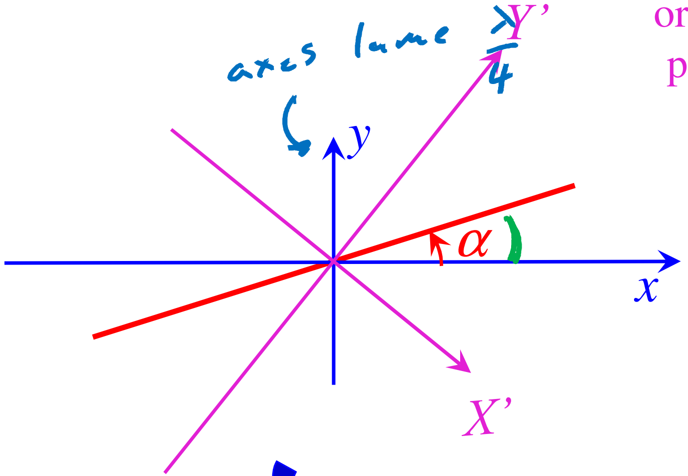


Desired orientation of polarization ellipse

$$\frac{D'_{0y}}{D'_{0x}} = r$$

Producing the desired state of polarization

1. Produce linearly polarized light with a polarizer

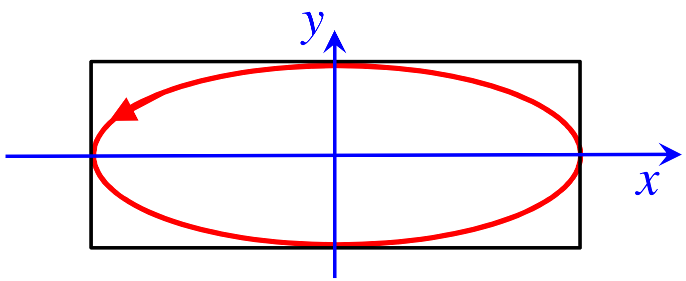


Desired orientation of polarization ellipse

2. Next step: use a quarter wave plate to change polarization to elliptical with the desired "aspect ratio" r

Orient quarter wave plate axes to get

$$r = \tan \alpha$$



Elliptical polarization with the right "shape" but wrong orientation

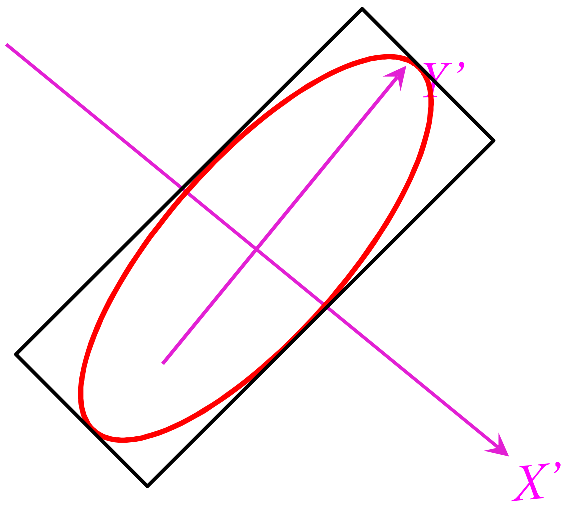
Producing the desired state of polarization

3. Next: use a half wave plate to change the polarization ellipse orientation

What should the half wave plate orientation be?

Half wave plate axis should bisect the angle between the current and desired ellipse axes!

Action of half wave plate



Polarization control in optical fibres

Optical fibres are made of isotropic media (glass). However, optical fibres exhibit *stress-induced birefringence*

When they are coiled (looped), they become anisotropic!

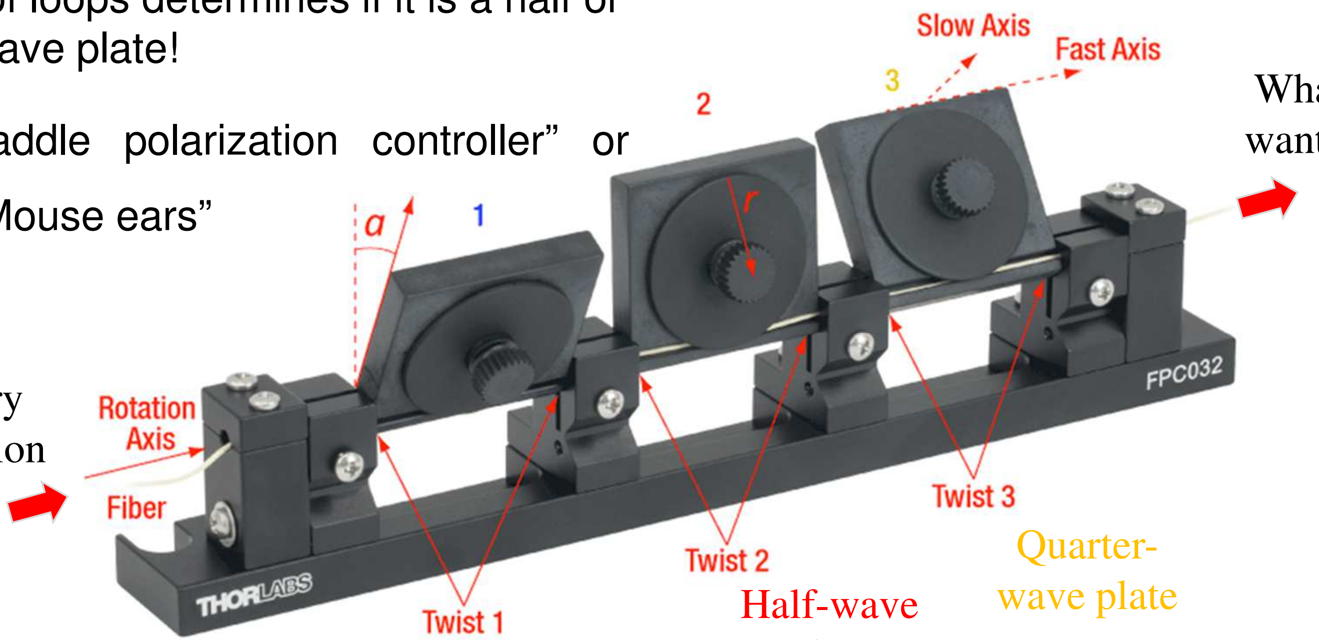
Can easily make quarter and half wave plates by looping fibre!

Number of loops determines if it is a half or quarter wave plate!

“Fiber paddle polarization controller” or

“Mickey Mouse ears”

Arbitrary polarization “in”



What you want “out”

Quarter-wave plate

Half-wave plate

Quarter-wave plate

https://youtu.be/5O7TL2SUAlo