Démonstration que ∂_μ est un bon quadrivecteur

$\partial'_\mu = \dfrac{\partial}{\partial x'^\mu} = \dfrac{\partial}{\partial x^\nu} \dfrac{\partial x^\nu}{\partial x'^\mu} = \Lambda_\mu{}^\nu \partial_\nu$ qui est la transfo^t de Lorentz d'un quadrivecteur covariant.

En effet

$$x^\nu = \begin{pmatrix} x^0 = ct \\ x^1 = x \\ x^2 = y \\ x^3 = z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix}$$

$\rightarrow x^0 = ct = \gamma(ct' + \beta x') = \gamma x'^0 + \beta\gamma x'^1$

$\dfrac{\partial x^0}{\partial x'^0} = \gamma$ et $\dfrac{\partial x^0}{\partial x'^1} = \beta\gamma$ de même $\dfrac{\partial x^1}{\partial x'^0} = \beta\gamma$ et $\dfrac{\partial x^1}{\partial x'^1} = \gamma$

On a donc $\partial'_0 = \dfrac{\partial}{\partial x^0}\dfrac{\partial x^0}{\partial x'^0} + \dfrac{\partial}{\partial x^1}\dfrac{\partial x^1}{\partial x'^0} = \gamma \dfrac{\partial}{\partial x^0} + \beta\gamma \dfrac{\partial}{\partial x^1}$

$= \gamma \partial_0 + \beta\gamma \partial_1$

$\partial'_1 = \dfrac{\partial}{\partial x^0}\dfrac{\partial x^0}{\partial x'^1} + \dfrac{\partial}{\partial x^1}\dfrac{\partial x^1}{\partial x'^1} = \beta\gamma \dfrac{\partial}{\partial x^0} + \gamma \dfrac{\partial}{\partial x^1} = \beta\gamma \partial_0 + \gamma \partial_1$

On a donc bien

$$\begin{pmatrix} \partial'_0 \\ \partial'_1 \\ \partial'_2 \\ \partial'_3 \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \partial_0 \\ \partial_1 \\ \partial_2 \\ \partial_3 \end{pmatrix} \Leftrightarrow \partial'_\mu = \Lambda_\mu{}^\nu \partial_\nu$$