Bonjour!

Points to remember Lecture 1:

Maxwell's equations in simple media with <u>no</u> free charges nor currents…

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.18) \qquad \nabla \cdot \mathbf{B} = 0 \quad (2.20)$$

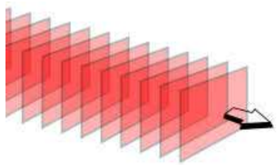
$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (2.19) \qquad \nabla \cdot \mathbf{D} = 0 \quad (2.21)$$

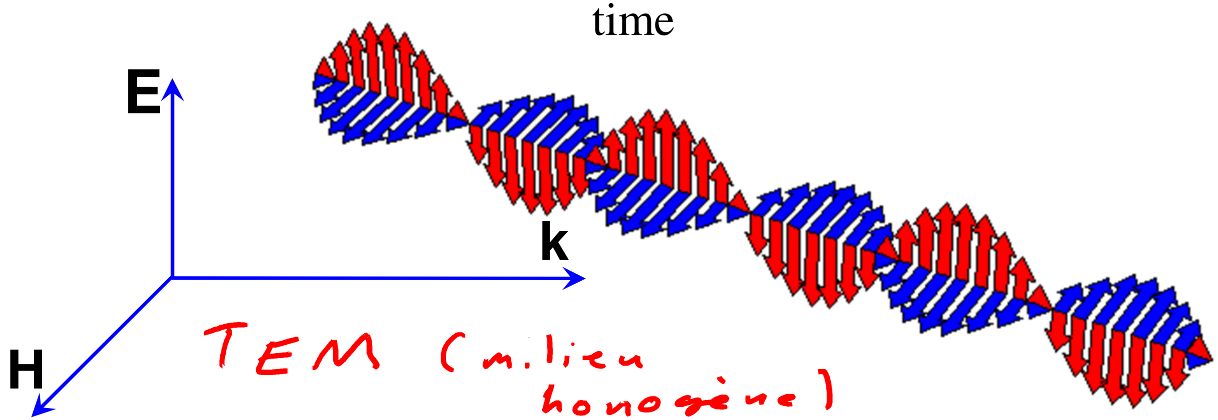
$$\boxed{\underbrace{\int_V d^3r \left(\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right)}_{\text{Change in stored energy per unit time}} = \underbrace{-\int_V d^3r\, \mathbf{j}_f \cdot \mathbf{E}}_{\text{-Work on free charges}} \underbrace{- \int_V d^3r\, \nabla \cdot (\mathbf{E} \times \mathbf{H})}_{\text{Energy leaving volume per unit time}}}$$

Poynting theorem

onde plane:

$$\vec{E} = \vec{\mathcal{E}}\, e^{i\vec{k}\cdot\vec{r} - i\omega t} + c.c.$$





TEM (milieu homogène)

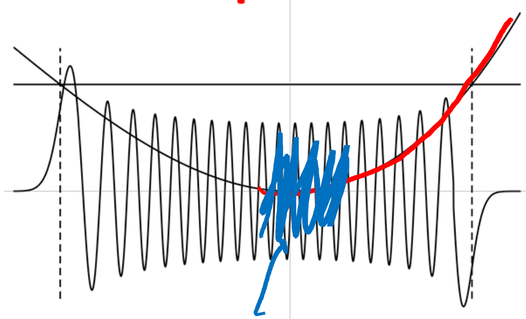
Lecture 2: Fresnel’s equations and applications, geometrical optics

Opt. géom

$\lambda \to 0$

Goals today

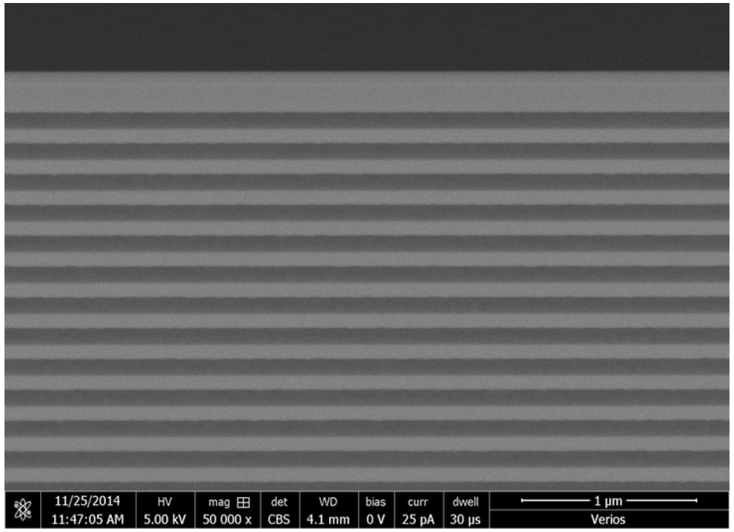
- Fresnel’s equations
- Applications of Fresnel’s equations: Fabry-Perot etalon, Bragg mirror
- Matrix Optics (geometrical optics)
- Relation between geometrical and wave optics: the eikonal equation



ε

phase

amplitude varie lentement par rapport à la phase



Refraction Matrix

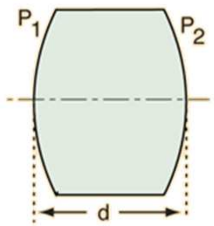
$$\begin{bmatrix} 1 & P \\ 0 & 1 \end{bmatrix}$$

P = surface power or lens power

Translation Matrix

$$\begin{bmatrix} 1 & 0 \\ -\dfrac{d}{n} & 1 \end{bmatrix}$$

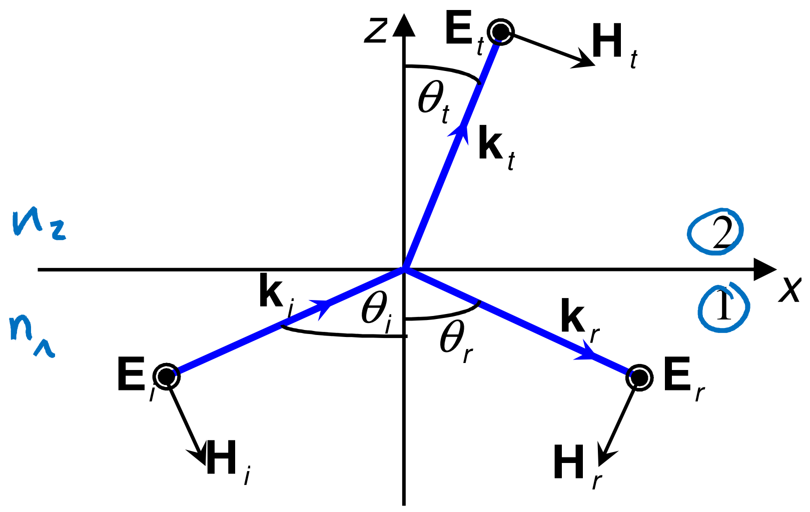
d = lens thickness or lens separation



$$\begin{bmatrix} 1 & P_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\dfrac{d}{n} & 1 \end{bmatrix}\begin{bmatrix} 1 & P_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-P_2\dfrac{d}{n} & P_1+P_2-P_1P_2\dfrac{d}{n} \\ -\dfrac{d}{n} & 1-P_1\dfrac{d}{n} \end{bmatrix}$$

System Matrix

Fresnel equations



Find the reflection and transmission coefficients.

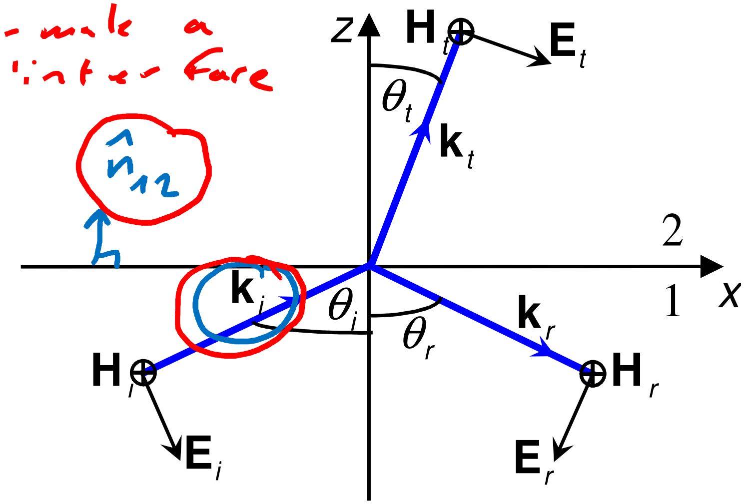
- How much of the wave is transmitted?
- How much of the wave is reflected?

Fresnel equations

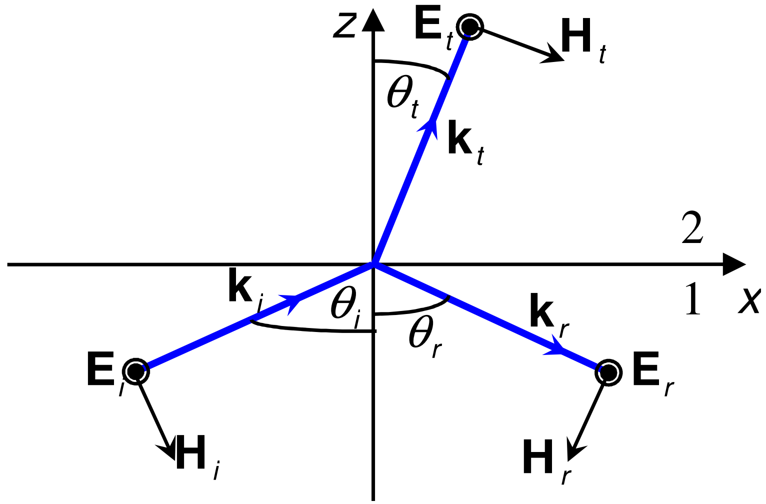
$\hat{k}_i$: dir d'un rayon incident

$\hat{n}_{12}$: normale à l'interface

détermine plane d'incidence



"p" polarisation, or TM, or ||



"s" polarisation , or TE, or ⊥

Fresnel equations: reflection and transmission coefficients

« À partir des éq'ns de Maxwell et cond. limites (voir TD)

$$r = \frac{\mathcal{E}_r}{\mathcal{E}_i}$$

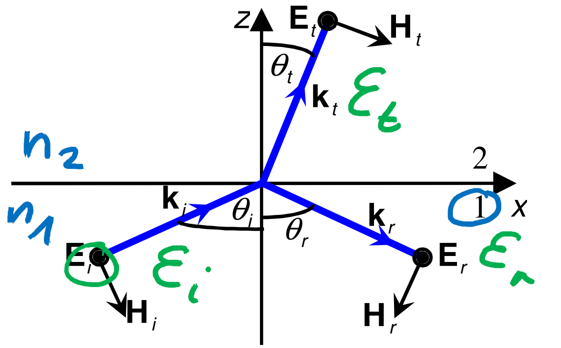
$$t = \frac{\mathcal{E}_t}{\mathcal{E}_i}$$

$$Z_1 = \sqrt{\frac{\mu_0}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{1}{n_1}$$

"s" polarization

2.66

2.67



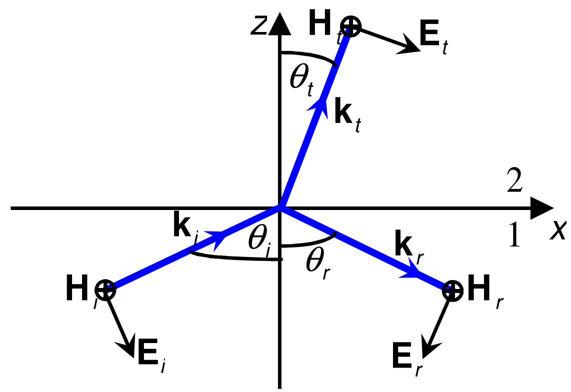
$$r_p = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_p = \frac{Z_1\cos\theta_i - Z_2\cos\theta_t}{Z_1\cos\theta_i + Z_2\cos\theta_t}$$

$$t_p = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_p = \frac{2Z_2\cos\theta_i}{Z_1\cos\theta_i + Z_2\cos\theta_t}$$

"p" polarization

2.61

2.62



Fresnel equations: reflection and transmission coefficients

$$r_s = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_s = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

$$t_s = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_s = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

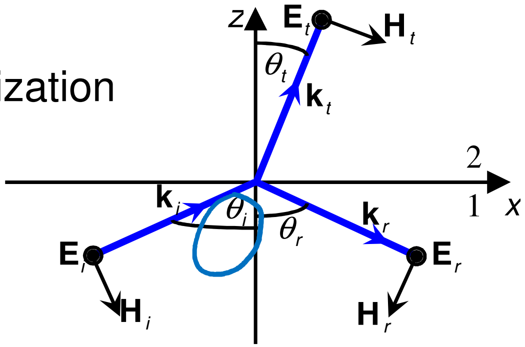
$$Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} =$$

In terms of the indices of refraction for an interface between two dielectrics: $\mu_1 = \mu_2 = \mu_0$

$\theta_i = 0$

Normal incidence

“s” polarization



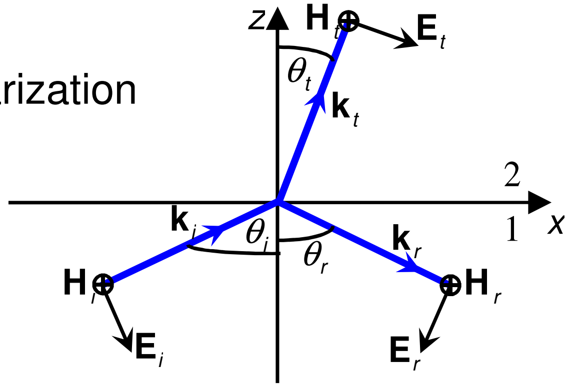
$$r_s = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_s = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} \qquad 2.66$$

$$t_s = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_s = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} \qquad 2.67$$

$$r_s = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_s = \frac{n_1 - n_2}{n_2 + n_1}$$

$$t_s = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_s = \frac{2n_1}{n_2 + n_1}$$

“p” polarization



$$r_p = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_p = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t} \qquad 2.61$$

$$t_p = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_p = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t} \qquad 2.62$$

$$r_p = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_p = \frac{n_2 - n_1}{n_1 + n_2}$$

$$t_p = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_p = \frac{2n_1}{n_1 + n_2}$$

Fresnel equations: air / perfect conductor interface

ε : décrit la réponse du milieu à un champs EM

$\varepsilon_m^{\text{idéal}} \to -\infty$

Normal incidence

$\theta_i = 0$

$$r_s = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_s = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$t_s = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_s = \frac{2Z_2}{Z_2 + Z_1}$$

$$r_p = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_p = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$t_p = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_p = \frac{2Z_2}{Z_1 + Z_2}$$

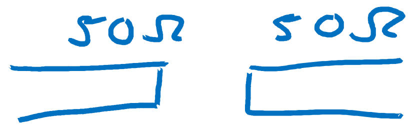
$$Z_m = \sqrt{\frac{\mu_0}{\varepsilon_m}} \longrightarrow 0$$

$|r| \to 1$

$t \to 0$

"Accord de phase"

"Impedance matching"



Normal incidence

$$r_s = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_s = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$t_s = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_s = \frac{2Z_2}{Z_2 + Z_1}$$

$$r_p = \left[\frac{\mathcal{E}_r}{\mathcal{E}_i}\right]_p = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$t_p = \left[\frac{\mathcal{E}_t}{\mathcal{E}_i}\right]_p = \frac{2Z_2}{Z_1 + Z_2}$$

Afin d'éviter des réflections ($r_p = r_s = 0$) il faut $Z_1 = Z_2$ (comme en électronique!)

$\vec{S}$: énergie/temps / surface [⊥ à la propagation]

I : intensité ; "irradiance" $\Rightarrow \boxed{I = \langle \vec{S} \rangle \cdot \hat{n}}$

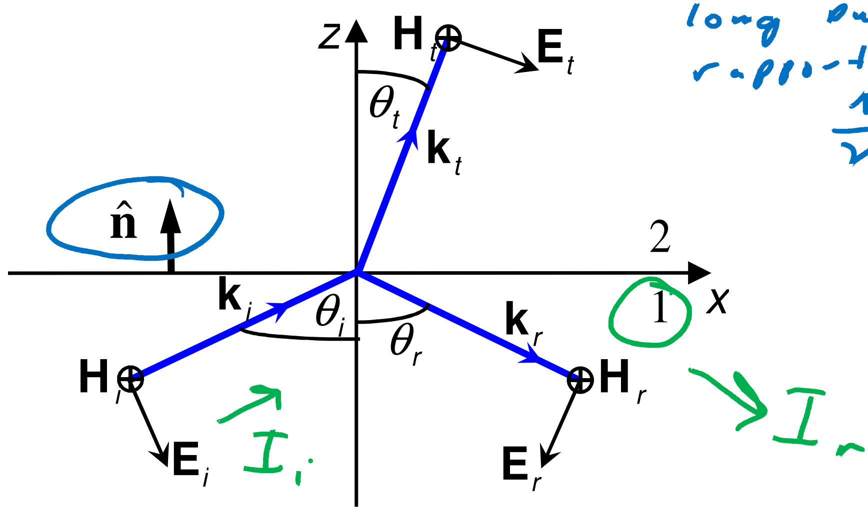
$\langle\ \rangle$ moyenne sur un temp long par rapport à $\frac{1}{\nu}$

Energy transport

How much energy (intensity) is **reflected**?

$$R = \frac{I_r}{I_i} = \left| \frac{\langle \mathbf{S}_r \rangle \cdot \hat{\mathbf{n}}}{\langle \mathbf{S}_i \rangle \cdot \hat{\mathbf{n}}} \right|$$

Recall: $\langle \mathbf{S} \rangle = \langle \mathbf{E} \times \mathbf{H} \rangle = \frac{2}{Z} \|\mathcal{E}\|^2 \hat{\mathbf{k}}$



$$R = \left| \frac{\cancel{2/Z_1}\, |\mathcal{E}_r|^2\, \cancel{\hat{k}_r \cdot \hat{n}}}{\cancel{2/Z_1}\, |\mathcal{E}_i|^2\, \cancel{\hat{k}_i \cdot \hat{n}}} \right|$$

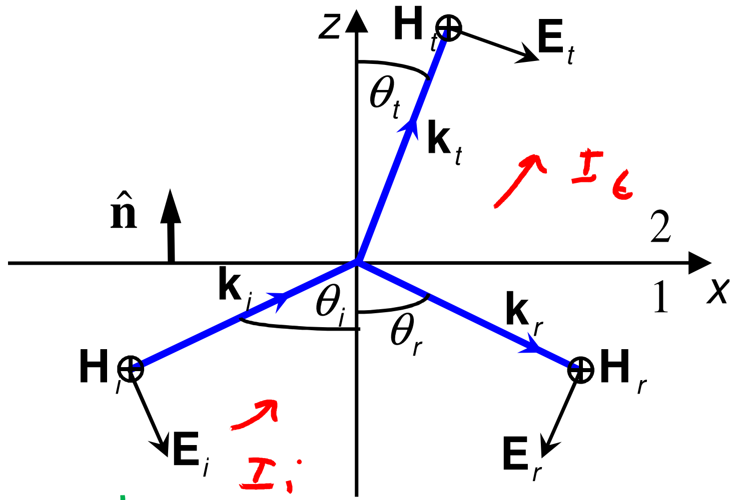
$$\boxed{R = \left| \frac{\langle \mathbf{S}_r \rangle \cdot \hat{\mathbf{n}}}{\langle \mathbf{S}_i \rangle \cdot \hat{\mathbf{n}}} \right| = \left| \frac{\mathcal{E}_r}{\mathcal{E}_i} \right|^2 = |r|^2} \quad (2.74)$$

Energy transport

$|\hat{k}_t \cdot \hat{n}| = \cos\theta_t$

How much energy (intensity) is **transmitted**?

$$T = \frac{I_t}{I_i} = \left|\frac{\langle \mathbf{S}_t\rangle \cdot \hat{\mathbf{n}}}{\langle \mathbf{S}_i\rangle \cdot \hat{\mathbf{n}}}\right|$$



I_t

n_2

n_1

I_i

$|\hat{k}_i \cdot \hat{n}|$
$= |\cos\theta_i|$

4

Recall: $$\langle \mathbf{S}\rangle = \langle \mathbf{E}\times\mathbf{H}\rangle = \frac{2}{Z}\left|\left|\mathcal{E}\right|\right|^2 \hat{\mathbf{k}}$$

$$T = \left| \frac{Z_1}{Z_2} \; \frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} \; \frac{\hat{k}_t \cdot \hat{n}}{\hat{k}_i \cdot \hat{n}} \right|$$

$$\boxed{T = \left|\frac{\langle \mathbf{S}_t\rangle \cdot \hat{\mathbf{n}}}{\langle \mathbf{S}_i\rangle \cdot \hat{\mathbf{n}}}\right| = \frac{Z_1\cos\theta_t}{Z_2\cos\theta_i}\left|\frac{\mathcal{E}_t}{\mathcal{E}_i}\right|^2 = \frac{Z_1\cos\theta_t}{Z_2\cos\theta_i}|t|^2} \quad (2.77)$$

Energy conservation

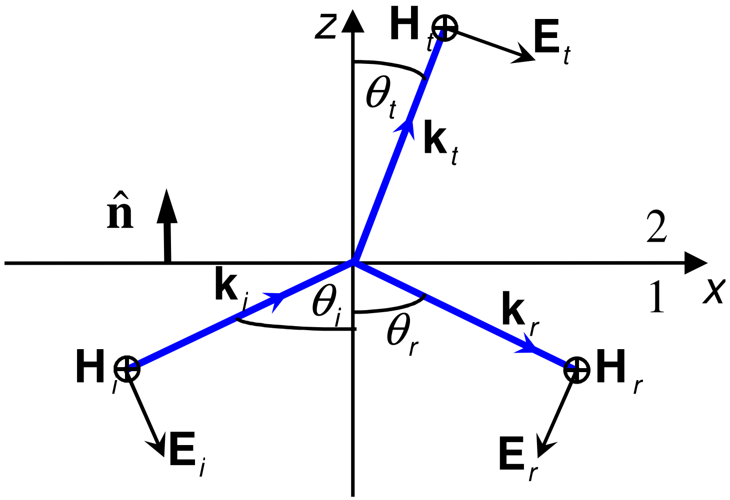
$$R = |r|^2$$

$$T = \frac{Z_1}{Z_2}\frac{\cos\theta_t}{\cos\theta_i}|t|^2$$

En utilisons les expressions trouvées

$$\Rightarrow \boxed{R + T = 1}$$

Energie conservée!



Si absorption : $R + T + A = 1$

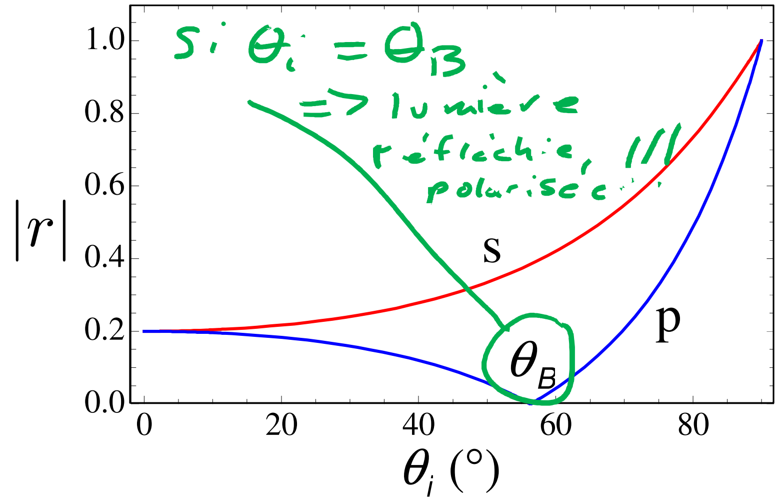
Reflection coefficient: amplitude and phase for an air/glass interface

Si $\theta_i = 0$

$\Rightarrow r = 0{,}2$

$\Rightarrow R = 4\%$

$$\boxed{n_1 < n_2}$$

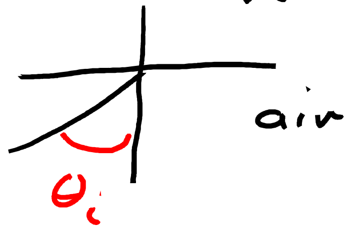


$$\theta_B = \arctan\frac{n_2}{n_1}$$

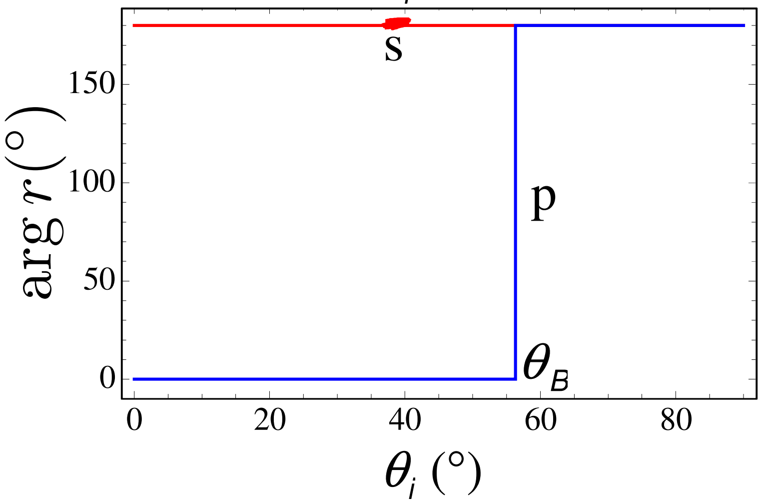
Angle de Brewster

$n_1 = 1$

$n_2 = 1.5$



$r_s, r_p \in \mathrm{R}$



$$\boxed{\mu_1 = \mu_2 = \mu_0}$$

Reflection coefficient: amplitude and phase for a glass/air interface

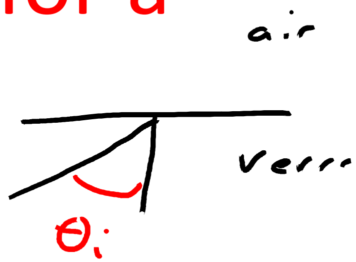
$\theta_i = 0$

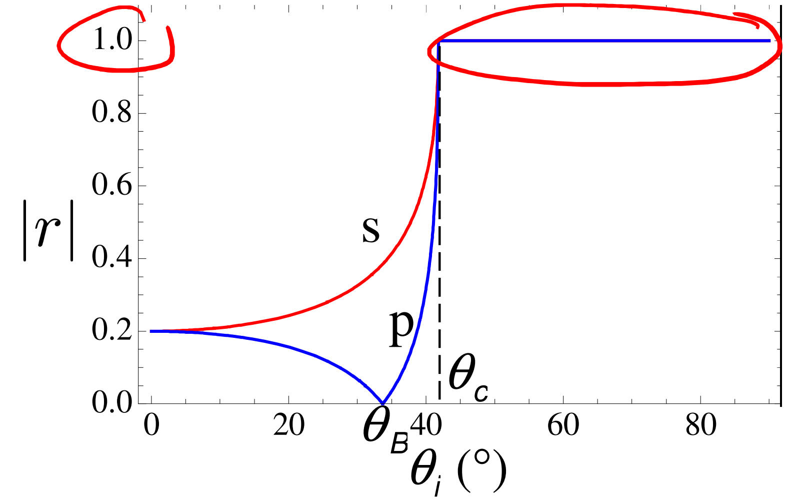
$r = 0{,}2$

$R = 0{,}04$

Vous perdez 8% si lunettes sans revêtement anti-reflet

$$\boxed{n_1 > n_2}$$





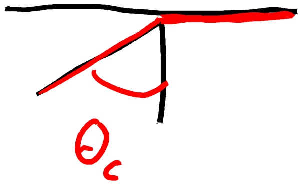
$n_1 = 1.5$

$n_2 = 1$

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$$\theta_B = \arctan\frac{n_2}{n_1}$$

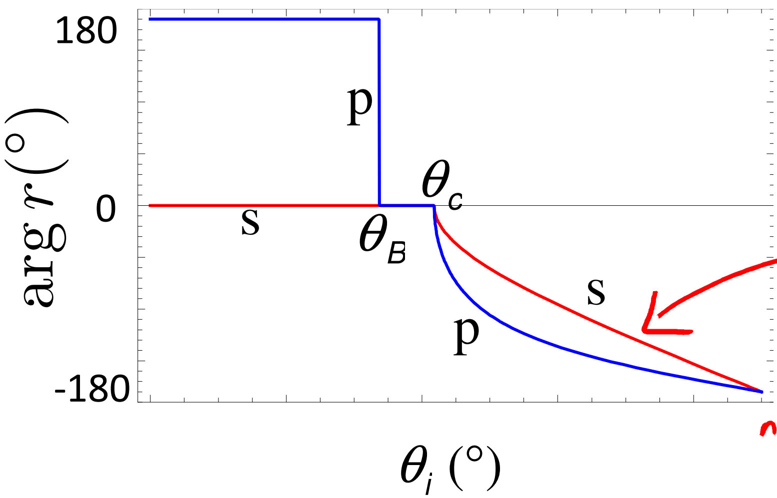
$r_s, r_p \in \mathbb{C}$



$$\boxed{\theta_c = \arcsin\frac{n_2}{n_1}}$$

$n_1 \sin\theta_c = n_2$

$$\boxed{\mu_1 = \mu_2 = \mu_0}$$



polarisation elliptique

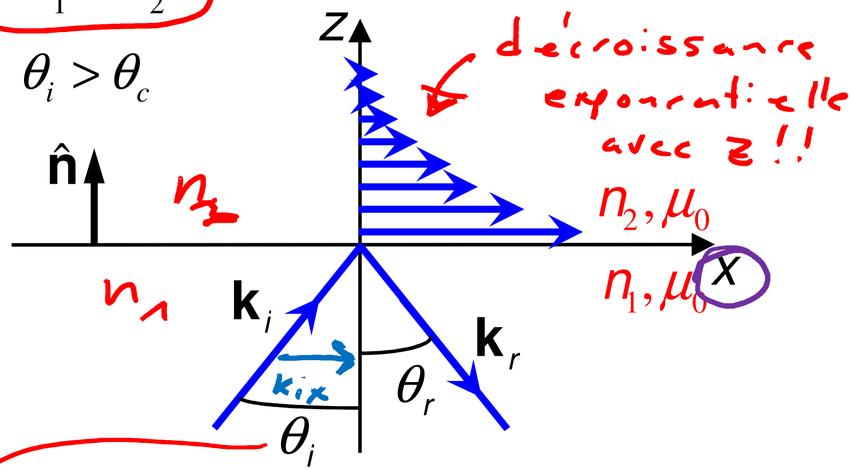
Total internal reflection

$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

Il faut toujours respecter les conditions aux limites

$n_1 > n_2$

$\theta_i > \theta_c$



$$\mathbf{E}_t(\mathbf{r},t) = \mathcal{E}_t \exp\left[-i\left(\omega t - \mathbf{k}_t \cdot \mathbf{r}\right)\right] + \text{c.c.}$$
$$= \mathcal{E}_t \exp\left[-i\left(\omega t - k_{tx}x - k_{tz}z\right)\right] + \text{c.c.}$$

$\Rightarrow$ Il faut donc $E_{ix} = E_{tx}$

$\Rightarrow k_{1x} = k_{tx}$

$k_{1x} = k_0 n_1 \sin\theta_i$

$k_2 = n_2 k_0$

$k_2^2 = k_{tx}^2 + k_{tz}^2$

$\theta_i > \theta_c$

$$k_{tz}^2 = n_2^2 k_0^2 - n_1^2 k_0^2 \sin^2\theta_i$$
$$= k_0^2 \underbrace{\left[n_2^2 - n_1^2 \sin^2\theta_i\right]}_{<0}$$

$\boxed{k_{tz} = i\kappa}$

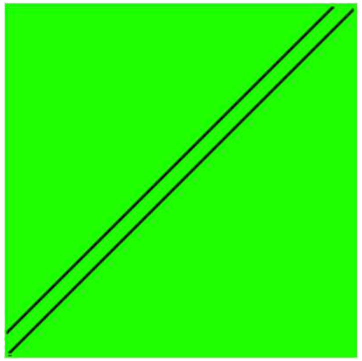
$\kappa \in \mathbb{R}$

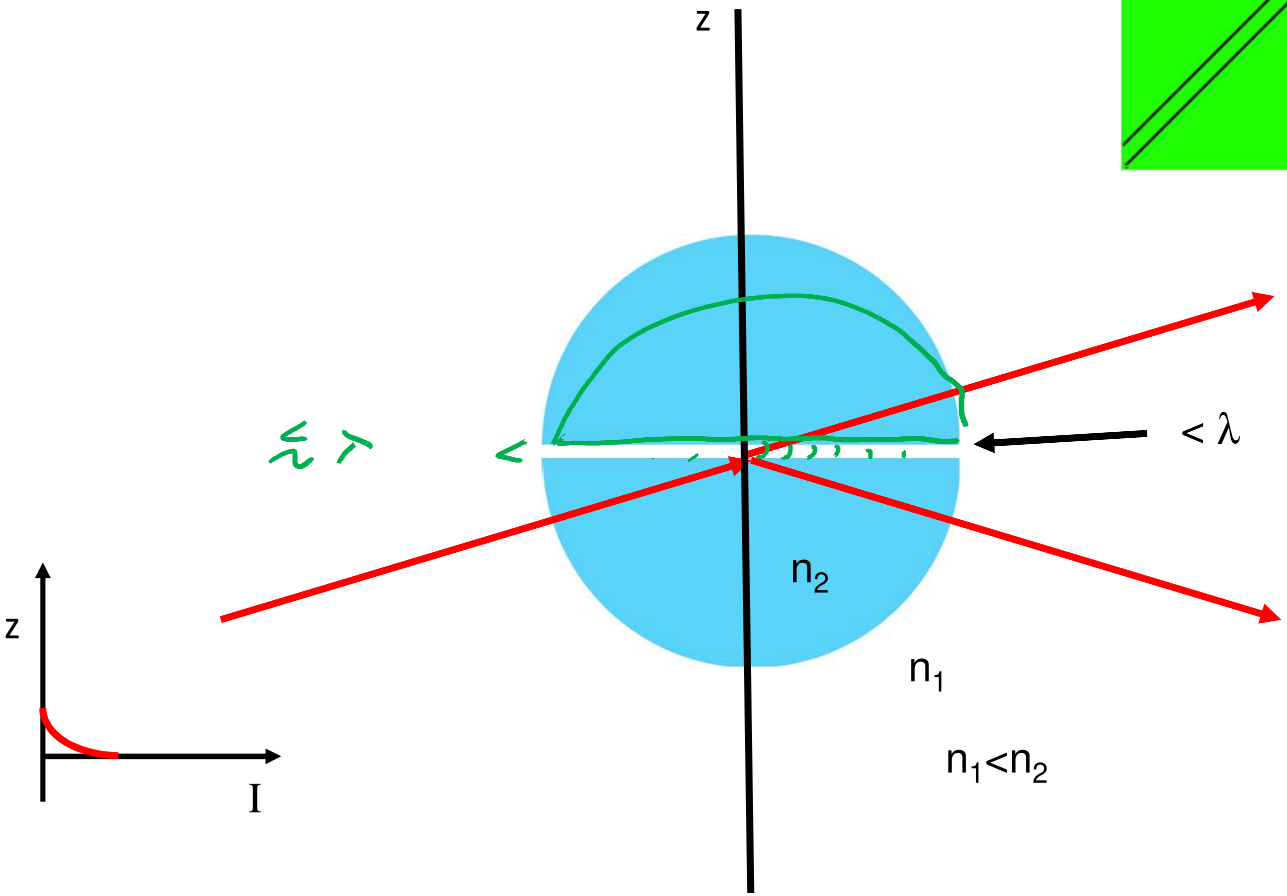
$$\boxed{\mathbf{E}_t(\mathbf{r},t) = \mathcal{E}_t \exp(-\kappa z) \exp\left[-i\left(\omega t - k_{tx}x\right)\right] + \text{c.c.}}$$

Evanescent wave: propagates in the x-direction, decays exponentially in the z-direction

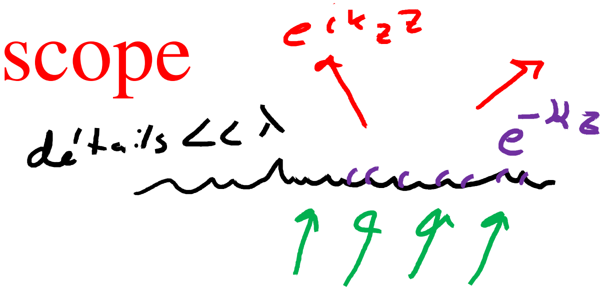
$\boxed{\mu_1 = \mu_2 = \mu_0}$

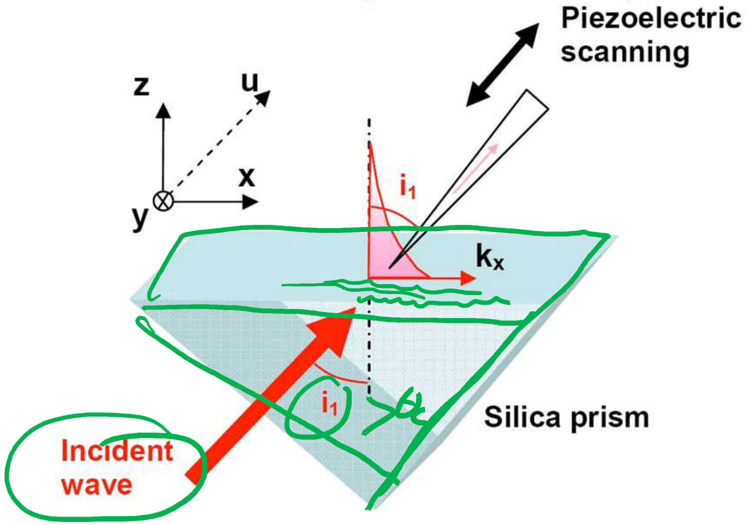
Frustrated total internal reflection



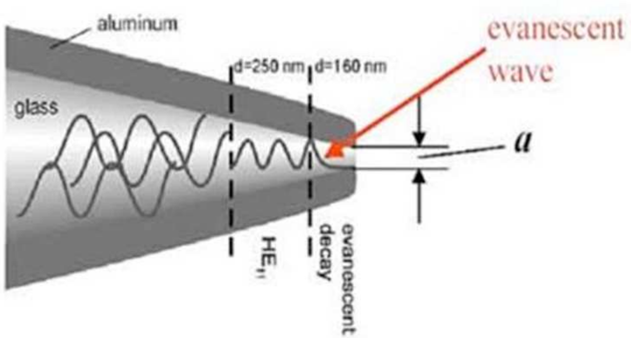


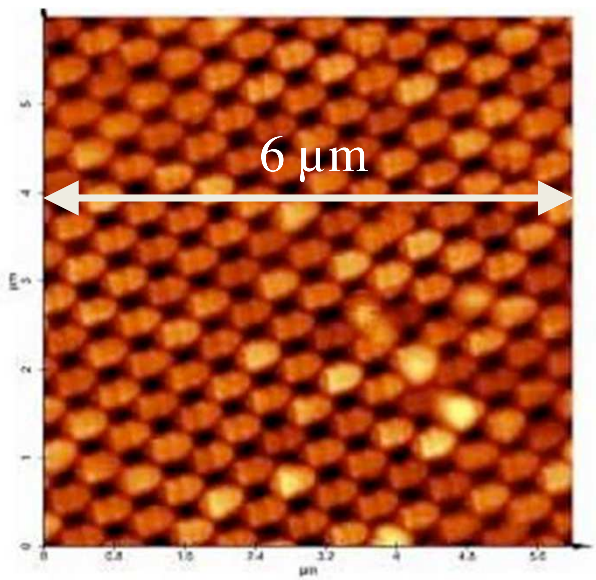
Scanning Near-Field Optical Microscope (SNOM): “optical tunneling”





http://iramis.cea.fr





This is a topographic picture showing a Scanning Near-Field Optical Microscopy (SNOM) image of a sub-micrometric triangular pattern of holes drilled on polymethyl methacrylate (PMMA) by electron beam lithography and wet etching, performed in the Materials and Microsystems Laboratory.

http://www.azonano.com

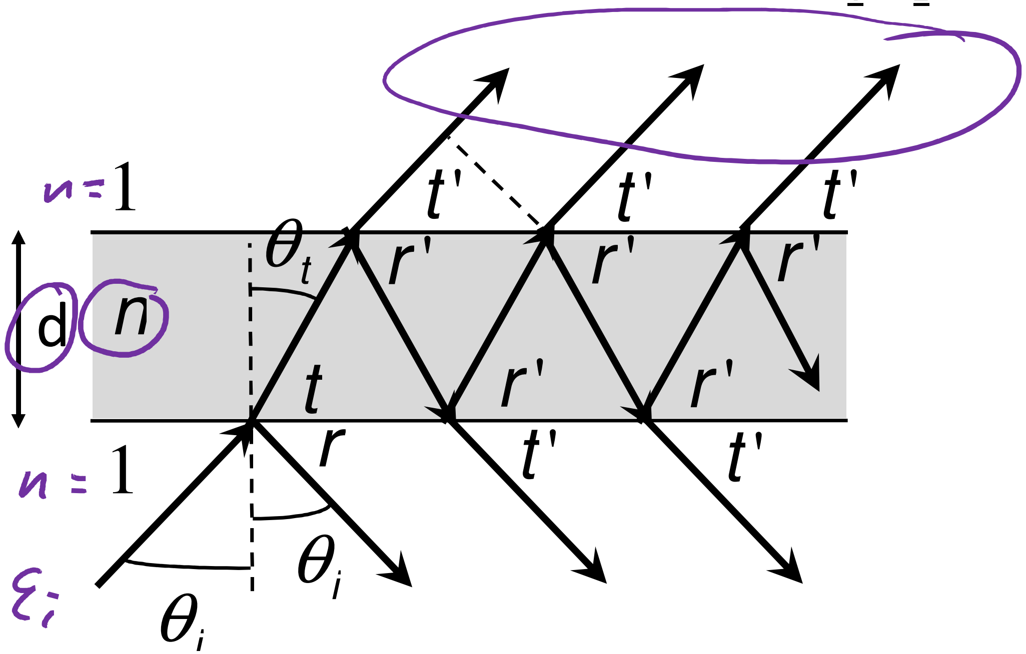
Fabry-Perot interferometer

↳ cavité laser
↳ spectro haute résolution

$t\,,\ r$: transmission and reflection coefficients, **$n_1 < n_2$**

$t',\ r'$: transmission and reflection coefficients, **$n_1 > n_2$**

$$\left| \frac{\mathcal{E}_t}{\mathcal{E}_i} \right|^2 = ?$$



Fabry-Perot: phase difference?

entre rayons sortants

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\Delta\phi = 2\ell n k_0 - k_0 a$$

$$\frac{a}{s} = \sin\theta_i \Rightarrow a = s\sin\theta_i$$

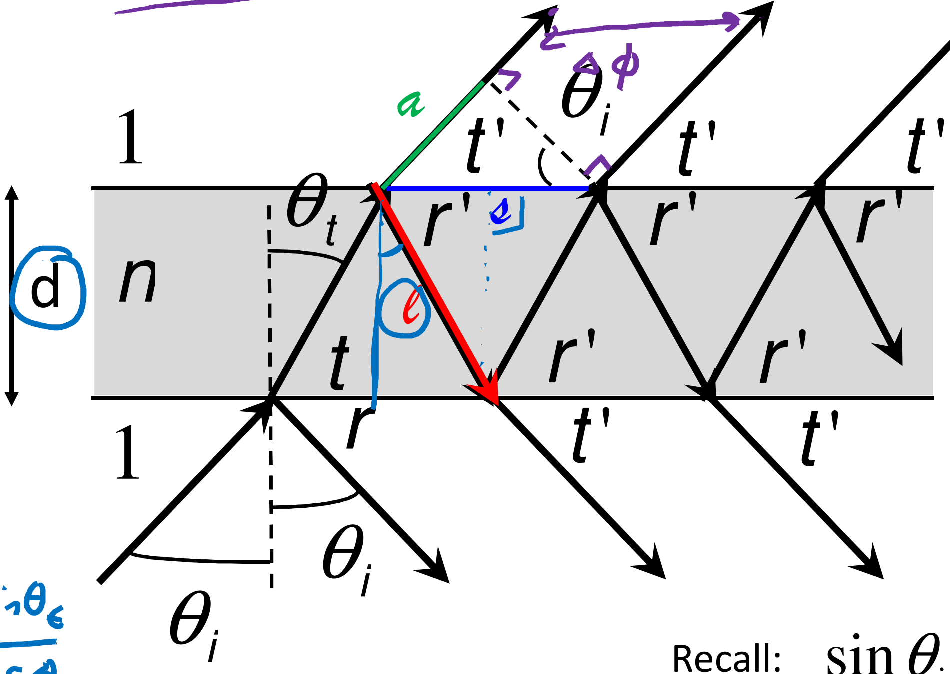
$$\frac{s/2}{\ell} = \sin\theta_t \Rightarrow s = 2\ell\sin\theta_t$$

$$\frac{d}{\ell} = \cos\theta_t \Rightarrow \ell = \frac{d}{\cos\theta_t}$$

$$\Delta\phi = 2k_0 n\frac{d}{\cos\theta_t} - k_0 \underbrace{\sin\theta_i}_{n\sin\theta_t} \frac{2d\sin\theta_t}{\cos\theta_t}$$

$$= \frac{2nk_0 d}{\cancel{\cos\theta_t}}\left[\underbrace{1-\sin^2\theta_t}_{\cos^{\cancel{2}}\theta_t}\right]$$

$$\boxed{\Delta\phi = 2nk_0 d\cos\theta_t}$$



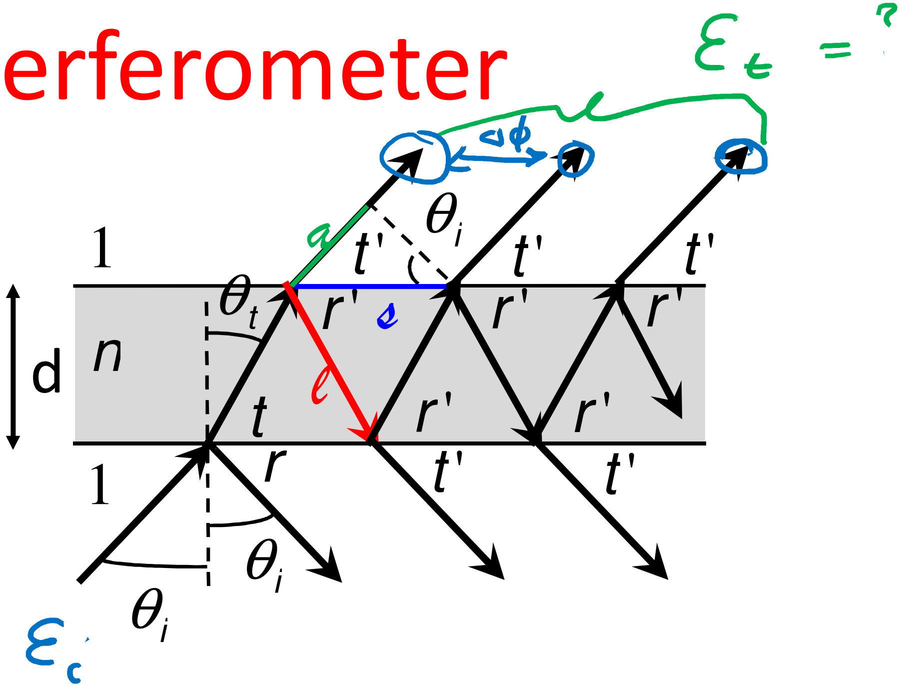
Recall: $\sin\theta_i = n\sin\theta_t$

Fabry-Perot interferometer

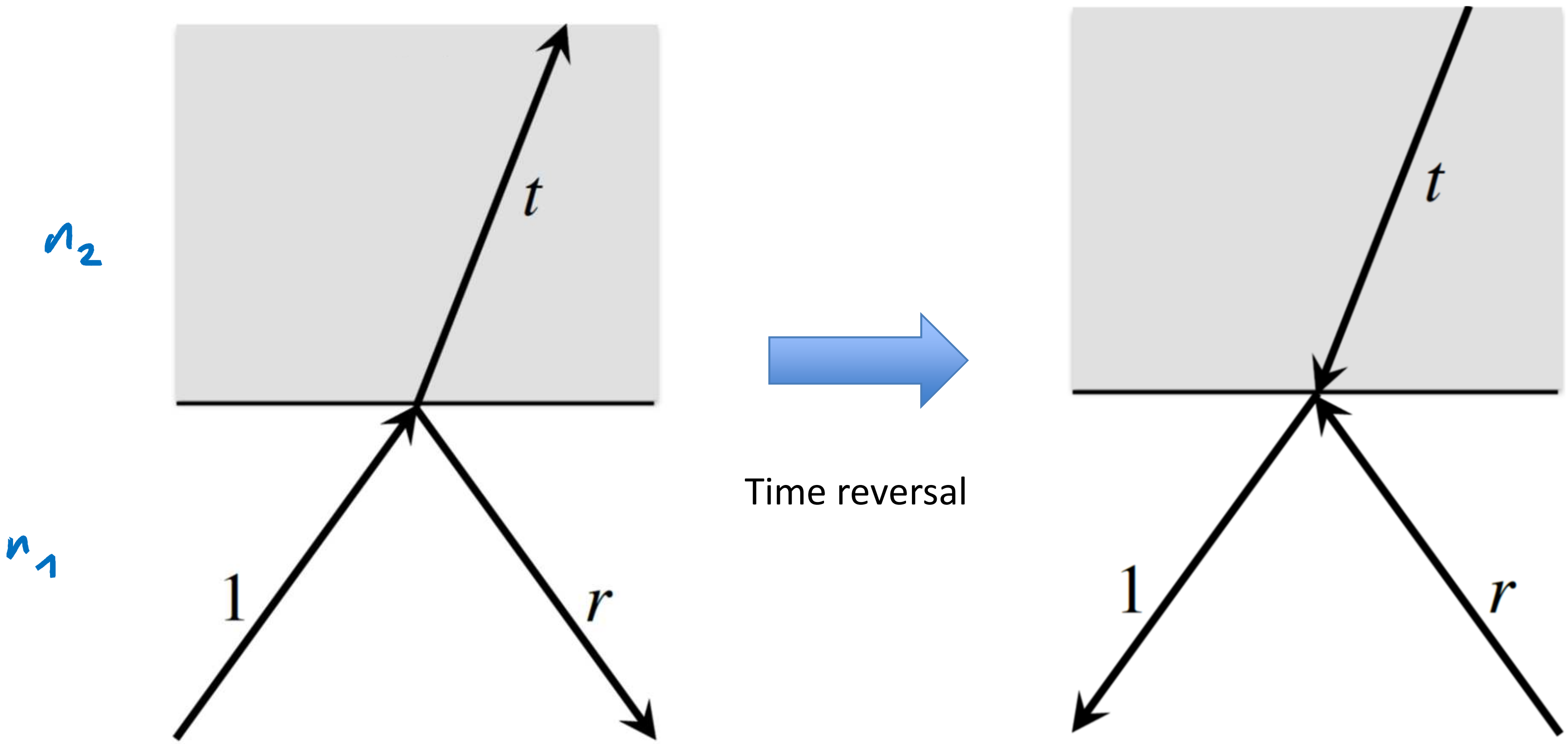
$$\mathcal{E}_t = \mathcal{E}_i \left[tt' + tt' r'^2 e^{i\Delta\phi} + tt' r'^4 e^{2i\Delta\phi} + \dots \right]$$

$$= \mathcal{E}_i \, tt' \sum_{p=0}^{\infty} \underbrace{r'^{2p} e^{ip\Delta\phi}}_{<1}$$

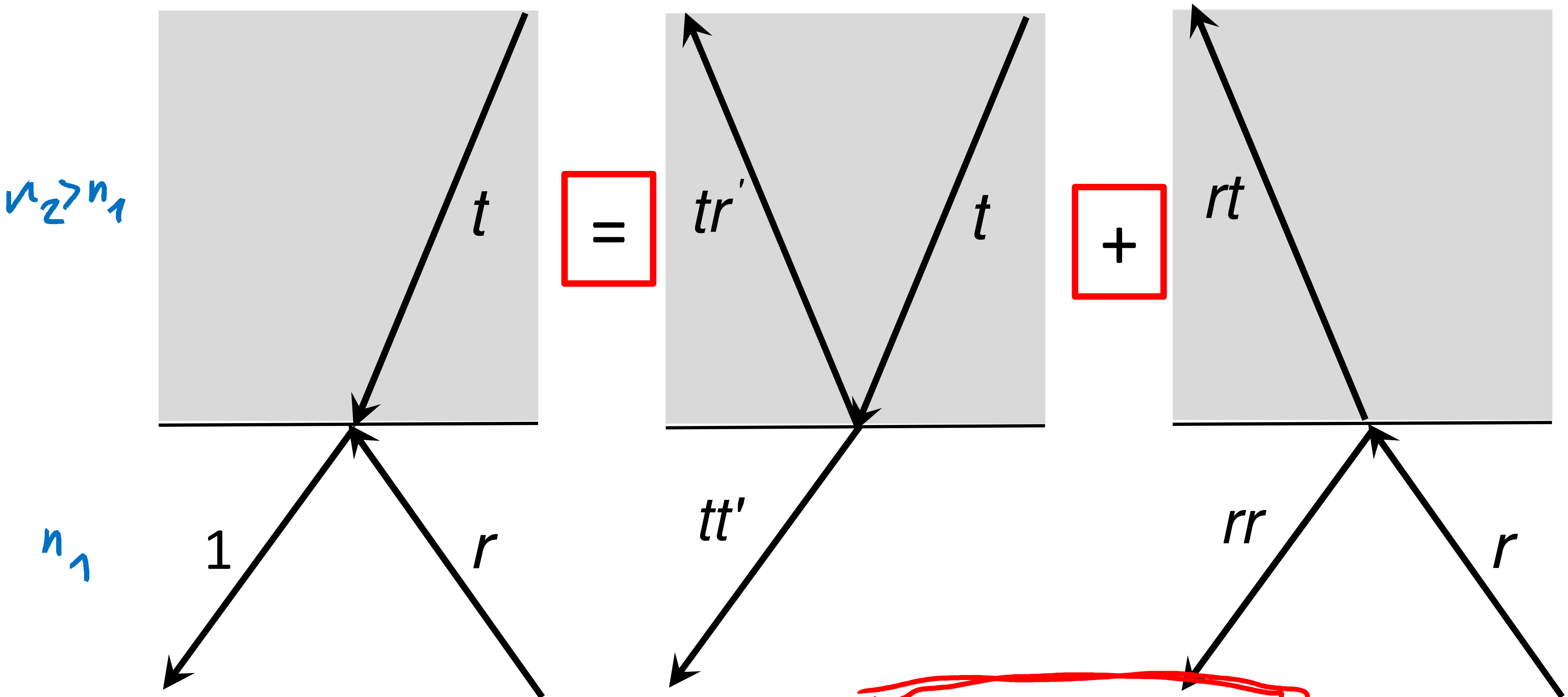
$$= \frac{\mathcal{E}_i \, tt'}{1 - r'^2 e^{i\Delta\phi}}$$



Stokes relations



Stokes relations



$$\boxed{\begin{gathered} tt' + r^2 = 1 \\ r' = -r \end{gathered}}$$

$$\cancel{t}r' + r\cancel{t} = 0$$

Lecture notes p.26

Fabry Perot interferometer

$$\Delta\phi = 2n\frac{\omega}{c}d\cos\theta_t$$

$$\mathcal{E}_t = \mathcal{E}_i \frac{tt'}{1-r'^2 e^{i\Delta\phi}} \qquad (2.92)$$

$$\begin{aligned} r^2 + tt' &= 1 \\ tr' + rt &= 0 \\ r &= -r' \\ tt' &= 1 - r^2 = 1 - R \end{aligned} \qquad (2.93)\text{-}(2.96)$$

$$\boxed{\frac{I_t}{I_i} = \left|\frac{\mathcal{E}_t}{\mathcal{E}_i}\right|^2 = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\frac{\Delta\phi}{2}}}$$

Transmission maximale

↪ à la résonance

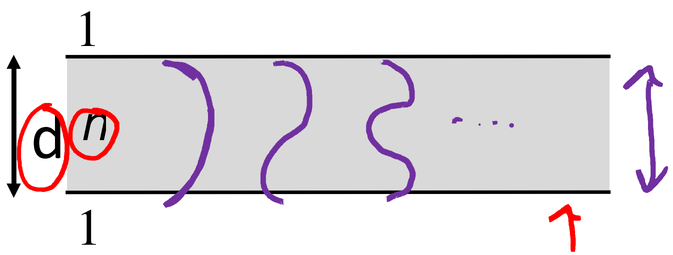
Resonance?

$$\sin\frac{\Delta\phi}{2} = 0$$

$$\Rightarrow \Delta\phi = 2m\pi \quad m \in \mathbb{Z}$$

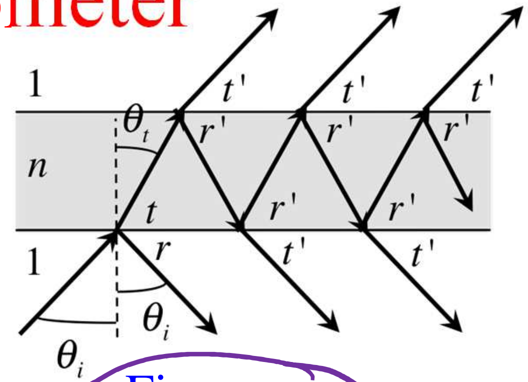
Considérons $\theta_i = 0$

$$d = m\frac{\lambda}{2n}$$



Fabry-Perot interferometer

$$\text{Finesse} = \frac{\text{"distance" entre pics}}{\text{largeur des pics}}$$

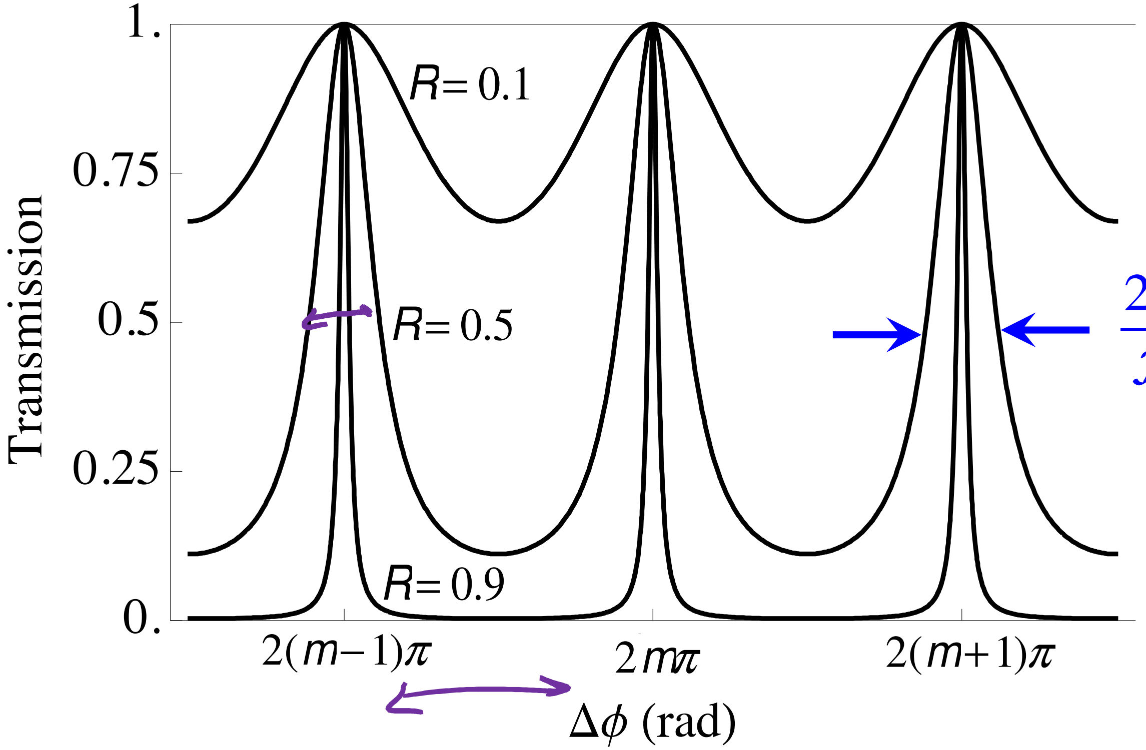


Finesse:

$$\boxed{\mathcal{F} = \frac{\pi\sqrt{R}}{1-R}}$$

À la résonance

$$I_{cavité} \approx \frac{\mathcal{F}}{\pi} I_i$$



$\frac{2\pi}{\mathcal{F}}$

Pour des applications on veut $R \nearrow\nearrow$ → comment faire?

$$\boxed{\begin{aligned} R &= \left(\frac{n-1}{n+1}\right)^2 \\ &= 0.25 \text{ for } n = 3 \end{aligned}}$$

semicond.

Highly reflective mirror

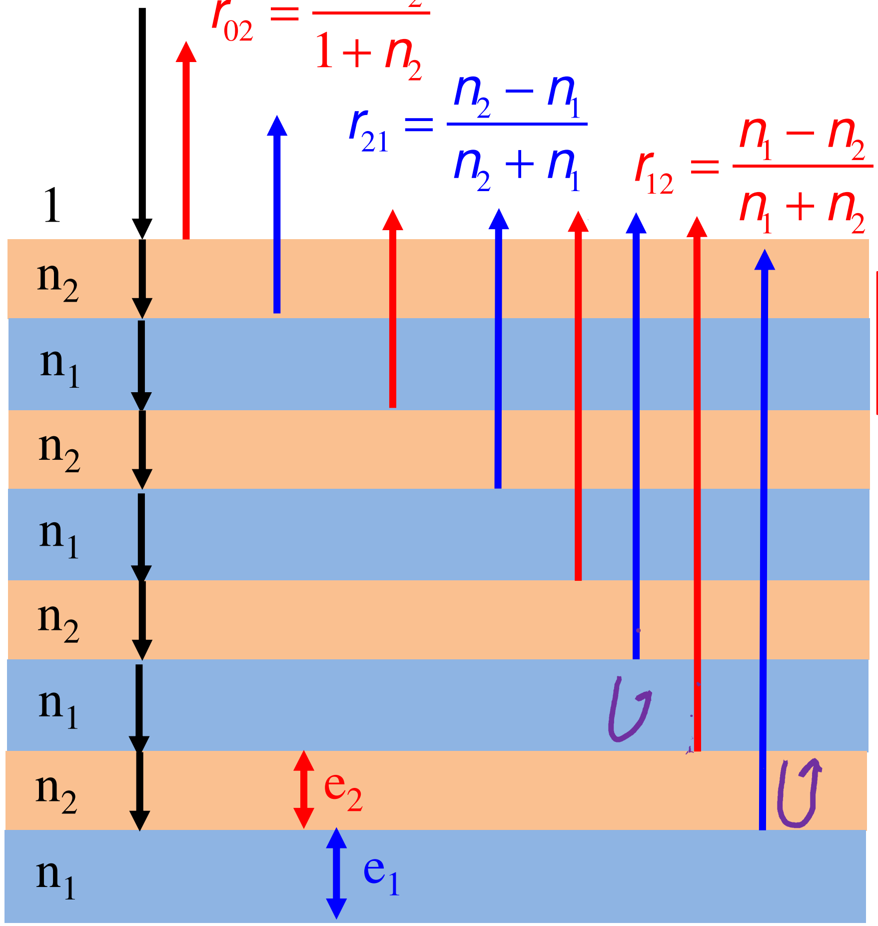
Metallic mirror: R ~ 99 %. Can we do better?

Try a dielectric thin film stack!

$$r_{02} = \frac{1-n_2}{1+n_2}$$

$$r_{21} = \frac{n_2-n_1}{n_2+n_1}$$

$$r_{12} = \frac{n_1-n_2}{n_1+n_2}$$



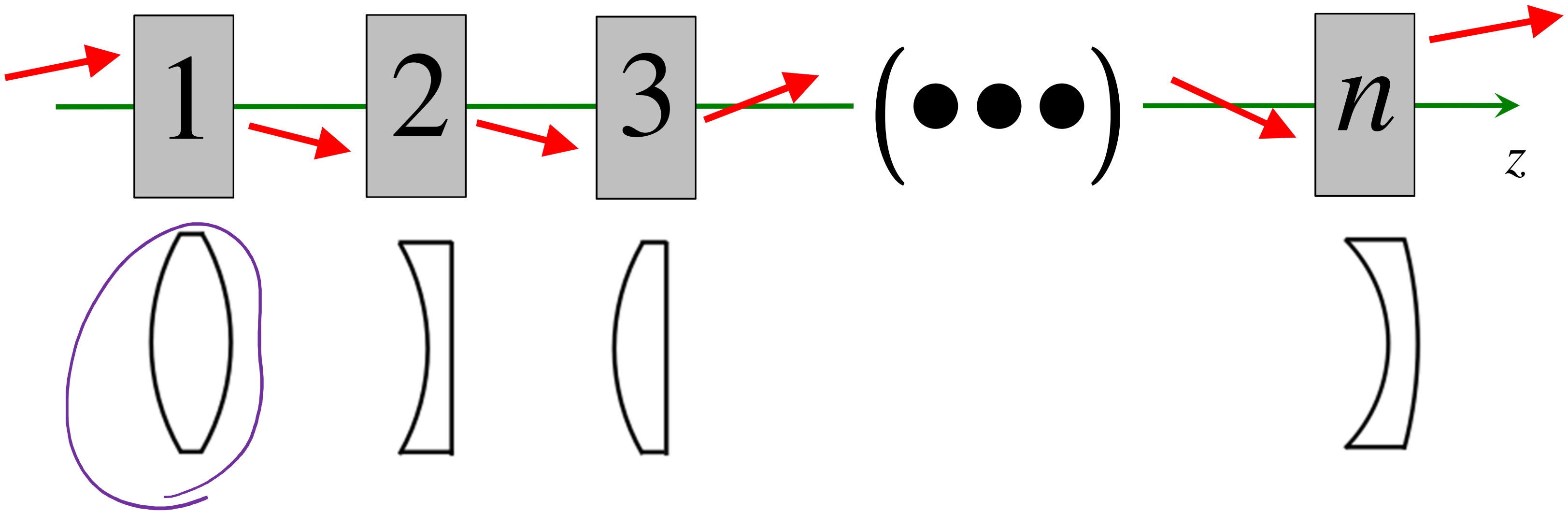
$\boxed{r_{21} = -r_{12}}$ => π

Need another π phase change for each round trip in a layer

$$\boxed{\frac{2\pi}{\lambda} n_1 2e_1 = \frac{2\pi}{\lambda} n_2 2e_2 = \pi}$$

Layer thickness = $\dfrac{\lambda}{4n}$

Matrix optics → optique géométrique



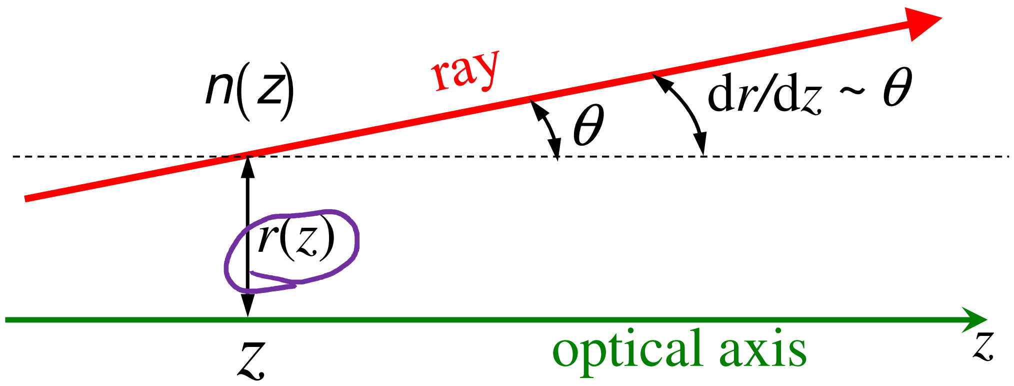
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \mathbf{M}_n \mathbf{M}_{n-1} \ldots \mathbf{M}_2 \mathbf{M}_1$$

Matrix optics (for paraxial rays)

"près"

rayons près de l'axe optique

$\theta \sim \sin\theta \sim \tan\theta$



Reduced slope

$$r'(z) \equiv n(z)\frac{\mathrm{d}r}{\mathrm{d}z}$$

$n(z)$: local index of refraction

Ray vector: $\mathbf{R}(z) \equiv \begin{pmatrix} r(z) \\ r'(z) \end{pmatrix}$

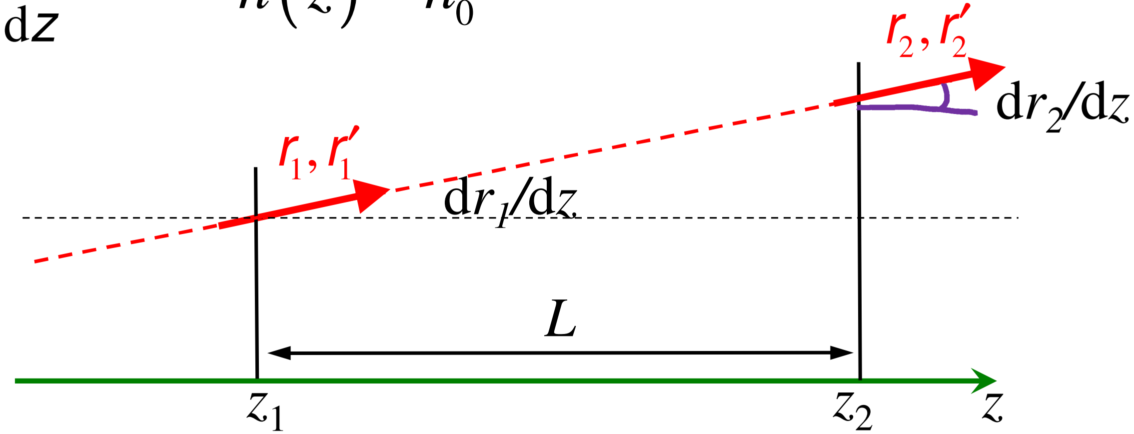
Matrix optics: free space propagation

Reduced slope

$$r'(z) \equiv n(z)\frac{\mathrm{d}r}{\mathrm{d}z} \qquad\qquad n(z) = n_0$$

$$\frac{r'}{n_0} = \frac{dr}{dz}$$

$$\boxed{r_1' = r_2'}$$



$$r = mz + b$$

à $z = z_1$

$$r_1 = \frac{dr_1}{dz} z_1 + b$$

$$b = r_1 - \frac{dr_1}{dz} z_1$$

à $z = z_2$

$$r_2 = \frac{dr_1}{dz} z_2 + r_1 - \frac{dr_1}{dz} z_1$$

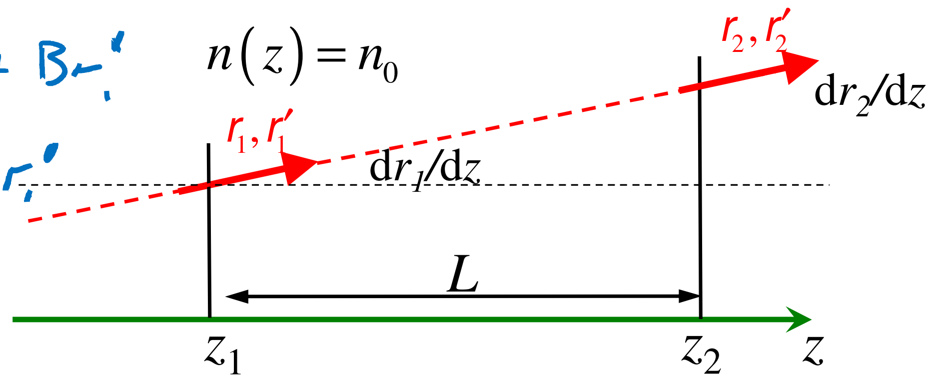
$$= r_1 + \frac{dr_1}{dz}\overbrace{(z_2 - z_1)}^{L}$$

$$\boxed{r_2 = r_1 + \frac{r_1'}{n_0} L}$$

Matrix optics: free space propagation

$$r_2 = r_1 + \frac{r_1'}{n_0} L = A r_1 + B r_1'$$

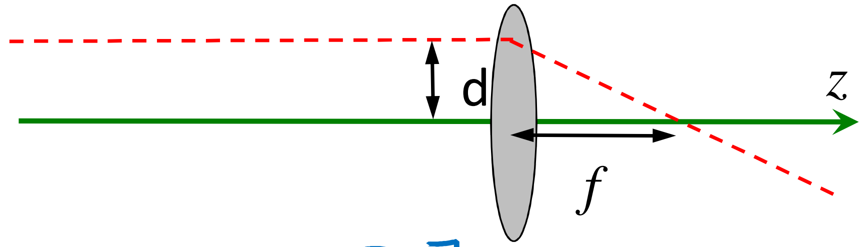
$$r_1' = r_2' = C r_1 + D r_1'$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix} = \begin{bmatrix} r_2 \\ r_2' \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & L/n_0 \\ 0 & 1 \end{bmatrix}$$

Matrix optics: thin lens of focal length *f* in air



Just before the lens:

$r_1 = d$

$r_1' = 0$

$$\begin{bmatrix} d \\ -\frac{d}{f} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} d \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} A &= 1 \\ C &= -\frac{1}{f} \end{aligned}}$$

$B = ?$

$D = ?$

Just after the lens:

$r_2 = d$

$r_2' = -\frac{d}{f}$

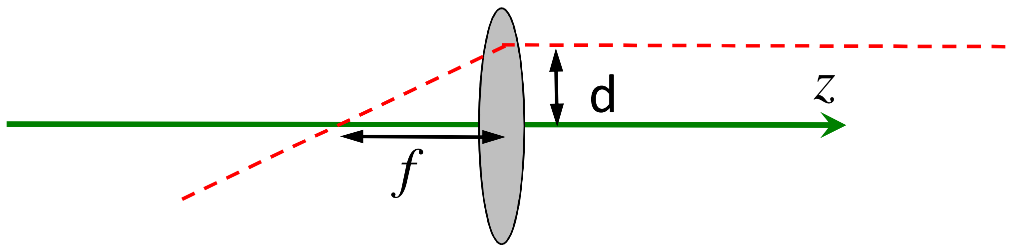
Matrix optics: thin lens of focal length f in air

$A = 1$; $C = -1/f$

Just before the lens:

$r_1 = d$

$r_1' = d/f$



$$\begin{bmatrix} d \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & B \\ -\frac{1}{f} & D \end{bmatrix} \begin{bmatrix} d \\ d/f \end{bmatrix}$$

Just after the lens:

$r_2 = d$

$r_2' = 0$

$$\Rightarrow \quad B = 0, \quad D = 1$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}}$$

$f > 0$ for a convergent lens