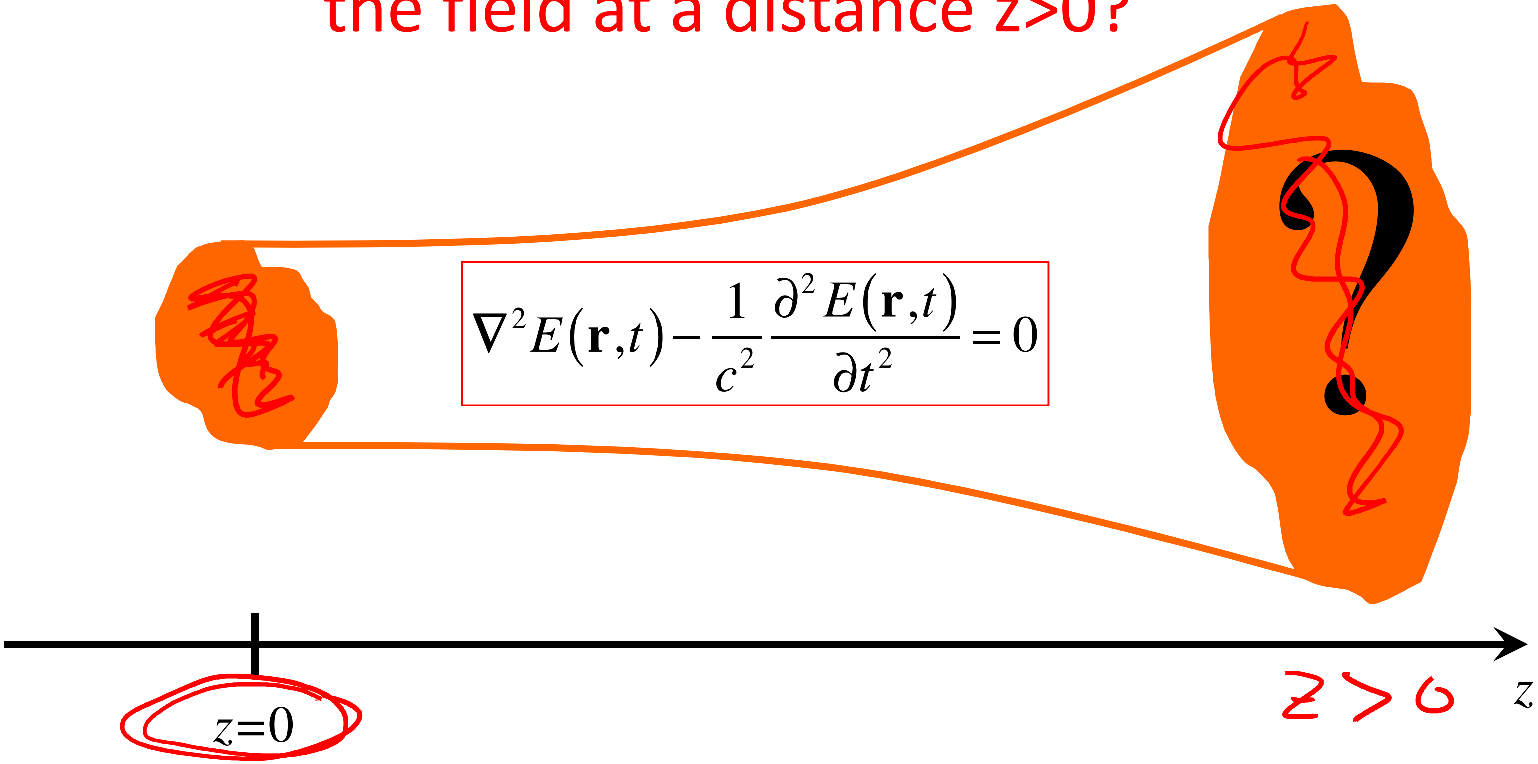
Bonjour!

Recall from last week: our goal

Knowing the distribution of the electric field on a plane at z=0 (e.g. on an aperture), can we find an expression for the field at a distance z>0?

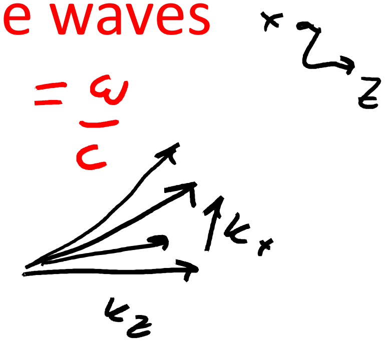


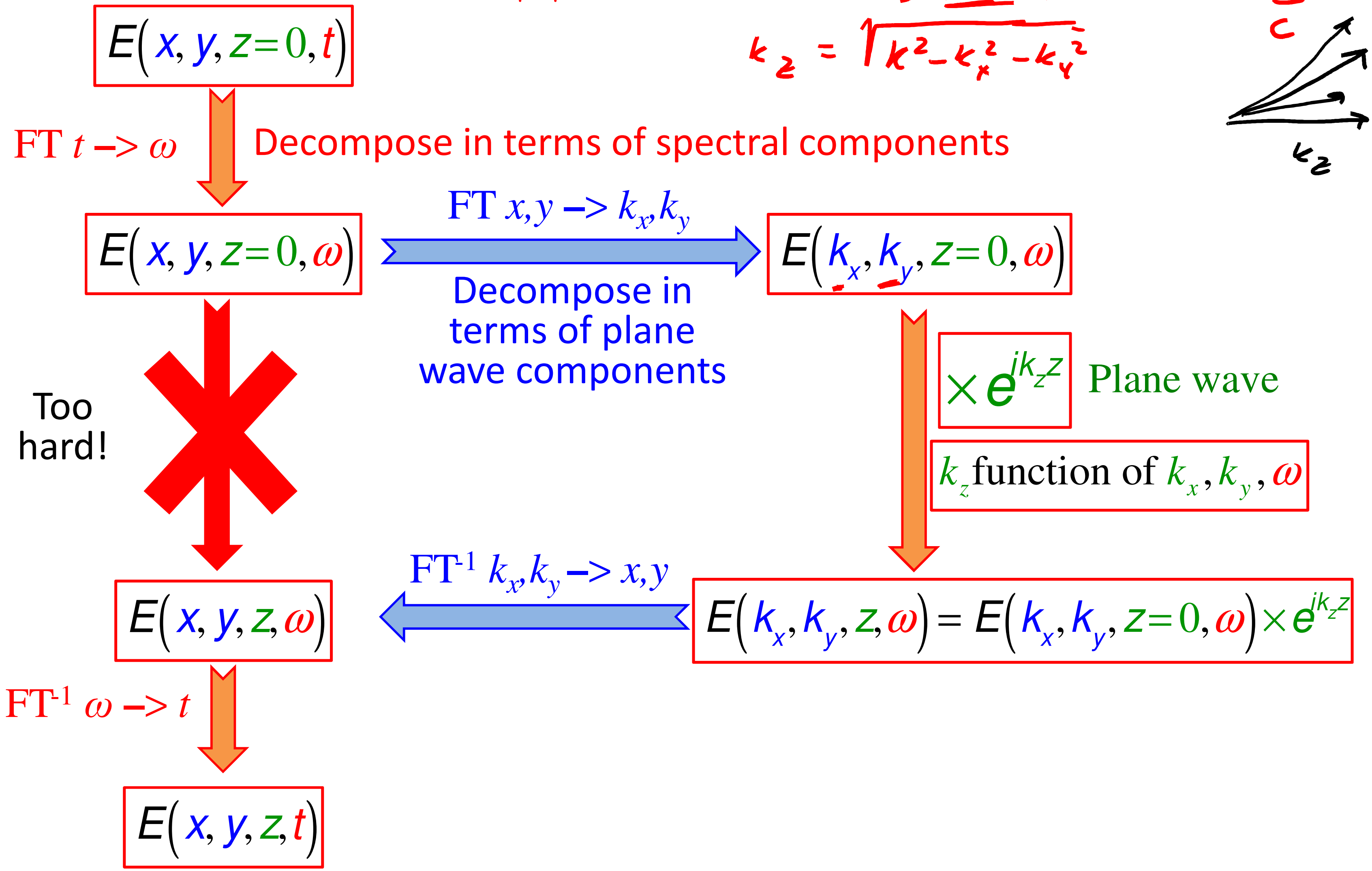
$$\boxed{\nabla^2 E(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2 E(\mathbf{r},t)}{\partial t^2} = 0}$$

Summary: express an arbitrary wave as a sum of plane waves (same $|k|$, different $k_x, k_y, k_z(k_x, k_y)$)

$|k| = \frac{\omega}{c}$

$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$





$E(x, y, z=0, t)$

FT $t \to \omega$ — Decompose in terms of spectral components

$E(x, y, z=0, \omega)$

FT $x,y \to k_x,k_y$ — Decompose in terms of plane wave components

$E(k_x, k_y, z=0, \omega)$

$\times e^{jk_z z}$ Plane wave

k_z function of k_x, k_y, ω

$E(k_x, k_y, z, \omega) = E(k_x, k_y, z=0, \omega) \times e^{jk_z z}$

FT^{-1} $k_x,k_y \to x,y$

$E(x, y, z, \omega)$

Too hard!

FT^{-1} $\omega \to t$

$E(x, y, z, t)$

Last week: express field as sum of plane waves

$|k| = \frac{2\pi}{\lambda}$

$$\boxed{E(x,y,z,\omega) = \frac{1}{(2\pi)^2}\iint dk_x\, dk_y\, E\left(k_x, k_y, z=0, \omega\right) e^{i\left(k_x x + k_y y + k_z z\right)}}$$

Ça se propage!

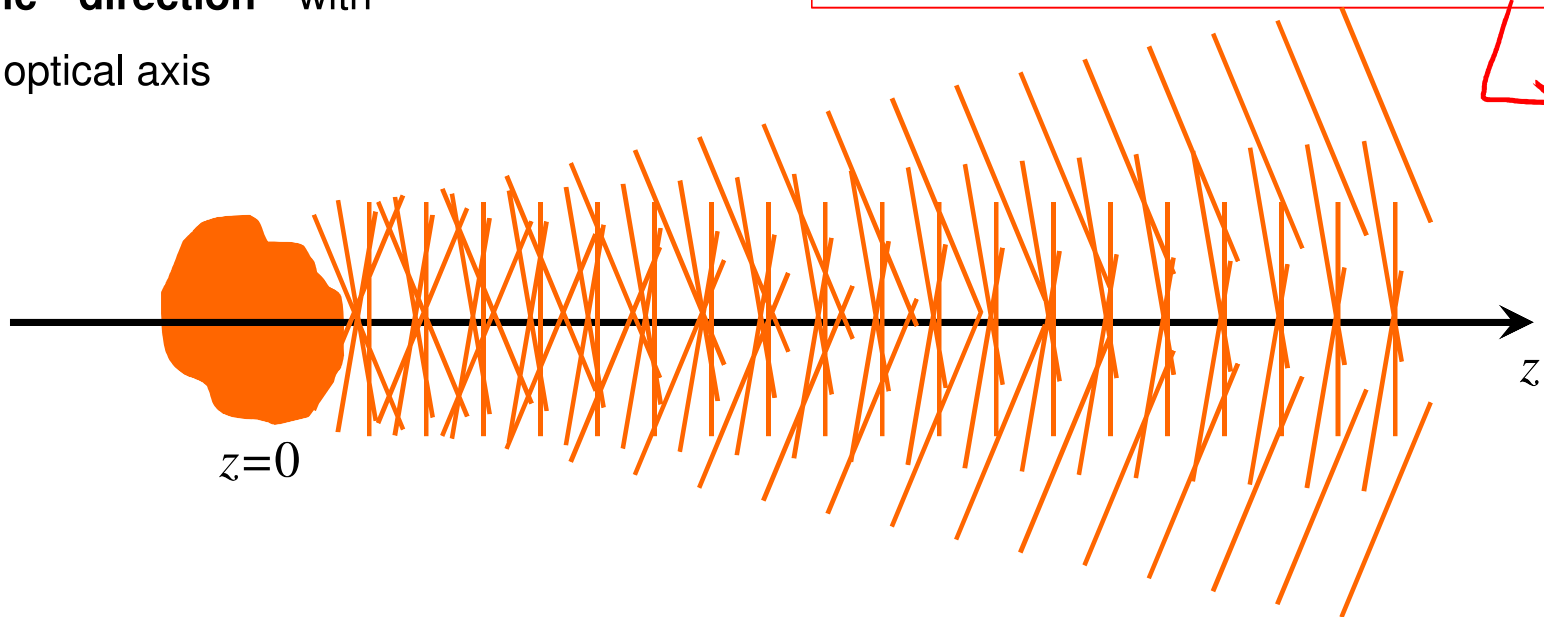
with: $$k_z = \begin{cases} \sqrt{\dfrac{\omega^2}{c^2} - k_x^2 - k_y^2} & \text{if} \quad \dfrac{\omega^2}{c^2} > k_x^2 + k_y^2 \\ i\sqrt{k_x^2 + k_y^2 - \dfrac{\omega^2}{c^2}} & \text{otherwise} \end{cases}$$

→ évanescent

⇒ Ne se propage PAS

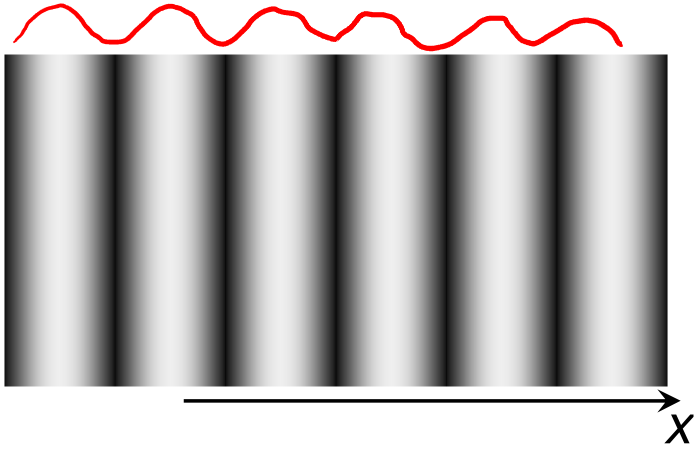
→ limite de diffraction

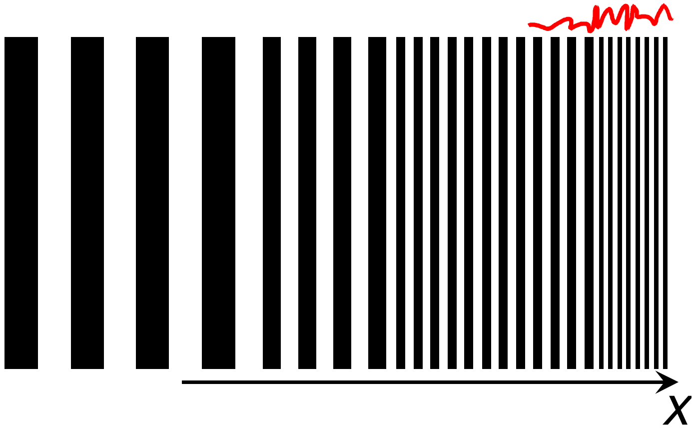
- Each wave in the sum corresponds to a **specific spatial frequency pair** k_x, k_y and propagates **in a specific direction** with respect to the optical axis

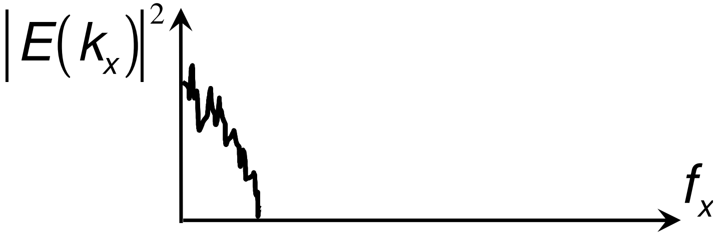


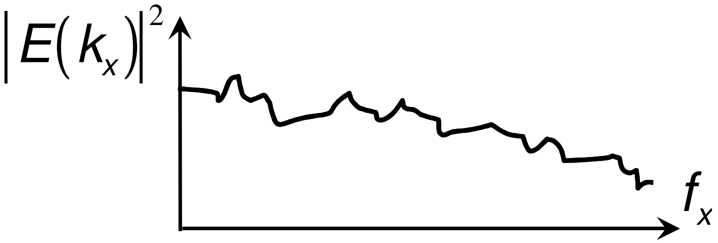
Concept of spatial frequencies

Which image has the most spatial frequency components?



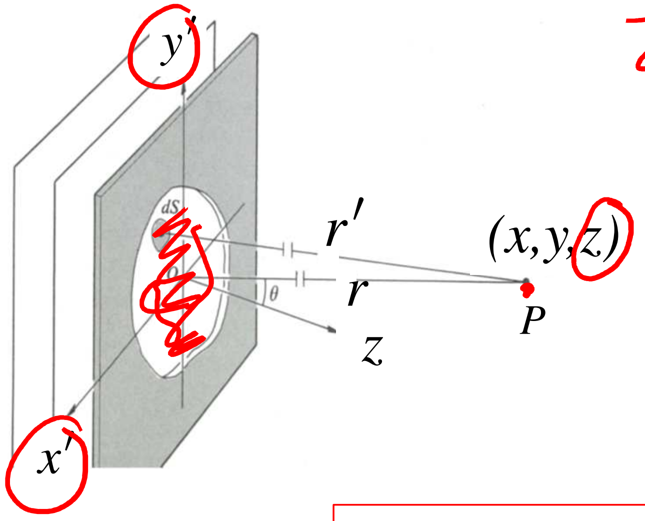






Recall: Fraunhofer approximation

$z \to \infty$



Fraunhofer condition:

Point of observation at a distance >> size of source

$z >> x', y'$

$$\boxed{E(x, y, z, \omega) = -\frac{i}{\lambda} E\left(k_x = \frac{kx}{z}, k_y = \frac{ky}{z}, z = 0, \omega\right)\frac{e^{ikz}}{z}}$$

Fourier transform as a function of *t*, *x* and *y* of the electric field

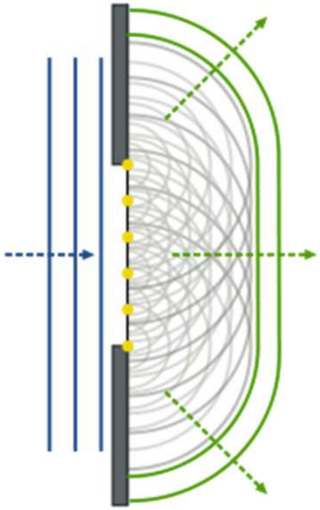
Far field diffraction = 2D spatial Fourier transform of incident field!!!

On fait des maths avec la lumière. ☺

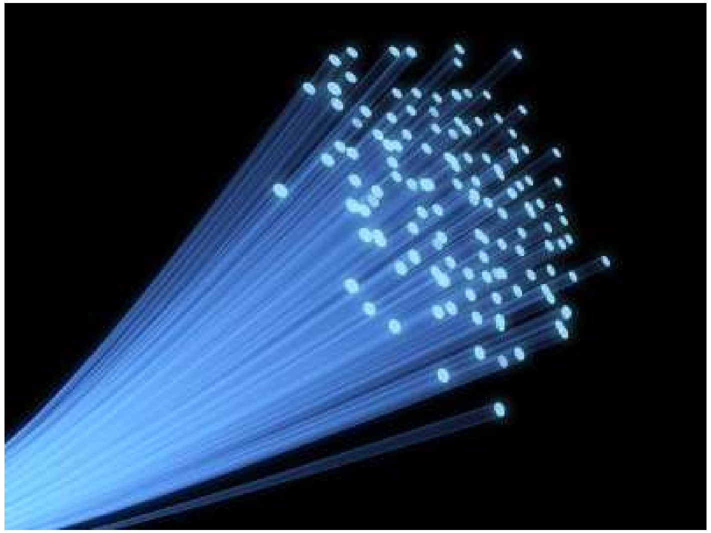
(More) diffraction and waveguides

Goals today:

- Express an arbitrary wave as a sum of *spherical* waves => "changement de base"
- Huygens-Fresnel principle
- Fresnel approximation—diffraction *before* the far-field
- Waveguides

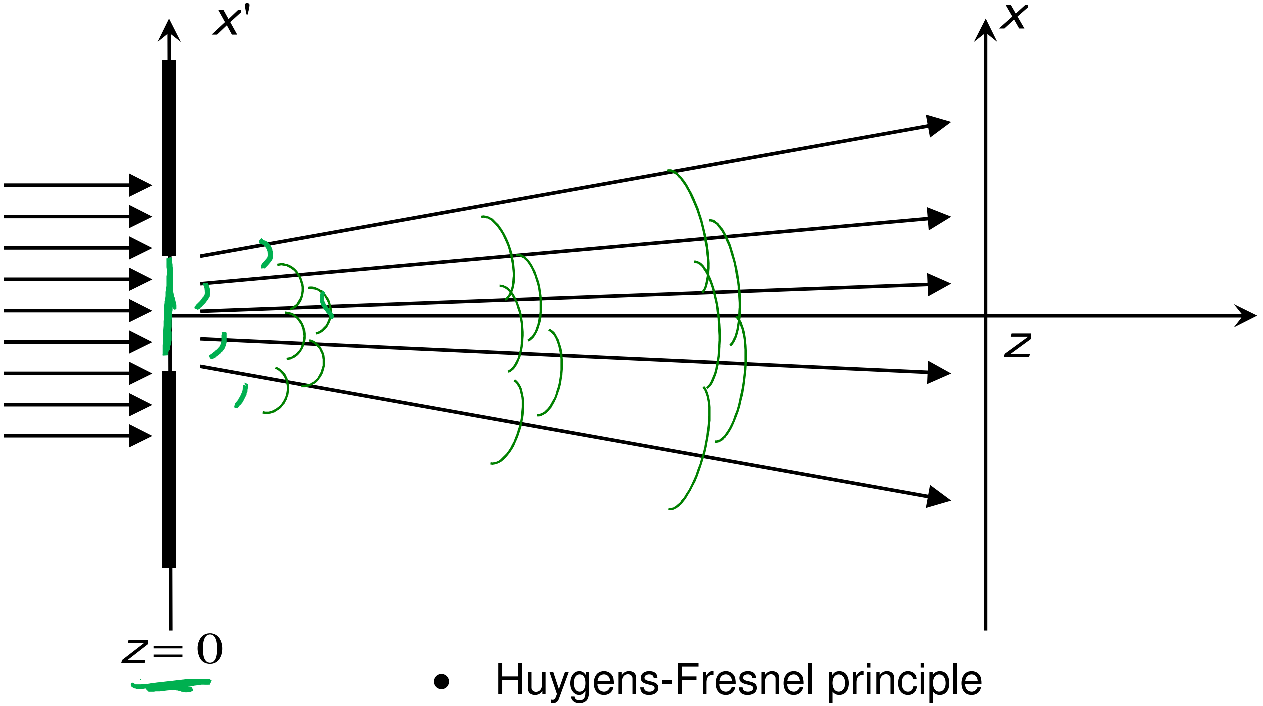


https://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle



https://qualitysurgicalrepairs.com/video_cameras__consoles___fiberoptic_cables

Now: express field as a sum of *spherical* waves



- Huygens-Fresnel principle
- Fresnel approximation—diffraction *before* the far-field

Arbitrary field as a sum of *spherical waves*: Rayleigh-Sommerfeld expression

$\vec{k} \equiv (k_x, k_y)$

$\vec{x} \equiv (x, y)$

$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$

Recall: Field as a sum of *plane* waves:

$$\boxed{E(x,y,z,\omega) = \frac{1}{(2\pi)^2}\iint dk_x dk_y \underbrace{E(k_x,k_y,z=0,\omega)}_{h(\vec{k})}\underbrace{e^{ik_z z}}_{g(\vec{k})} e^{i(k_x x + k_y y)}}$$

$TF^{-1}\{h(\vec{k})\, g(\vec{k})\}$

$$TF^{-1}\{h(\vec{k})\cdot g(\vec{k})\} = TF^{-1}\{h(\vec{k})\} \circledast TF^{-1}\{g(\vec{k})\}$$

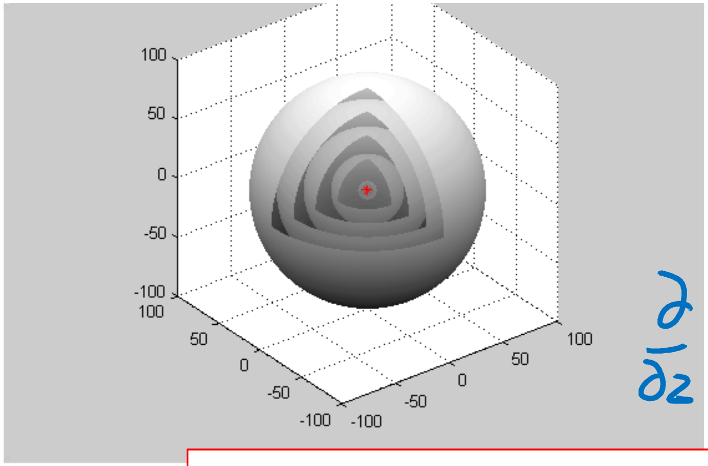
"The Fourier transform of a product is equal to the convolution of the separate Fourier transforms".

$$= h(\vec{x}) \circledast g(\vec{x})$$

$$= \iint_{-\infty}^{\infty} h(\vec{x}')\, g(\vec{x}-\vec{x}')\, d\vec{x}'$$

$$E(x,y,z,\omega) = E(x,y,z=0,\omega) \circledast \underbrace{TF^{-1}\{e^{ik_z z}\}}$$

Notre mission : trouver ceci



Weyl plane wave decomposition of a spherical wave

$$\frac{\partial}{\partial z}\;\boxed{\frac{\exp(ikr)}{r} = \frac{\overset{-1}{\cancel{i\,i}}}{2\pi}\iint dk_x \overset{\cancel{k_z}}{} dk_y \frac{\exp\left[i\left(k_x x + k_y y + k_z|z|\right)\right]}{\cancel{k_z}}} \qquad (3.18)$$

$$\boxed{E(x,y,z,\omega) = E(x,y,z=0,\omega) * FT^{-1}\left\{e^{ik_z z}\right\}}$$

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

In the search to find $FT^{-1}\left\{e^{ik_z z}\right\}$ calculate $\boxed{\frac{\partial}{\partial z}\frac{\exp(ikr)}{r}}$

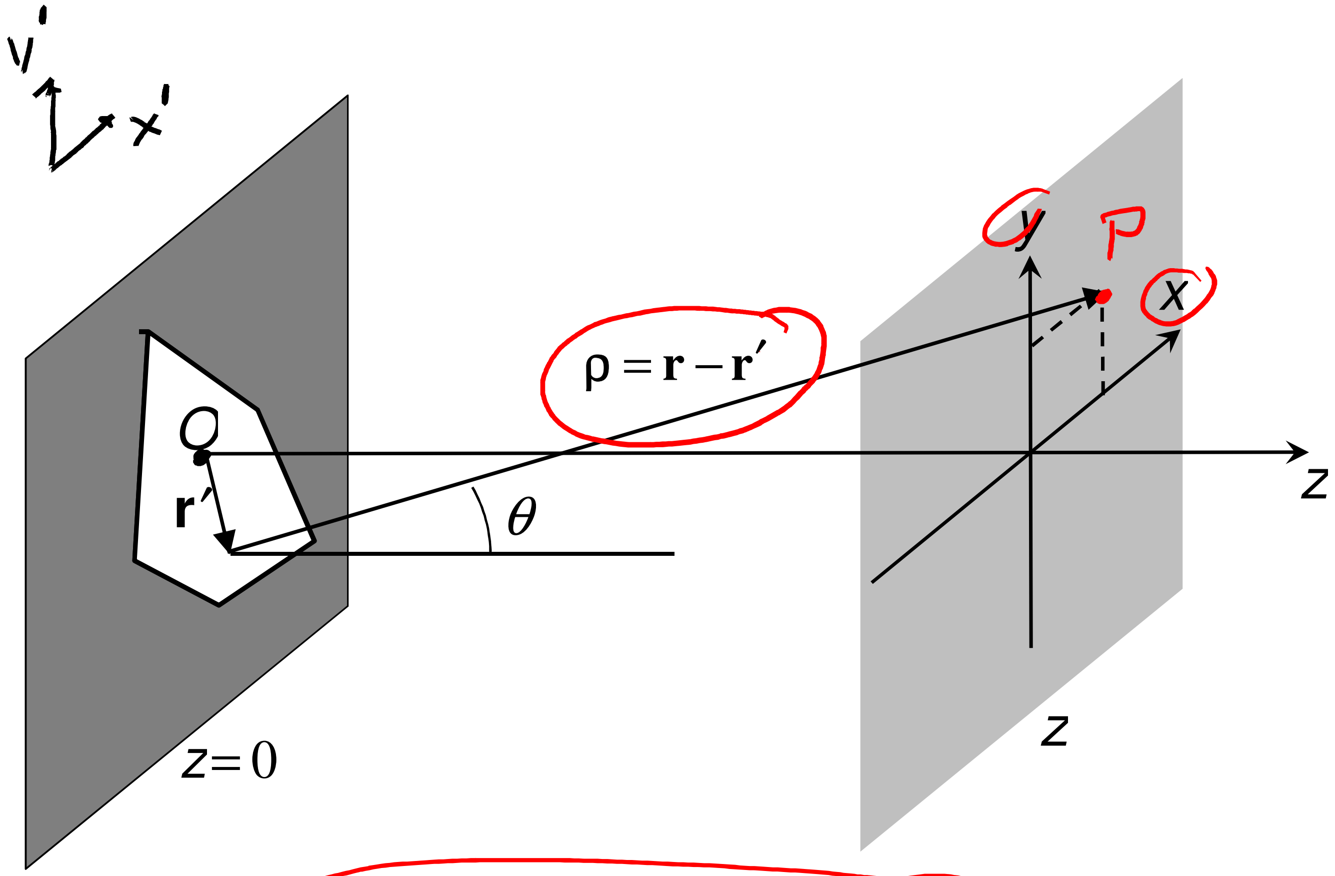
$$= -\frac{1}{2\pi}\iint dk_x\, dk_y\, e^{ik_x x + ik_y y}\, e^{ik_z z}$$

$$= -2\pi\ TF^{-1}\left\{e^{ik_z z}\right\}$$ Youhou !

$$E(x,y,z,\omega) = E(x,y,z=0,\omega) \circledast \left\{-\frac{1}{2\pi}\frac{\partial}{\partial z}\frac{e^{ikr}}{r}\right\}$$

Geometry

entre
$\vec{r}$: origin
at
point P



$$\rho(\mathbf{r},\mathbf{r}') = \|\mathbf{r}-\mathbf{r}'\| = \sqrt{(x-x')^2 + (y-y')^2 + z^2}$$

Towards the Rayleigh-Sommerfield relation

$$E(x,y,z,\omega) = E(x,y,z=0,\omega) * \left(-\frac{1}{2\pi}\frac{\partial}{\partial z}\frac{\exp(ikr)}{r}\right)$$

$$h(\vec{x}) * g(\vec{x}) = \int_{-\infty}^{\infty} d\vec{x}'\, h(\vec{x}')\, g(\vec{x}-\vec{x}') \qquad \rho(\mathbf{r},\mathbf{r}') = \|\mathbf{r}-\mathbf{r}'\| = \sqrt{(x-x')^2+(y-y')^2+z^2}$$

$$E(x,y,z,\omega) = \iint_{-\infty}^{\infty} dx'dy'\, E(x',y',z=0,\omega)\left[-\frac{1}{2\pi}\frac{\partial}{\partial z}\frac{e^{ik\rho(\vec{r},\vec{r}')}}{\rho(\vec{r},\vec{r}')}\right]$$

- Relation Rayleigh-Sommerfeld

Towards the Huygens-Fresnel principle

$$\boxed{E(x,y,z,\omega) = -\frac{1}{2\pi}\iint dx' dy' E(x',y',z=0,\omega)\frac{\partial}{\partial z}\frac{\exp\left[ik\rho(\mathbf{r},\mathbf{r}')\right]}{\rho(\mathbf{r},\mathbf{r}')}} \qquad (3.20)$$

Rayleigh-Sommerfeld relation

$$\frac{\partial \rho}{\partial z} = \frac{1}{\cancel{2}}\frac{1}{\rho}\cancel{2}z = \frac{z}{\rho}$$

Find: $\frac{\partial}{\partial z}\frac{e^{ik\rho}}{\rho}$ $\qquad \rho(\mathbf{r},\mathbf{r}') = \left\|\mathbf{r}-\mathbf{r}'\right\| = \sqrt{(x-x')^2+(y-y')^2+z^2}$

$$\frac{\partial}{\partial z}\frac{e^{ik\rho}}{\rho} = -\frac{1}{\rho^2}e^{ik\rho}\frac{\partial \rho}{\partial z} + ik\frac{e^{ik\rho}}{\rho}\frac{\partial \rho}{\partial z} = \frac{z}{\rho}\frac{e^{ik\rho}}{\rho}\left[-\frac{1}{\rho}+ik\right]$$

$$E(x,y,z,\omega) = -\frac{1}{2\pi}\iint dx' dy' E(x',y',z=0,\omega)\frac{\exp\left[ik\rho(\mathbf{r},\mathbf{r}')\right]}{\rho(\mathbf{r},\mathbf{r}')^2}z\left(-\frac{1}{\rho(\mathbf{r},\mathbf{r}')}+ik\right)$$

$\rho(\mathbf{r},\mathbf{r}') = \|\mathbf{r}-\mathbf{r}'\| = \sqrt{(x-x')^2+(y-y')^2+z^2}$

$\rho \geq z$

Huygens-Fresnel Principle

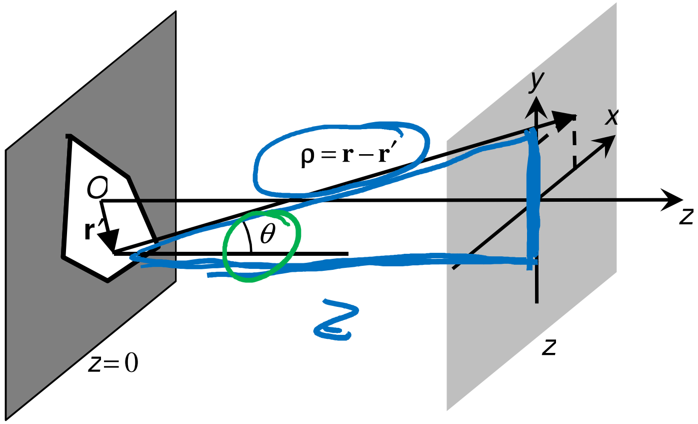
$k = 2\pi/\lambda$

$z >> \lambda$ => PAS d'ondes évanescentes

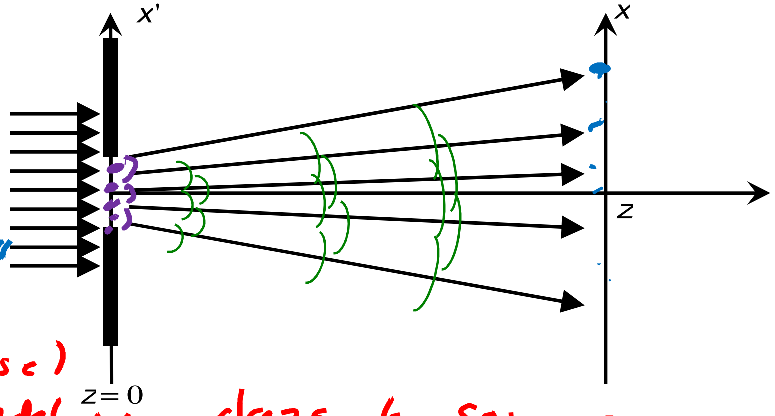
$$E(x,y,z,\omega) = -\frac{1}{2\pi}\iint dx'dy' E(x',y',z=0,\omega)\frac{\exp[ik\rho(\mathbf{r},\mathbf{r}')]}{\rho(\mathbf{r},\mathbf{r}')^2} z\left(-\frac{1}{\rho(\mathbf{r},\mathbf{r}')}+ik\right)$$

$-\frac{1}{\cancel{2\pi}} \cdot \frac{i\cancel{2\pi}}{\lambda} = -\frac{i}{\lambda}$

↳ $\frac{z}{z} - \frac{1}{z} + \frac{i2\pi}{\lambda}$



$\frac{z}{\rho} = \cos\theta$ "facteur d'obliquité"



amplitude (et phase) de chaque ondelette dans la somme

$$E(x,y,z,\omega) = -\frac{i}{\lambda}\iint dx'dy' E(x',y',z=0,\omega)\frac{\exp[ik\rho(\mathbf{r},\mathbf{r}')]}{\rho(\mathbf{r},\mathbf{r}')}\cos\theta \qquad (3.24)$$

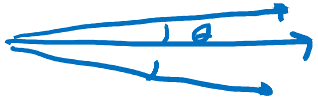
→ ondes sphériques centrées en $\vec{r}'$

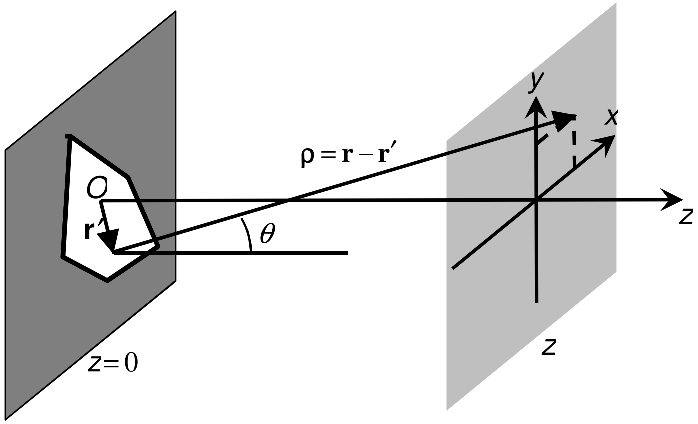
ondelette

Huygens-Fresnel principle

→ interférence

Fresnel approximation





- Valid "before the far field" (to be defined more precisely).

$$\rho(\mathbf{r},\mathbf{r}') = \|\mathbf{r}-\mathbf{r}'\| = \sqrt{(x-x')^2+(y-y')^2+z^2}$$

on ne considère que des ondes 'près' de l'axe optique (paraxiale)

$$E(x,y,z,\omega) = -\frac{i}{\lambda}\iint dx'dy' E(x',y',z=0,\omega)\frac{\exp[ik\rho(\mathbf{r},\mathbf{r}')]}{\rho(\mathbf{r},\mathbf{r}')}\underline{\underline{\cos\theta}} \qquad (3.24)$$

$\cos\theta \approx 1$

$$\frac{1}{\rho} \approx \frac{1}{z}$$

Doit fait plus d'attention pour le terme $e^{ik\rho}$

Négligé dans l'approx. Fraunhofer

$$\rho = z\left[1 + \frac{x^2+y^2}{z^2} + \frac{x'^2+y'^2}{z^2} - \frac{2xx'+2yy'}{z^2}\right]^{1/2}$$

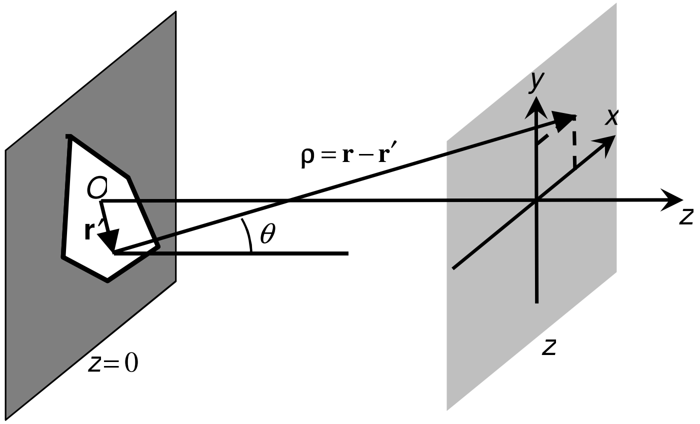
Fresnel: on garde tous les termes. mais on fait un DL

DL

$$\rho = z\left(1 + \frac{1}{2}\frac{(x-x')^2}{z^2} + \frac{1}{2}\frac{(y-y')^2}{z^2}\right)$$

$z^2 \gg (x-x')^2, (y-y')^2$

Fresnel approximation



$$\rho(\mathbf{r},\mathbf{r}') = \|\mathbf{r}-\mathbf{r}'\| = \sqrt{(x-x')^2+(y-y')^2+z^2}$$

$$\rho(\mathbf{r},\mathbf{r}') \simeq z\left[1+\frac{1}{2}\left(\frac{x-x'}{z}\right)^2+\frac{1}{2}\left(\frac{y-y'}{z}\right)^2\right]$$

$$E(x,y,z,\omega) = -\frac{i}{\lambda}\iint dx'dy' E(x',y',z=0,\omega)\frac{\exp\left[ik\rho(\mathbf{r},\mathbf{r}')\right]}{\rho(\mathbf{r},\mathbf{r}')}\cos\theta \qquad (3.24)$$

$$\Rightarrow \boxed{E(x,y,z,\omega) \simeq -\frac{i}{\lambda}\frac{e^{ikz}}{z}\iint dx'dy' E(x',y',z=0,\omega)\, e^{\frac{ik}{2z}(x-x')^2+\frac{ik}{2z}(y-y')^2}}$$

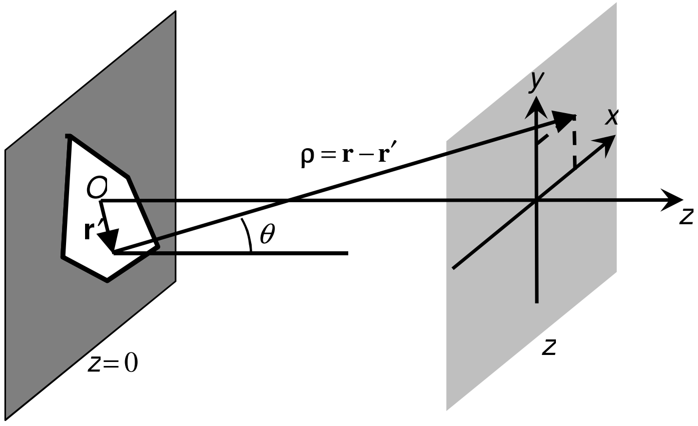
Diffraction de Fresnel

$z >> |x-x'|, |y-y'|$

Fresnel approximation: summary

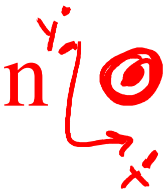
$$E(x,y,z,\omega)=-\frac{ie^{ikz}}{\lambda z}\iint dx'dy'E(x',y',z=0,\omega)\exp\left\{i\frac{k}{2z}\left[(x-x')^2+(y-y')^2\right]\right\}$$

Fraunhofer



Fresnel : fronts d'ondes de forme parabolique!

Connection to Fraunhofer approximation



$z >> x', y'$

Starting with

Diffraction Fresnel

$$E(x,y,z,\omega) = -\frac{ie^{ikz}}{\lambda z}\iint dx'dy' E(x',y',z=0,\omega)\exp\left\{i\frac{k}{2z}\underbrace{\left[(x-x')^2+(y-y')^2\right]}\right\} \qquad (3.48)$$

$$\frac{k}{2z}\left[(x-x')^2+(y-y')^2\right] = \frac{k}{2z}\left(\boxed{\underbrace{x^2+y^2}_{\text{Ne contribue qu'un terme de phase}} + \cancel{x'^2+y'^2}} - 2xx' - 2y\underline{y'}\right)$$

→ 0 pour Fraunhofer

$z >> x', y'$

$$\Rightarrow E(x,y,z,\omega) = -\frac{i}{\lambda}\frac{e^{ikz}}{z}\iint dx'dy' E(x',y',0,\omega)\, e^{-ik\frac{x}{z}x' - ik\frac{y}{z}y'}$$

$$\boxed{E(x,y,z,\omega) = -\frac{i}{\lambda}E\left(k_x = \frac{kx}{z}, k_y = \frac{ky}{z}, z=0, \omega\right)\frac{e^{ikz}}{z}}$$

Fraunhofer
(ça marche !)

Link between Fresnel diffraction and the plane wave expansion

Fresnel Difraction:

$$E(x,y,z,\omega) = -\frac{ie^{ikz}}{\lambda z}\iint dx'dy' \underline{E(x',y',z=0,\omega)} \exp\left\{ i\frac{k}{2z}\left[(x-x')^2+(y-y')^2\right]\right\}$$

$$E(x,y,z,\omega) = E(x',y',z=0,\omega) \circledast h_{fresnel}(x,y,z,\omega)$$

Fresnel Diffraction = convolution of the field at $z = 0$ with the transfer function

$$h_{\text{Fresnel}}(x,y,z,\omega) = -\frac{ie^{ikz}}{\lambda z}\exp\left[i\frac{k}{2z}\left(x^2+y^2\right)\right]$$

The FT of this transfer function is:

$$h_{\text{Fresnel}}(k_x,k_y,z,\omega) = e^{ikz}\exp\left[-i\frac{z}{2k}\left(k_x^2+k_y^2\right)\right]$$

$$F(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right) \longleftrightarrow F(\omega) = \sqrt{2\pi}\sigma\exp\left(-\frac{\omega^2\sigma^2}{2}\right)$$

$t \to x,y$

$\frac{1}{\sigma^2} \to -\frac{ik}{z}$

$$E(k_x,k_y,z,\omega) = E(k_x,k_y,z=0,\omega)\bullet h_{Fresnel}(k_x,k_y,z,\omega)$$

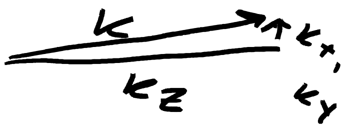
Link between Fresnel diffraction and the plane wave expansion

Fresnel: $E\left(k_x,k_y,z,\omega\right)=E\left(k_x,k_y,z=0,\omega\right)\bullet h_{Fresnel}(k_x,k_y,z,\omega)$ $\boxed{h_{\text{Fresnel}}\left(k_x,k_y,z,\omega\right)=e^{ikz}\exp\left[-i\frac{z}{2k}\left(k_x^2+k_y^2\right)\right]}$

Plane wave expansion:

$$E\left(k_x,k_y,z,\omega\right)=E\left(k_x,k_y,z=0,\omega\right)e^{ik_z z}$$

On applique les approx. de Fresnel



approx. paraxiale

Plane wave expansion = product of field at $z = 0$ and transfer function

$z >> \lambda$

$$h_{\text{plane\_waves}}\left(k_x,k_y,z,\omega\right)=\begin{cases}\exp\left[ikz\sqrt{1-\frac{k_x^2}{k^2}-\frac{k_y^2}{k^2}}\right] & if\ \ k_x^2+k_x^2<k^2\\ 0 & \text{otherwise for } \underline{z\gg\lambda}\quad (\text{evanescent waves})\end{cases}$$

PAS de

car approx. paraxiale

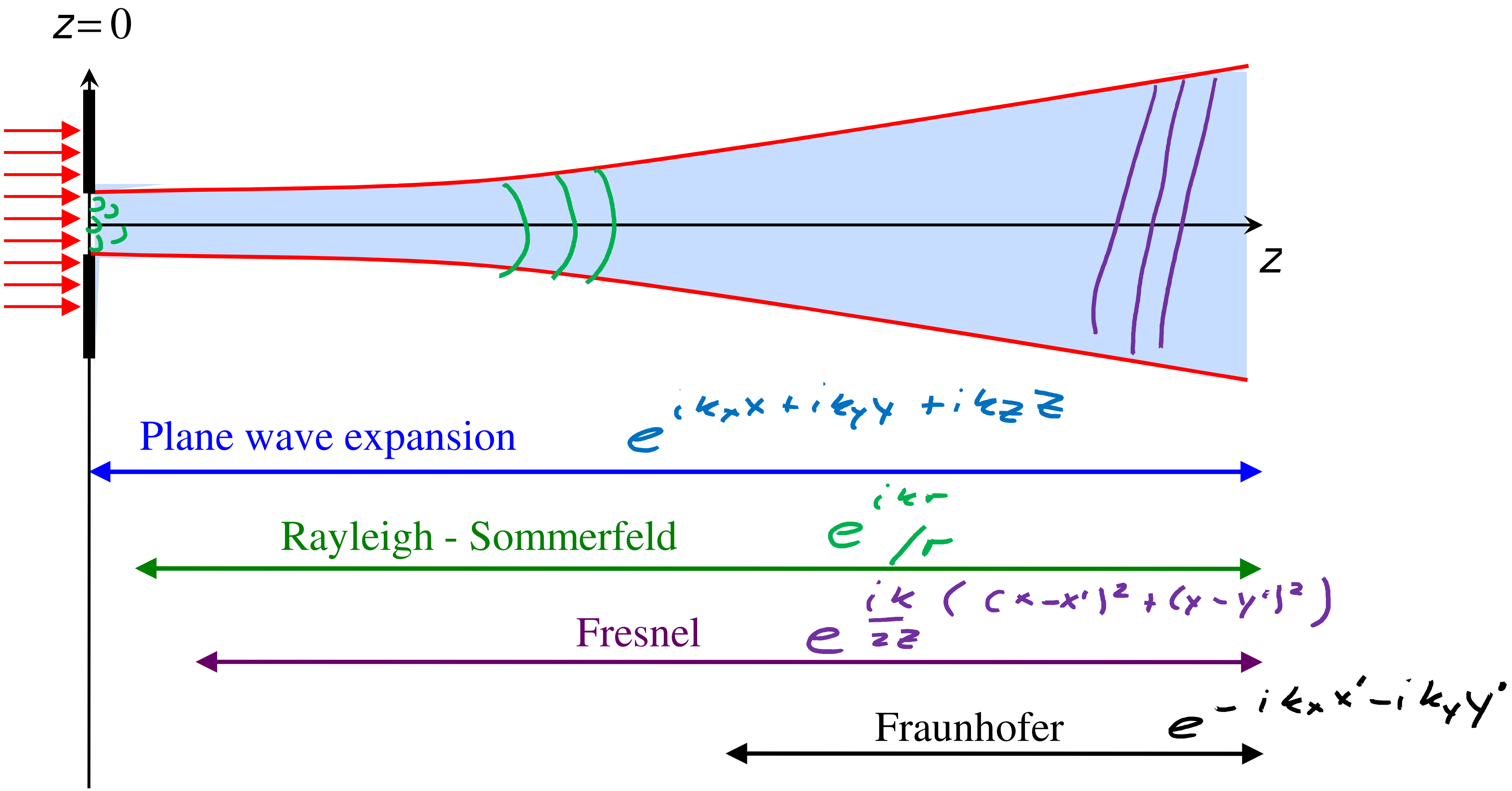
$k >> k_x, k_y$

$$\sqrt{1-\frac{k_x^2}{k^2}-\frac{k_y^2}{k^2}} \approx 1-\frac{1}{2}\frac{(k_x^2+k_y^2)}{k^2}$$

$$h_{\text{plane\_waves}}\left(k_x,k_y,z,\omega\right)\simeq e^{ikz}\exp\left[-i\frac{z}{2}\left(\frac{k_x^2}{k}+\frac{k_y^2}{k}\right)\right]=h_{\text{Fresnel}}\left(k_x,k_y,z,\omega\right)$$

Thus this confirms that the Fresnel approximation is valid for k_x, $k_y << k$,
i.e., for *small diffraction angles* => PARAXIAL APPROXIMATION

Conclusion: validity of the different formulations for diffraction



Plane wave expansion $e^{ik_x x + ik_y y + ik_z z}$

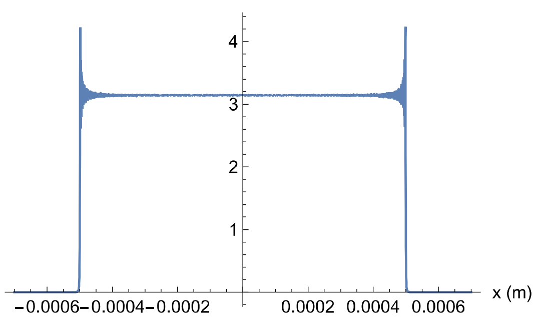
Rayleigh - Sommerfeld e^{ikr}/r

Fresnel $e^{\frac{ik}{2z}\left((x-x')^2+(y-y')^2\right)}$

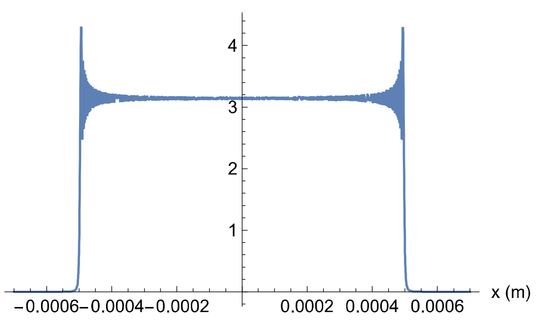
Fraunhofer $e^{-ik_x x' - ik_y y'}$

Example: Fresnel diffraction for a slit of width w =1 mm; λ = 0.5 µm

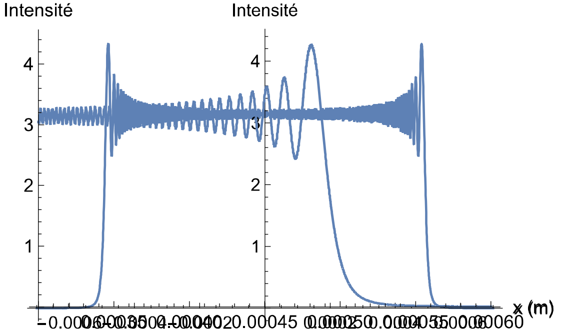
z = 10 µm



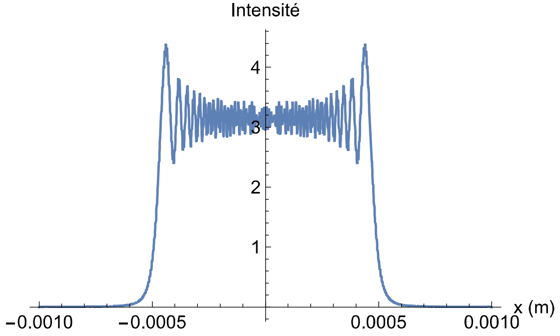
z = 100 µm



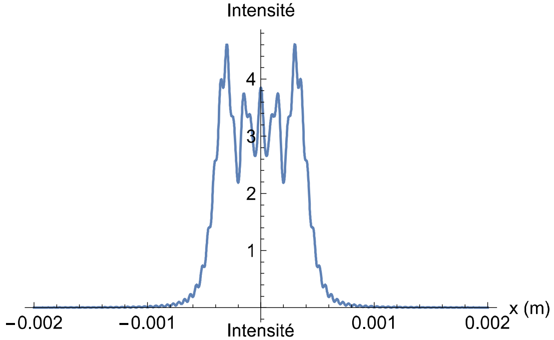
z = 1 mm



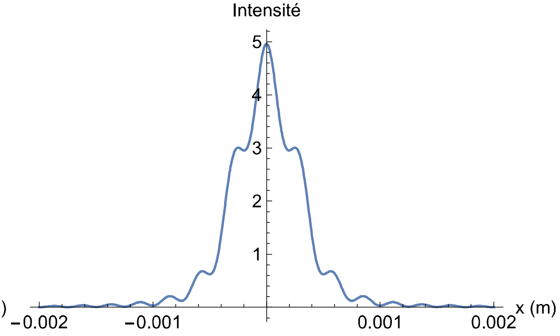
z = 1 cm



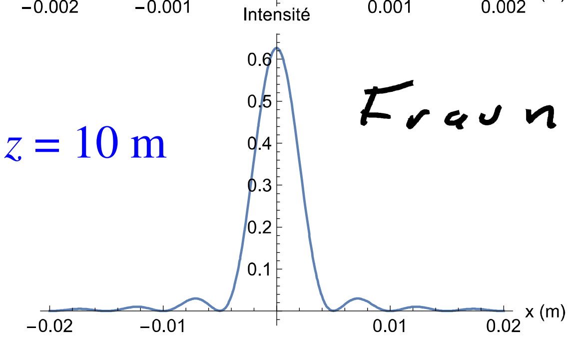
z = 10 cm



z = 50 cm

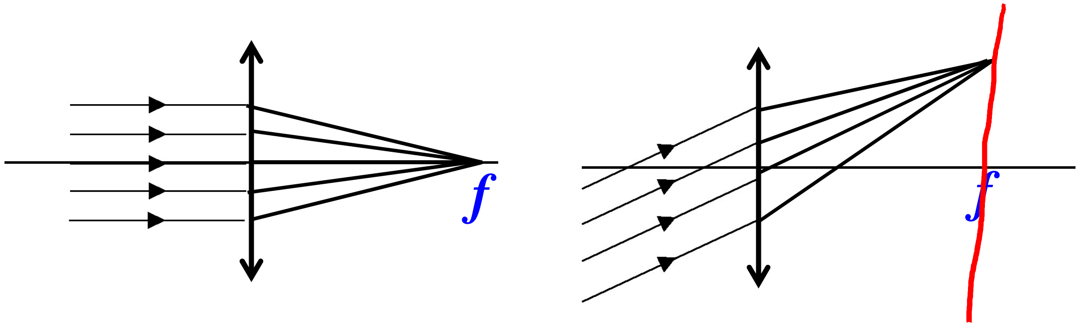


z = 10 m



Fraunhofer

What does a lens do?



plan arrière
focal ou plan Fourier

"Amène l'infini
au plan Fourier"

Diffraction : qui y-t-il à ∞ ?

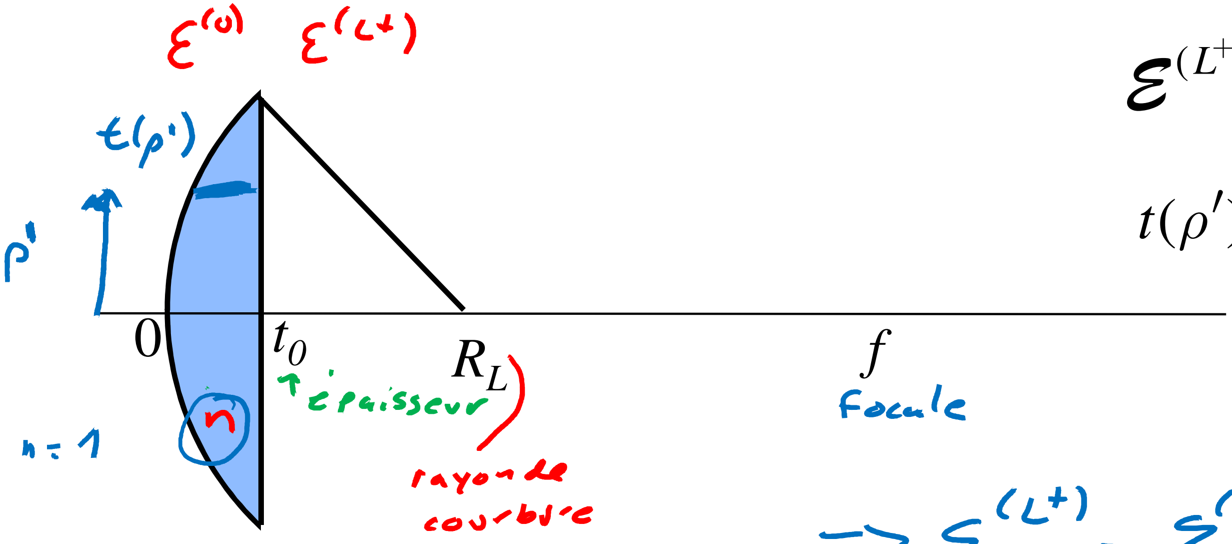
$\Rightarrow$ Diffraction de Fraunhofer

ou : TF du champs à $z = 0$.

Quel est le lien entre champs
à $z = 0$ et plan focal arrière ??

Trouver

What does a lens do? Transfer function pour lentille



$$\rho' = \left(x'^2 + y'^2\right)^{1/2}$$

$$\mathcal{E}^{(L^+)} = \mathcal{E}^{(0)}\, \mathrm{e}^{ik(t_0 - t)}\, e^{inkt}$$

$$t(\rho') = \sqrt{R_L^2 - \rho'^2} - \left(R_L - t_0\right)$$

⇒ Régime paraxiale

$\rho' < R_L$

DL

$$\boxed{t(\rho') \approx t_0 - \frac{\rho'^2}{2R_L}}$$

$$\Rightarrow \mathcal{E}^{(L^+)} = \mathcal{E}^{(0)}\, e^{+ik\frac{\rho'^2}{2R_L}}\, e^{\cancel{inkt_0} - ink\frac{\rho'^2}{2R_L}}$$

$-1/f$

$$= \mathcal{E}^{(0)}\, e^{+ik\frac{\rho'^2}{2}\left[\frac{1-n}{R_L}\right]}$$

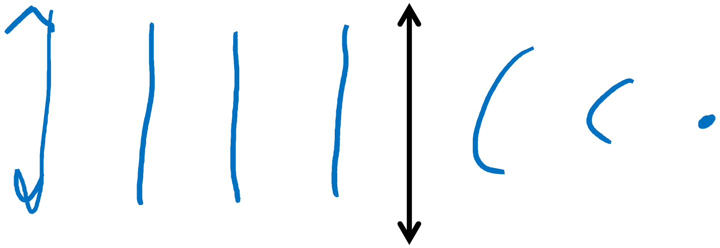
$$\boxed{\mathcal{E}^{(L^+)} = \mathcal{E}^{(0)}\, \underbrace{e^{-ik\frac{\rho'^2}{2f}}}} \quad \left[e^{inkt_0}\right]$$

→ Fonc. de transfert lentille

Thin lens approximation:
neglect e^{inkt_0} term.

Recall: $$f = \frac{\overset{=1}{n_1}}{\underset{n}{n_2} - n_1} R_L$$

What does a lens do?



- Ideal thin lens acts as a plane wave to paraxial spherical wave converter

$$\boxed{t(x', y') = \exp\left[-i\frac{k}{2f}\left(\underline{x'^2} + \underline{y'^2}\right)\right]}$$

Field in the focal plane of a lens

Recall: Fresnel diffraction:

$$E(x,y,z,\omega) = -\frac{ie^{ikz}}{\lambda z}\iint dx'dy' E(x',y',z=0,\omega)\exp\left\{i\frac{k}{2z}\left[(x-x')^2+(y-y')^2\right]\right\}$$

$$E(x,y,z,\omega) = E(x,y,z=0,\omega) * h_{Fresnel}(x,y,z,\omega)$$

$$h_{\text{Fresnel}}(x,y,z,\omega) = -\frac{ie^{ikz}}{\lambda z}\exp\left[i\frac{k}{2z}\left(x^2+y^2\right)\right]$$

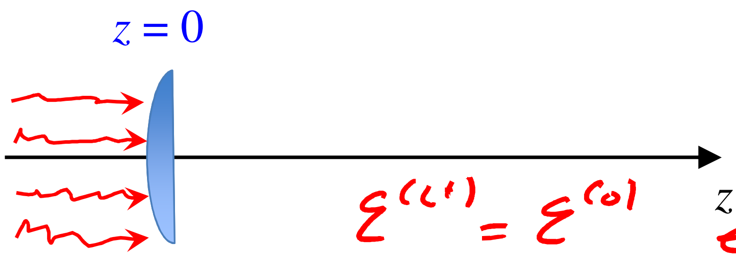
$$E(k_x,k_y,z,\omega) = E(k_x,k_y,z=0,\omega)\bullet h_{Fresnel}(k_x,k_y,z,\omega)$$

$$h_{\text{Fresnel}}(k_x,k_y,z,\omega) = e^{ikz}\exp\left[-i\frac{z}{2k}\left(k_x^2+k_y^2\right)\right]$$

Consider first: input field at lens

Incident monochromatic field:

$$\mathcal{E}(x,y,0)e^{-i(\omega t-kz)} + c.c.$$



$$\mathcal{E}^{(1)} = \mathcal{E}^{(0)}$$

$$e^{-ik\frac{\rho'^2}{2f}}$$

$$\mathcal{E}(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z}\int dx'dy'\,\overbrace{t(x',y')\mathcal{E}(x',y',0)}\exp\left[\frac{ik}{2z}\left((x-x')^2+(y-y')^2\right)\right]$$

Field in the focal plane of a lens

$$t(x',y') = \exp\left(-i\frac{k}{2f}\left(x'^2+y'^2\right)\right)$$

$$\mathcal{E}(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z}\int dx'dy' t(x',y')\mathcal{E}(x',y',0)\exp\left[\frac{ik}{2z}\left(\left(x-x'\right)^2+\left(y-y'\right)^2\right)\right]$$

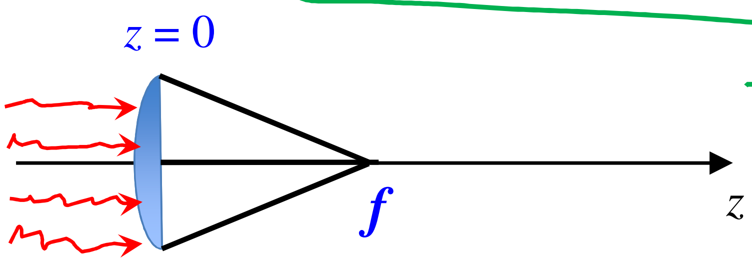
$$\mathcal{E}(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z}\iint dx'dy'\, e^{-i\frac{k}{2f}\cancel{(x'^2+y'^2)}}\,\mathcal{E}(x',y',0)\, e^{\cancel{+i\frac{k}{2z}(x'^2}+y'^2)}\, e^{+\frac{ik}{2z}(x^2+y^2)} * e^{-ik\frac{x}{z}x'-ik\frac{y}{z}y'}$$

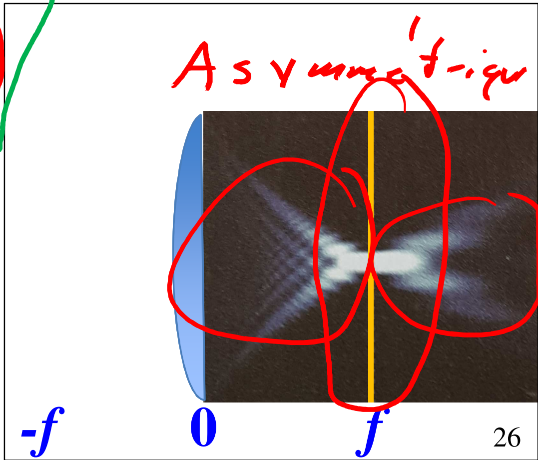
$$z = f$$

$$\Rightarrow \mathcal{E}(x,y,f) = \underbrace{-\frac{i}{\lambda}\frac{e^{ikf}}{f}e^{\frac{ik}{2f}(x^2+y^2)}}_{\equiv A = A(x,y)}\iint dx'dy'\,\mathcal{E}(x',y',z=0)\,e^{-ik\frac{x}{f}x'-ik\frac{y}{f}y'}$$

$$\boxed{\mathcal{E}(x,y,f) = A(x,y)\;\mathcal{E}\left(k_x=\frac{kx}{f}, k_y=\frac{ky}{f}, 0\right)}$$

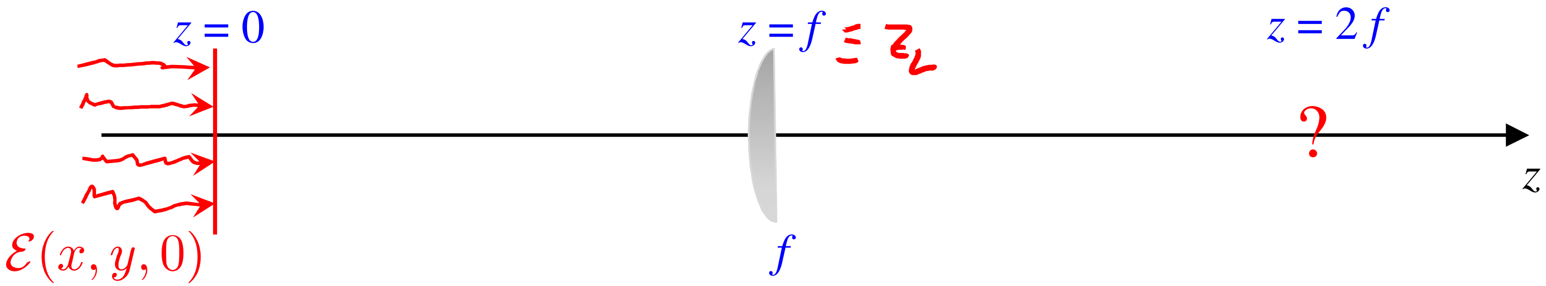
Dans le plan focal, on a un TF "inexact" car $A = A(x,y)$





Field in the focal plane of a lens

Consider now: input field in object focal plane



Recall: $E\left(k_x,k_y,z,\omega\right)=E\left(k_x,k_y,z=0,\omega\right)\bullet h_{Fresnel}\left(k_x,k_y,z,\omega\right)$ with $h_{\text{Fresnel}}\left(k_x,k_y,z,\omega\right)=e^{ikz}\exp\left[-i\frac{z}{2k}\left(k_x^2+k_y^2\right)\right]$

Find field just before lens:

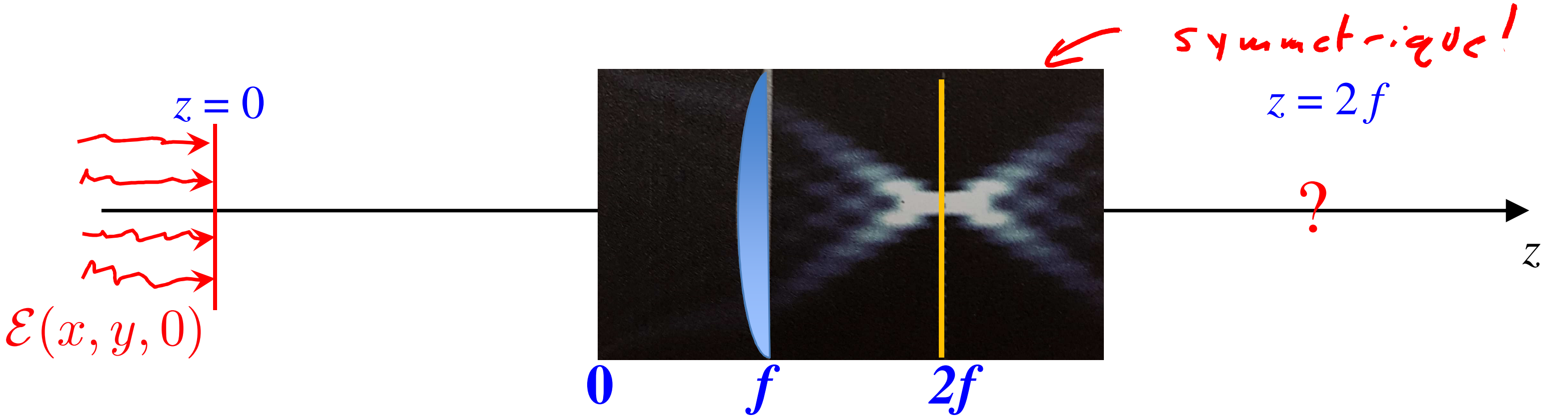
$$\mathcal{E}(k_x,k_y,z=f^-)=\mathcal{E}(k_x,k_y,z=0)h_{\text{Fresnel}}(k_x,k_y,f) \text{ with } h_{\text{Fresnel}}(k_x,k_y,f)=e^{ikf}\exp\left[-i\frac{f}{2k}(k_x^2+k_y^2)\right]$$

From last slide, just after lens: $\mathcal{E}(x,y,z_L+f)=-\frac{i}{\lambda}\frac{e^{ikf}}{f}\exp\left[\frac{ik}{2f}\left(x^2+y^2\right)\right]\mathcal{E}(k_x=\frac{kx}{f},k_y=\frac{ky}{f},z_L)$

maintenant

$z_L=f$

Field in the focal plane of a lens



$$\frac{f}{2k} k_x^2 = \frac{\cancel{f}}{2\cancel{k}} \frac{k^{\cancel{2}} x^2}{f^{\cancel{2}}} = \frac{k x^2}{2k}$$

$\mathcal{E}(k_x, k_y, z = f^-) = \mathcal{E}(\underline{k_x, k_y, z} = 0) h_{\text{Fresnel}}(k_x, k_y, f)$ with $h_{\text{Fresnel}}(k_x, k_y, f) = e^{ikf} \underline{\exp\left[-i\frac{f}{2k}(k_x^2 + k_y^2)\right]}$

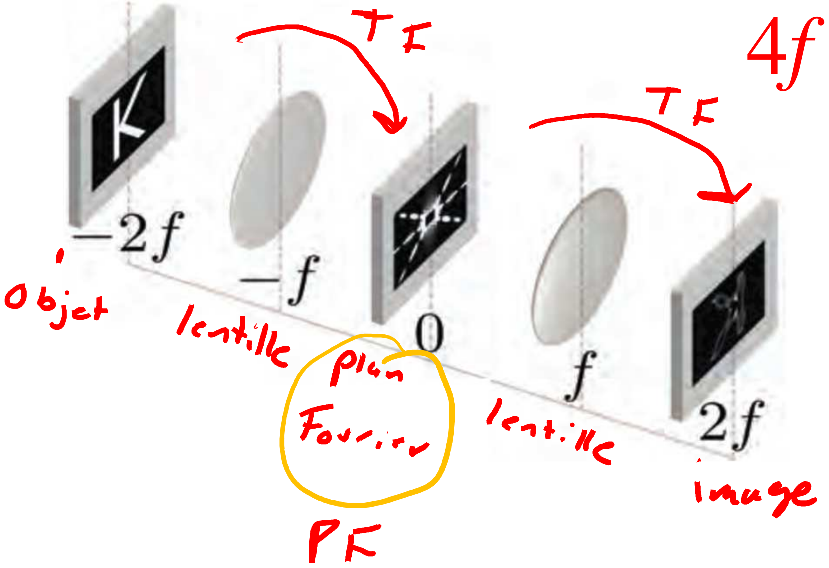
From last slide, just after lens: $\mathcal{E}(x, y, z_L + f) = -\frac{i}{\lambda}\frac{e^{ikf}}{f}\exp\left[\frac{ik}{2f}\left(x^2 + y^2\right)\right]\mathcal{E}(k_x = \frac{kx}{f}, k_y = \frac{ky}{f}, z_L)$ $z = f$

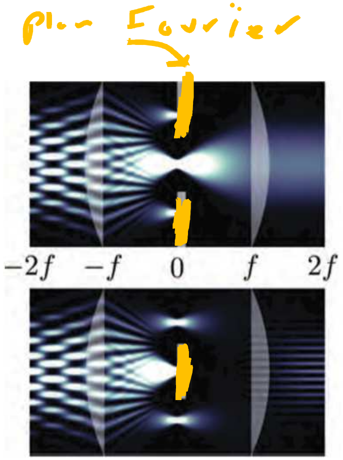
$$\mathcal{E}(x, y, 2f) = -\frac{i}{\lambda}\frac{e^{ikf}}{f} \cancel{e^{\frac{ik}{2f}(x^2+y^2)}} \underline{\mathcal{E}(k_x, k_y, z=0)} e^{ikf} \underline{\cancel{e^{-i\frac{f}{2k}(k_x^2+k_y^2)}}}$$

$$\boxed{\mathcal{E}(x, y, 2f) = B \, \mathcal{E}\left(k_x = \frac{kx}{f}, k_y = \frac{ky}{f}, 0\right) \qquad B \neq B(x, y)}$$

TF exact!

$4f$ spatial filtre



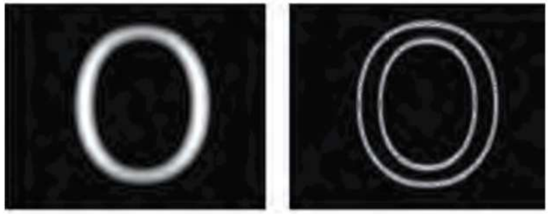


Low pass filtre

High pass filtre

Low pass

High pass

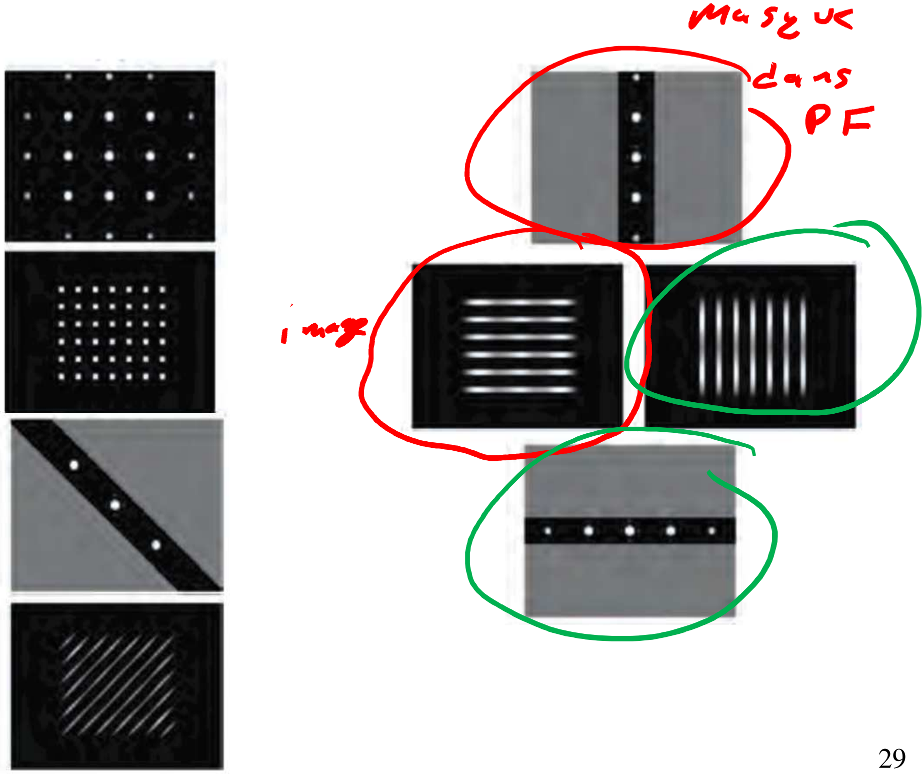


Fourier plane

Output plane (image)

Fourier plane

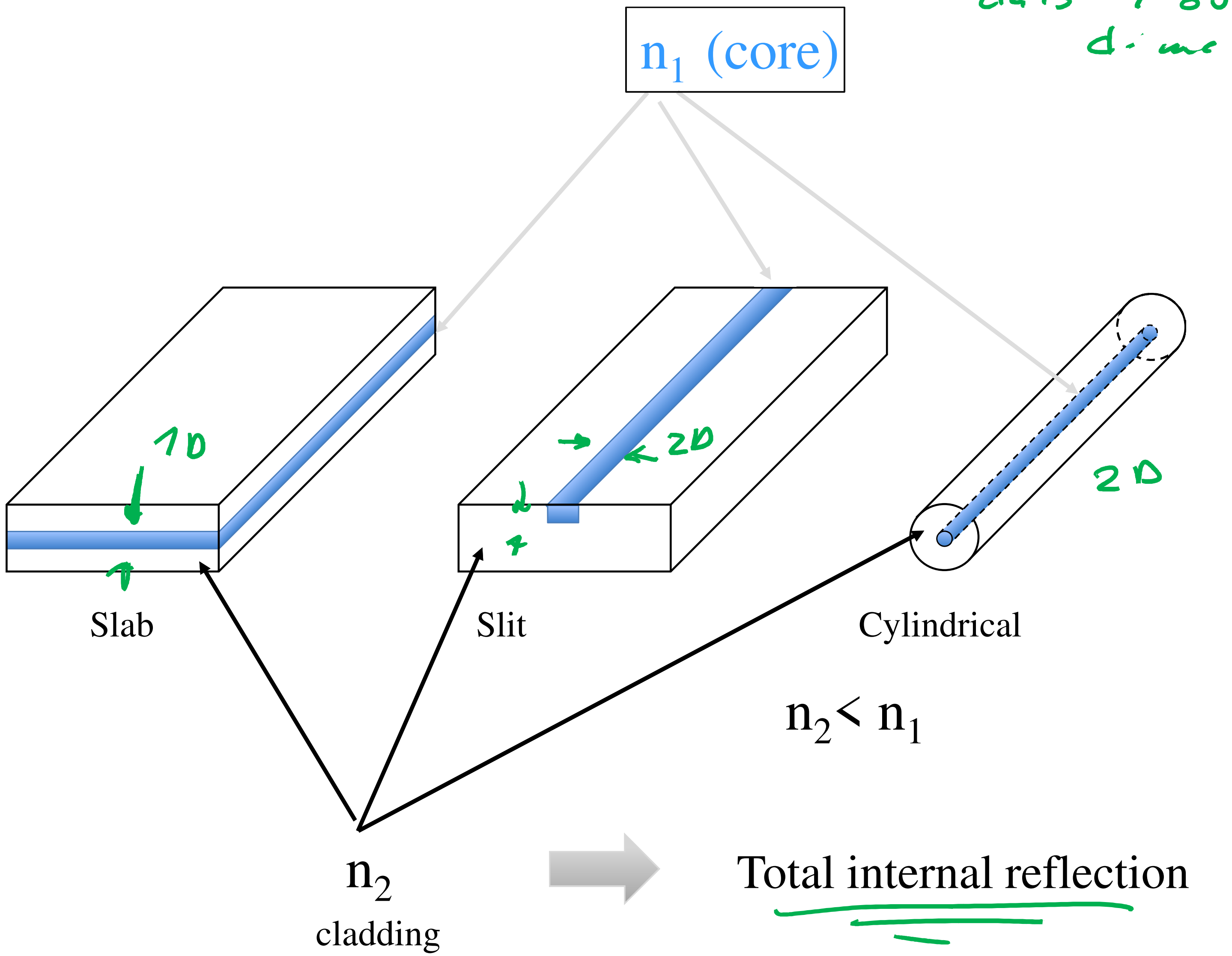
Output plane (image)



Diélectriques

Waveguides

→ restreint faisceau dans 1 ou 2 dimensions



n_1 (core)

Slab

Slit

Cylindrical

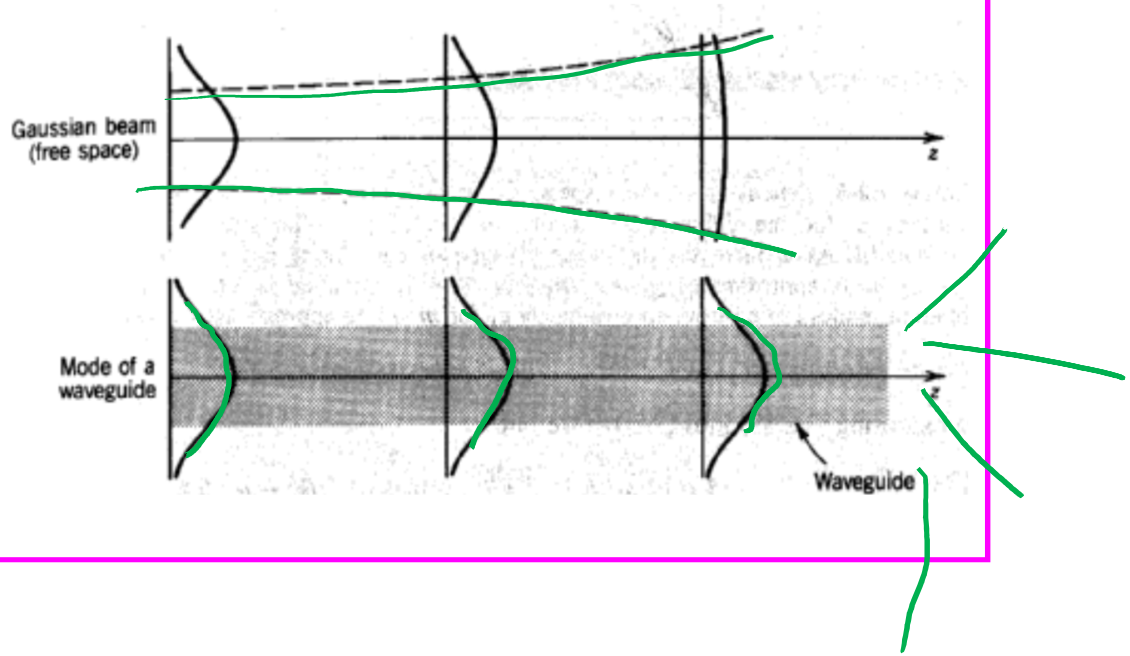
$n_2 < n_1$

n_2
cladding

Total internal reflection

Why study waveguides?

Avoid diffraction



Why study waveguides?

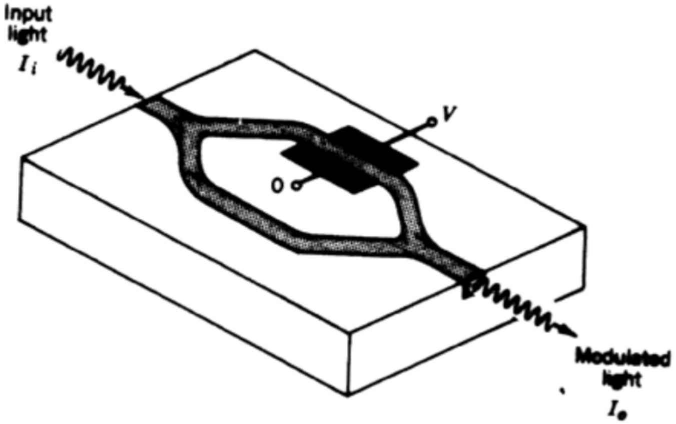
Electrical response time

$$\boxed{\tau = RC}$$

$$\boxed{C = \frac{\varepsilon A}{d}}$$

Towards integrated optoelectronics…

Small
=
Fast

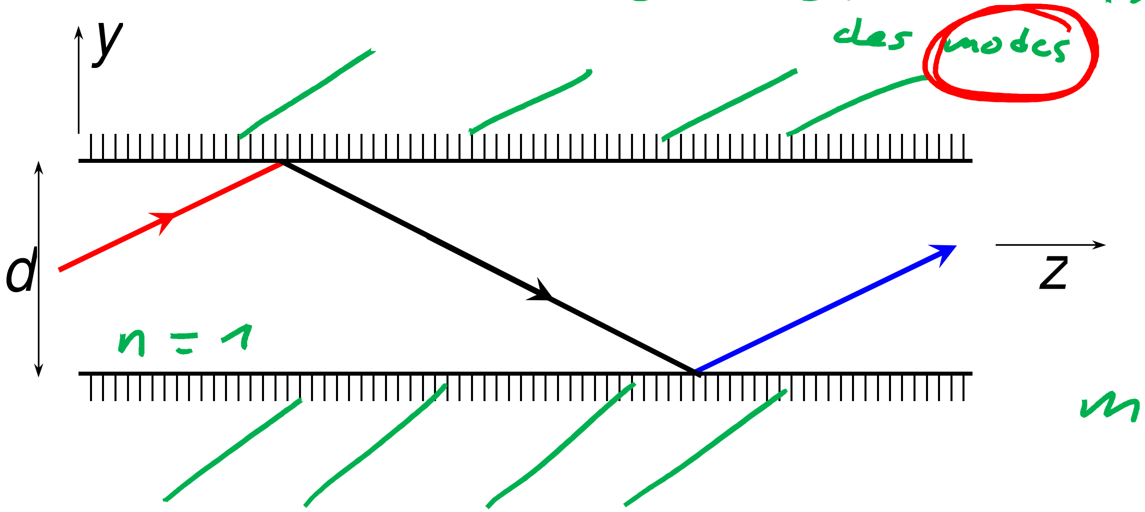


Metallic planar waveguide

E field polarized in x direction

↳ TE

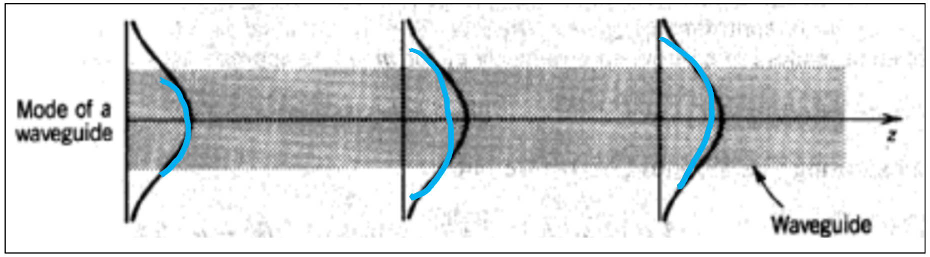
On va chercher des modes transverse électrique

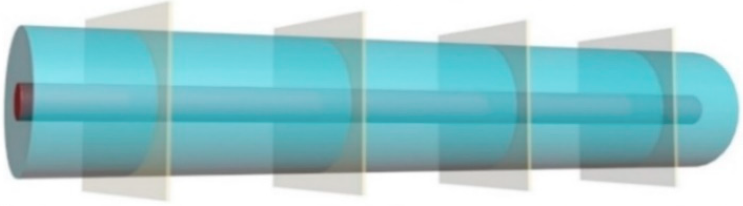


métal parfait

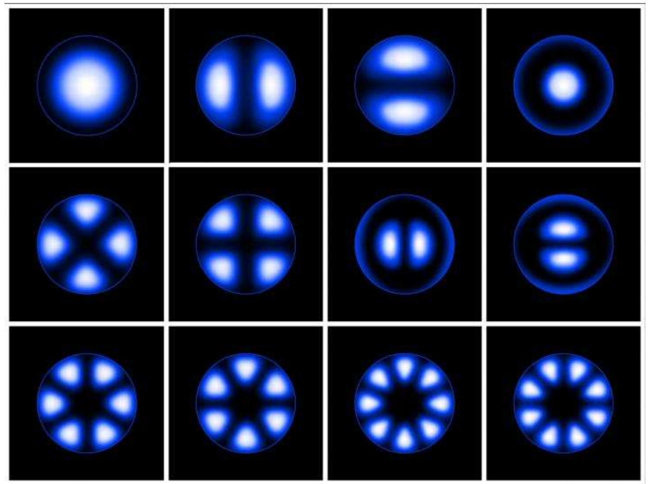
Optical modes

mode optique:
- énergie spécifique
- distribution transverse spécifique du champ
- angle d'incidence spécifique



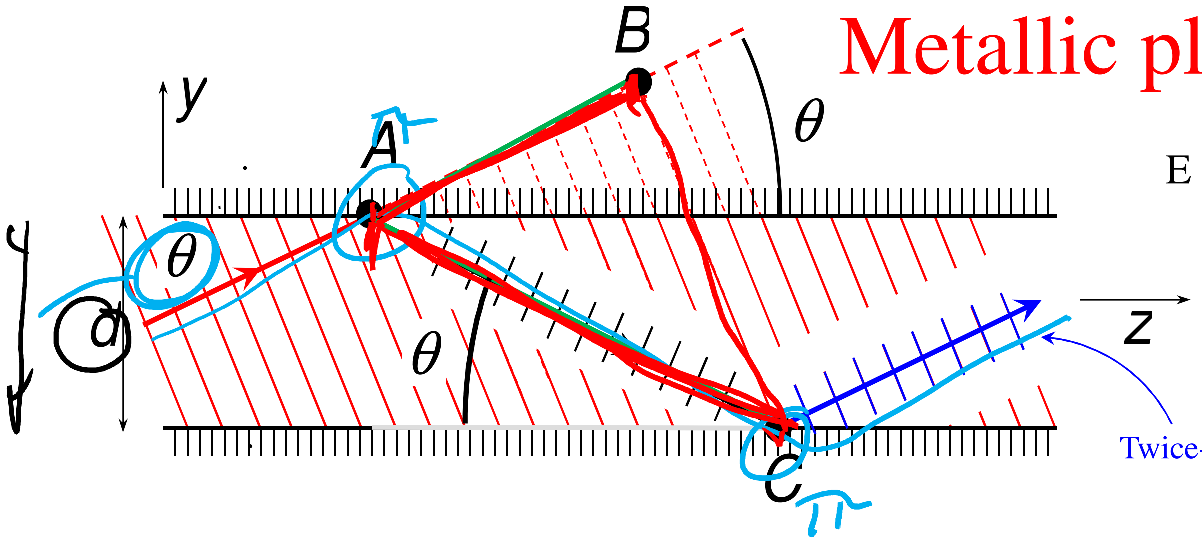


mode ordre "0"



https://www.photonics.com/Articles/Large-Mode-Area_Optical_Fibers_Maintain/a62269

Metallic planar waveguide



E field polarized in x direction

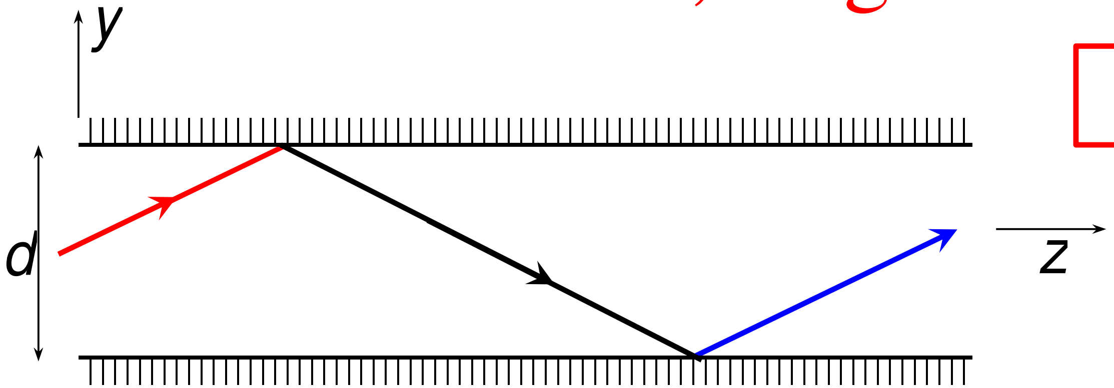
Mode: Twice-reflected wave must be identical to the incident wave

$\Delta\phi = 2q\pi = 2\pi + k(\overline{AC} - \overline{AB})$

$$\sin\theta_m = m\frac{\lambda}{2d}$$

$m = 1, 2 \ldots$

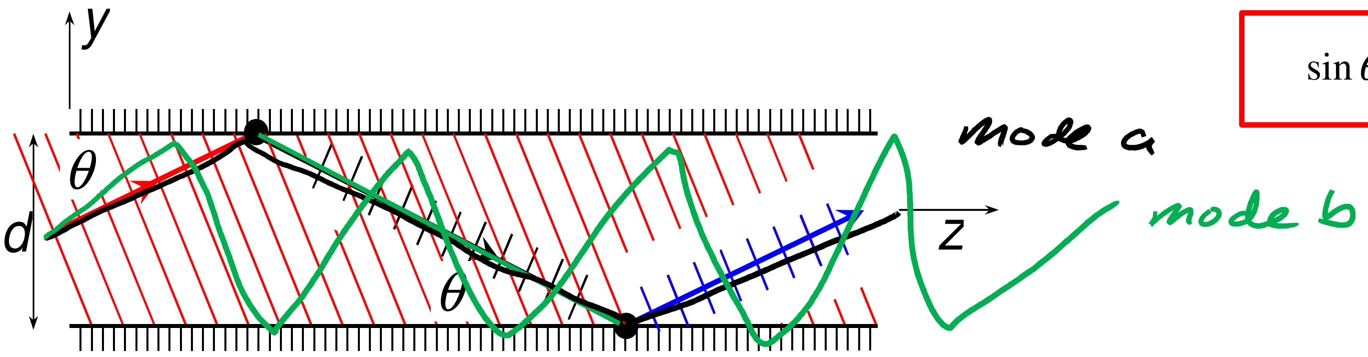
Fundamental mode, single mode waveguides



$$\sin\theta_m = m\frac{\lambda}{2d} \quad m = 1, 2, 3....$$

Single mode if $2d > \lambda > d$ ➡ Micron-sized waveguides

Propagation constants, group and phase velocities



$$\boxed{\sin\theta_m = m\frac{\lambda}{2d} \quad m = 1,\ 2,\ 3\ldots.}$$

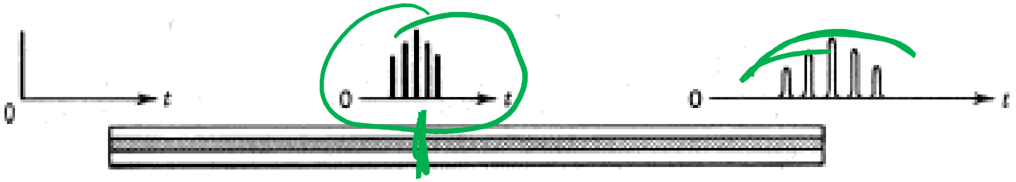
$$k_y^{(m)} = k\sin\theta_m$$

$$\beta_m = k_z^{(m)} = k\cos\theta_m$$

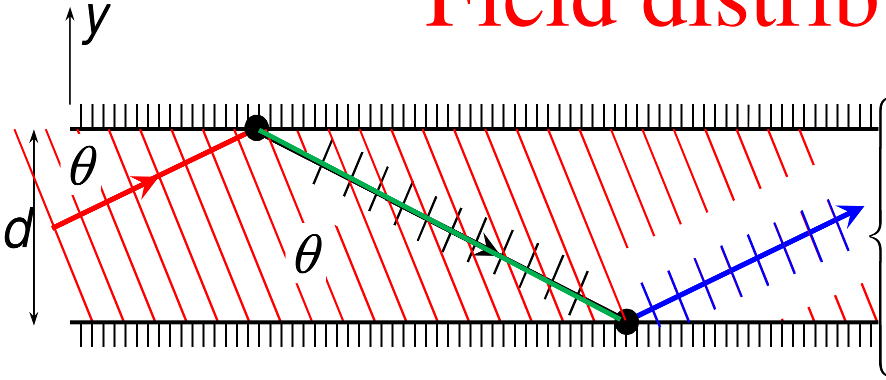
$$v_\varphi^{(m)} = \frac{\omega}{\beta_m}$$

$$v_g^{(m)} = \frac{d\omega}{d\beta_m}$$

v_g depends on m:
intermodal
dispersion



Field distributions



Mode of order *m*

$$\begin{cases} k_y^{(m)} = k\sin\theta_m = m\dfrac{\pi}{d} \\ \beta_m = k_z^{(m)} = k\cos\theta_m = \sqrt{k^2 - m^2\dfrac{\pi^2}{d^2}} \end{cases}$$

Mode of order *m*: interference between plane wave with wave vectors $\left(0, k_y^{(m)}, \beta_m\right)$ and $\left(0, -k_y^{(m)}, \beta_m\right)$, in such a way that the fields cancel at the mirrors.

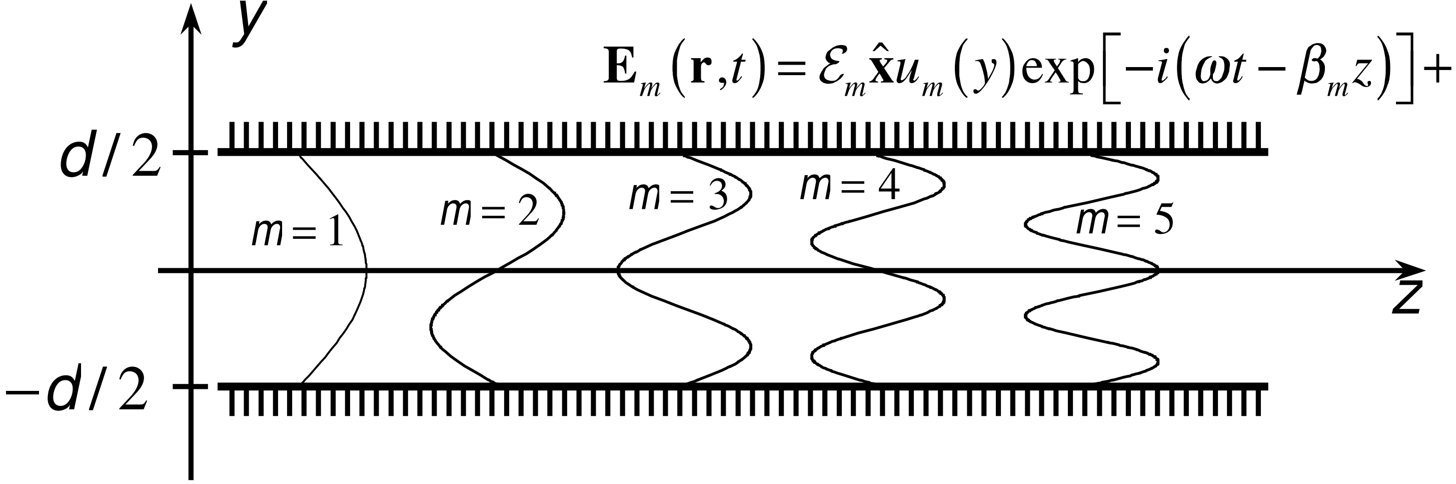
$$\mathbf{E}_m(\mathbf{r},t) = \mathcal{E}_m \hat{\mathbf{x}} u_m(y) \exp\left[-i\left(\omega t - \beta_m z\right)\right] + \text{c.c.}$$

$$u_m(y) = \begin{cases} \sqrt{\dfrac{2}{d}}\cos\dfrac{m\pi y}{d} & \text{for } m = 1,3,5\ldots \\ \sqrt{\dfrac{2}{d}}\sin\dfrac{m\pi y}{d} & \text{for } m = 2,4,6\ldots \end{cases}$$

Field distributions

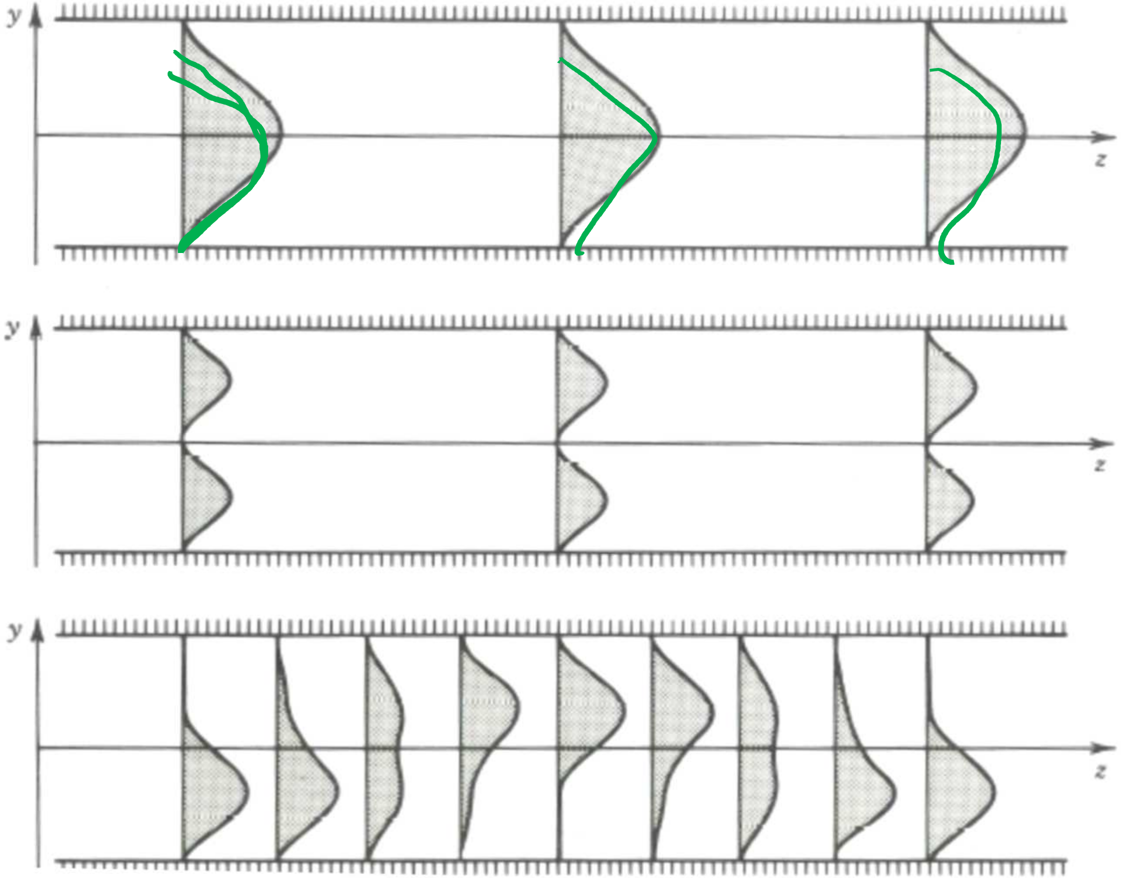
Electric field distributions

$$\mathbf{E}_m(\mathbf{r},t)=\mathcal{E}_m\hat{\mathbf{x}}u_m(y)\exp\left[-i\left(\omega t-\beta_m z\right)\right]+\text{c.c.}$$



Metallic planar waveguide

Multimode fields



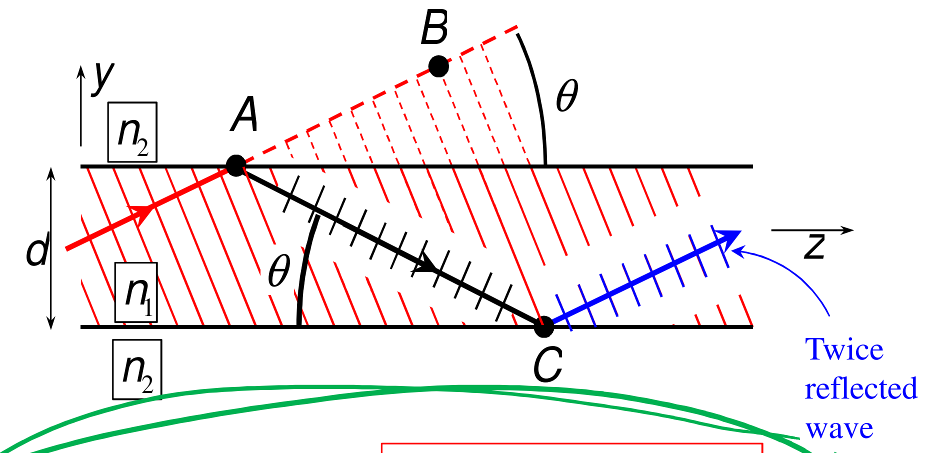
Mode 1

Mode 2

Mode 1+2

Saleh and Teich, *Fundamentals of Photonics, p. 247*

Planar dielectric waveguide



- Medium with index n_1 between two media with lower indices
- 2D problem: invariant along the x direction
- Propagation in the *yz* plane
- Electric field in the *x* direction

Total internal reflection: $\boxed{\theta < \theta_c = \arccos\frac{n_2}{n_1}}$

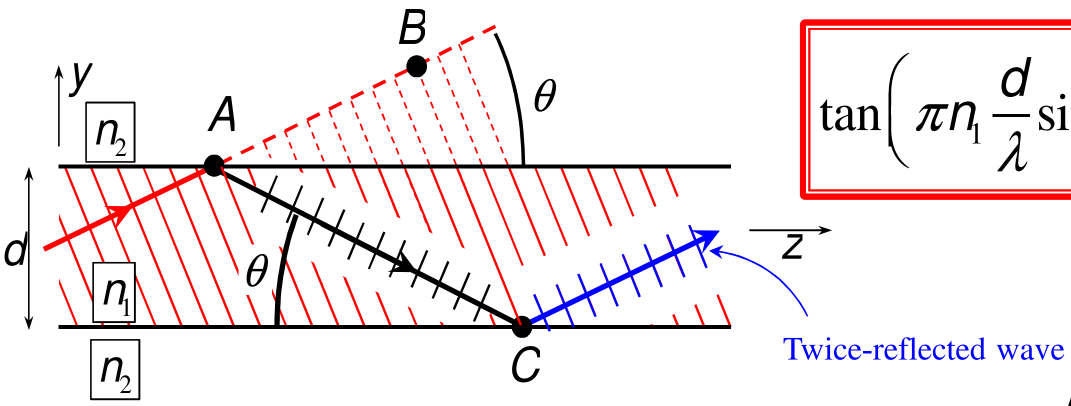
with $\boxed{\tan\frac{\varphi_r^{\mathrm{TE}}}{2} = -\sqrt{\frac{\sin^2\theta_c}{\sin^2\theta} - 1}}$

"Self-consistency condition"

$$\boxed{\frac{2\pi n_1}{\lambda} 2d\sin\theta + 2\varphi_r = 2m\pi}$$

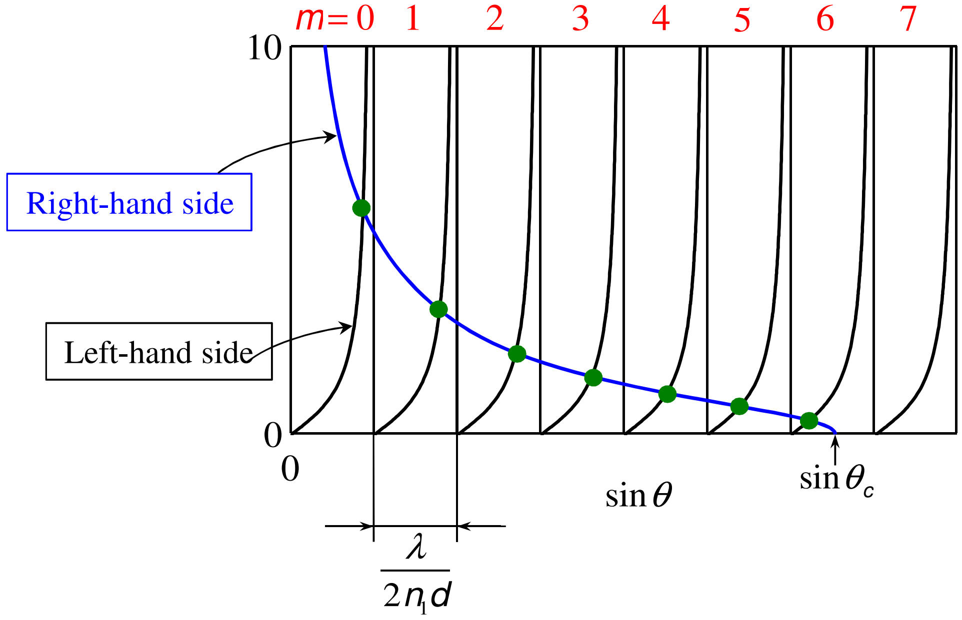
$$\boxed{\tan\left(\pi n_1 \frac{d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \sqrt{\frac{\sin^2\theta_c}{\sin^2\theta} - 1}}$$

Planar dielectric waveguide



$$\boxed{\tan\left(\pi n_1 \frac{d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \sqrt{\frac{\sin^2\theta_c}{\sin^2\theta}-1}}$$

$$\beta_m = k_z^{(m)} = \frac{2\pi n_1}{\lambda}\cos\theta_m$$



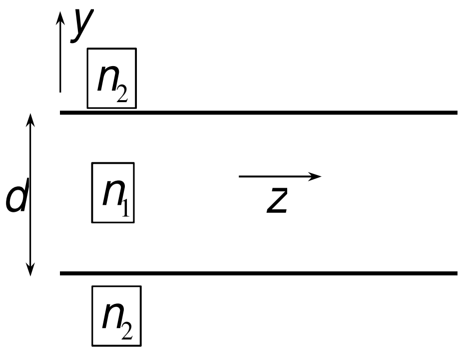
$$\boxed{\frac{2\pi n_2}{\lambda} \le \beta_m \le \frac{2\pi n_1}{\lambda}}$$

There is always at least one mode.

Monomode if: $\boxed{\frac{\lambda}{2n_1d} > \sin\theta_c}$

$$\boxed{\frac{2d}{\lambda}\sqrt{n_1^2 - n_2^2} < 1}$$

Planar dielectric waveguide

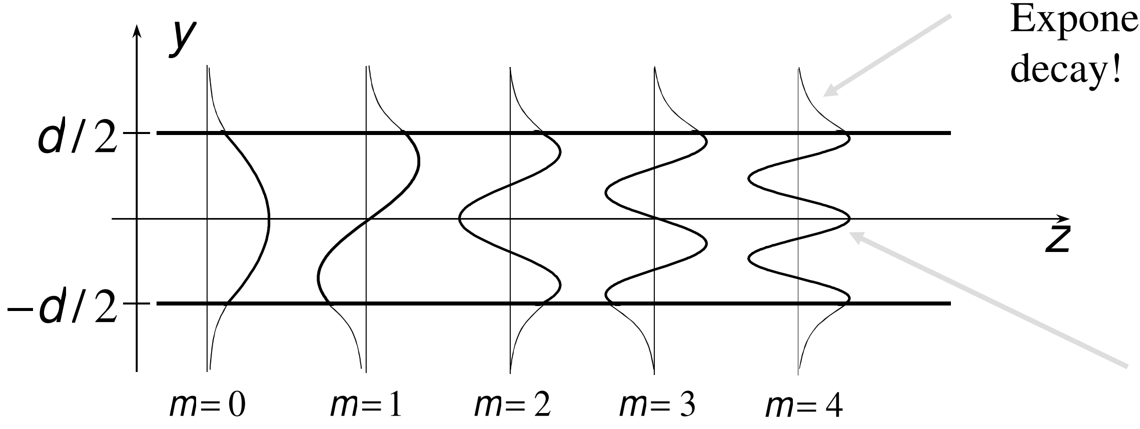


Propagation equation

$$\nabla^2 \mathbf{E}_m(\mathbf{r},t) - \frac{n^2(\mathbf{r})}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}_m(\mathbf{r},t) = 0$$

with $\mathbf{E}_m(\mathbf{r},t) = A\left[\mathcal{E}_m \hat{\mathbf{x}} u_m(y) \exp\left[-i(\omega t - \beta_m z)\right] + \text{c.c.}\right]$

$$\boxed{\frac{d^2 u_m(y)}{dy^2} + \left[n^2(y)\frac{\omega^2}{c^2} - \beta_m^2\right] u_m(y) = 0}$$

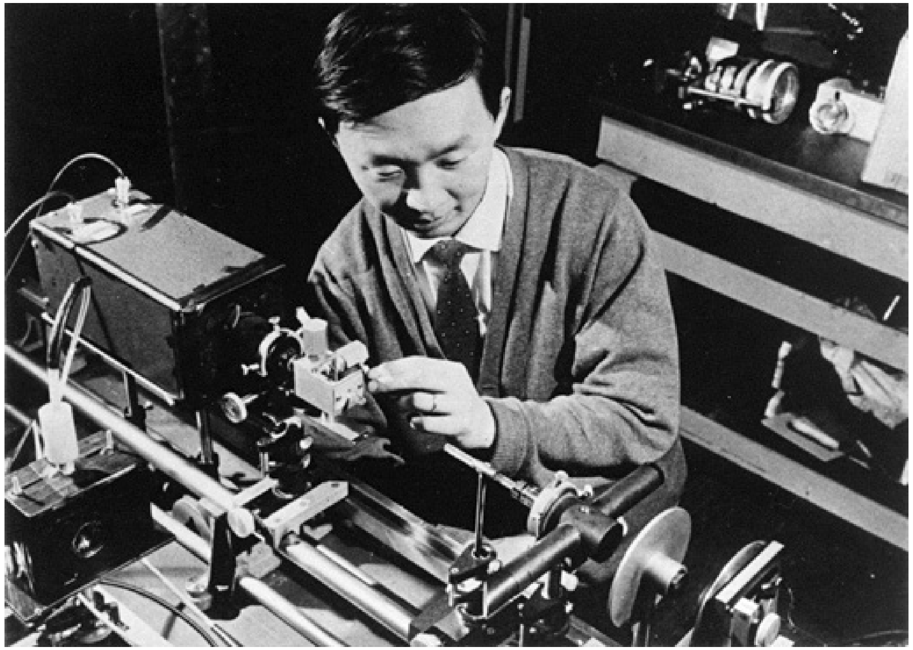


Exponential decay!

Even or odd!

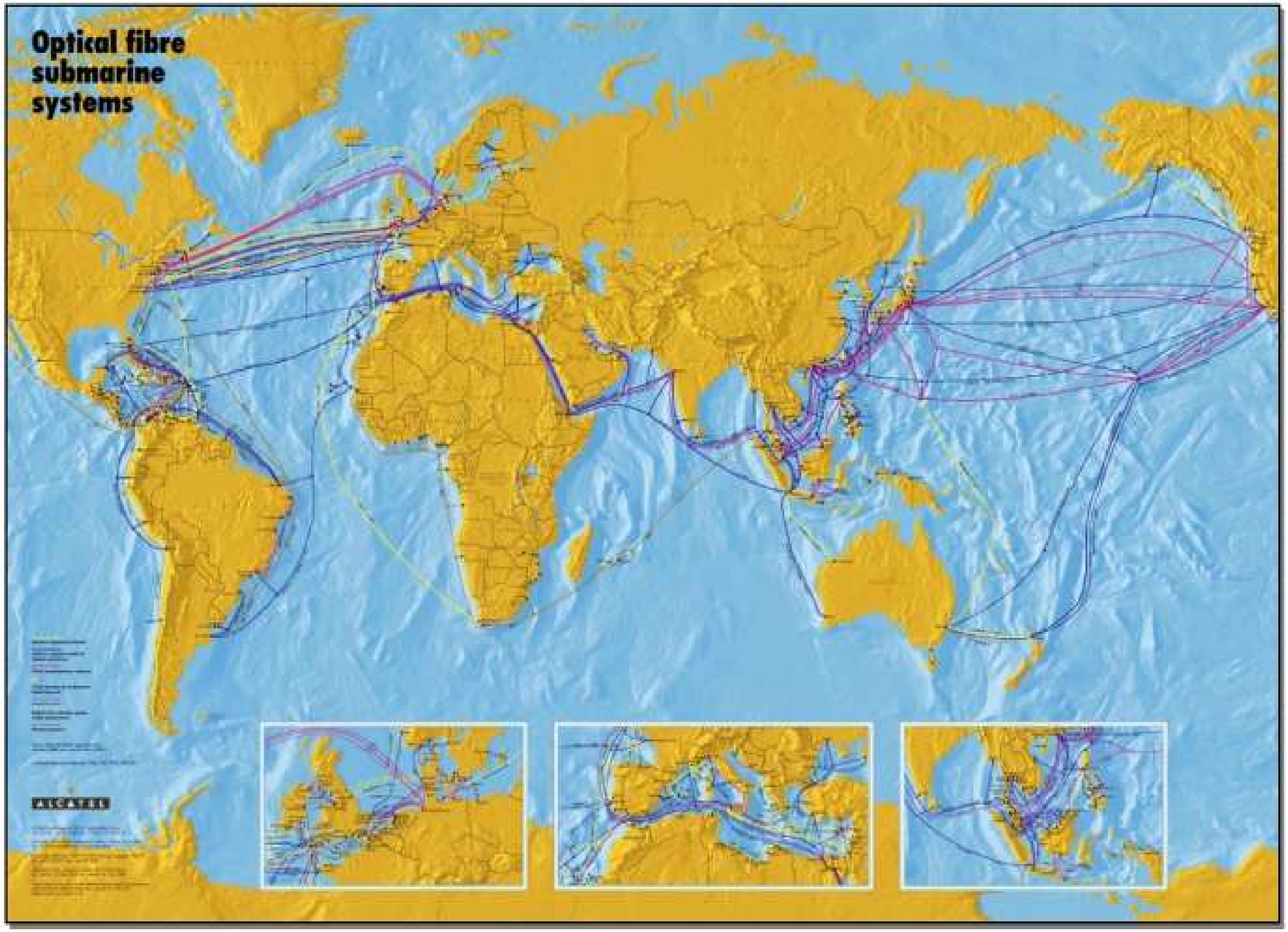
Nobel in Physics 2009 : Charles K. Kao

"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"



http://nobelprize.org/nobel_prizes/physics/laureates/2009/index.html

World fiber optics network



Reading

- Huygens-Fresnel principle, Rayleigh-Sommerfeld:
  - J. W. Goodman, *Introduction to Fourier Optics (4th edition), p. 58-65*
  - A.E. Siegman, *Lasers,* p.632-633
- Fresnel approximation:
  - Siegman, p.633-635; Goodman p. 78-84
- Waveguides:
  - B.E.A. Saleh and M.C. Teich, *Fundamentals of Photonics, p. 239-255*

Conclusion

- Huygens-Fresnel = diffraction in terms of spherical waves
- Rayleigh-Sommerfeld, Fresnel (= paraxial), and Fraunhofer approximations
- Metallic waveguides: based on reflection at metal surfaces. Problem of losses for non-ideal metals.
- Dielectric waveguides:
  - very low losses in the IR;
  - may be used to miniaturize opto-electronic components, thus increasing their bandwidth and decreasing their consumption;
  - along with the laser, are at the origin of the internet.