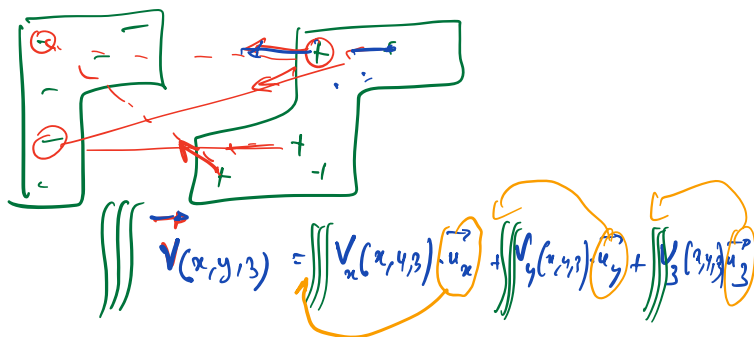


$$[F] = [mg] = [m][g]$$

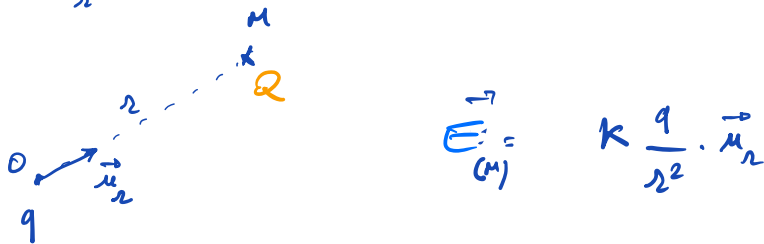
$$= M \cdot L \cdot T^{-2}$$

SI :  $1N = 1 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$

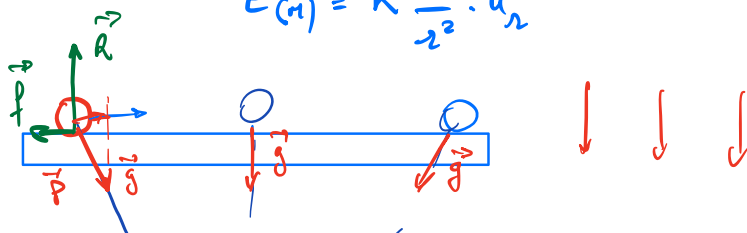


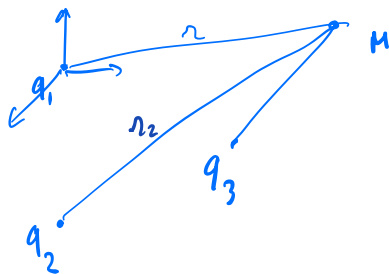
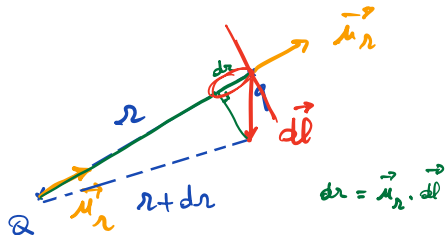
$$\vec{u}_x \cdot \iiint V_x(x,y,z) \cdot dx \cdot dy \cdot dz$$

$$\vec{OM} = \vec{r} = r \cdot \vec{u}_r$$



$$\vec{E}(M) = k \frac{q}{r^2} \cdot \vec{u}_r$$



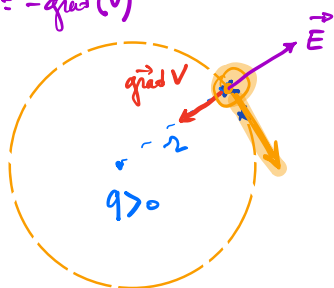


$$V(x, y, z) = V_1(x, y, z) + V_2(x, y, z) + V_3(x, y, z)$$

$$V_1 = k \frac{q_1}{r} = k \frac{q_1}{\sqrt{x^2 + y^2 + z^2}}$$

$$\vec{E} = -\text{grad}(V(x, y, z)) = - \begin{pmatrix} \frac{\partial V(x, y, z)}{\partial x} \\ \frac{\partial V(x, y, z)}{\partial y} \\ \frac{\partial V(x, y, z)}{\partial z} \end{pmatrix}$$

$$\vec{E} = -\text{grad}(V)$$



$$V(r, \theta, \varphi) = k \frac{q}{r}$$

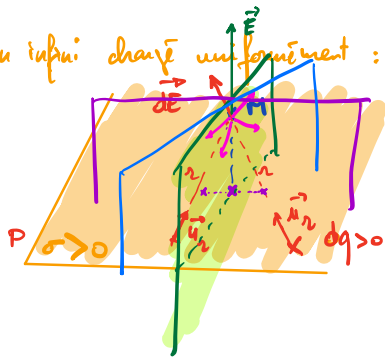
$$\begin{cases} \frac{\partial V}{\partial r} \neq 0 \\ \frac{\partial V}{\partial \theta} = 0 \\ \frac{\partial V}{\partial \varphi} = 0 \end{cases}$$

$$\text{grad } V = \begin{pmatrix} \frac{\partial V}{\partial r} = -\frac{kq}{r^2} \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{E} = -\text{grad}(V) = \begin{pmatrix} \frac{kq}{r^2} \\ 0 \\ 0 \end{pmatrix}$$

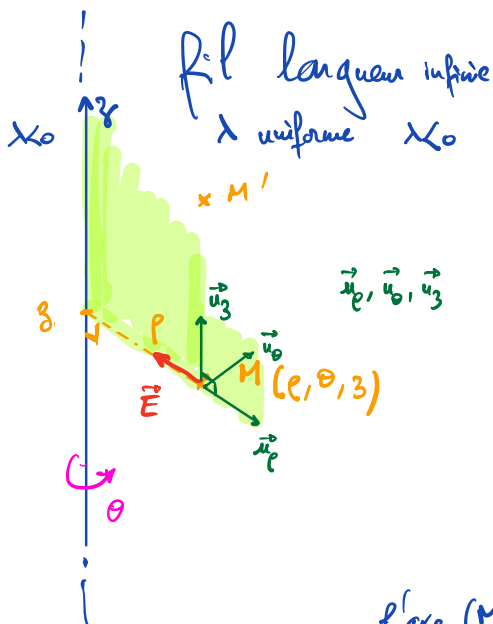
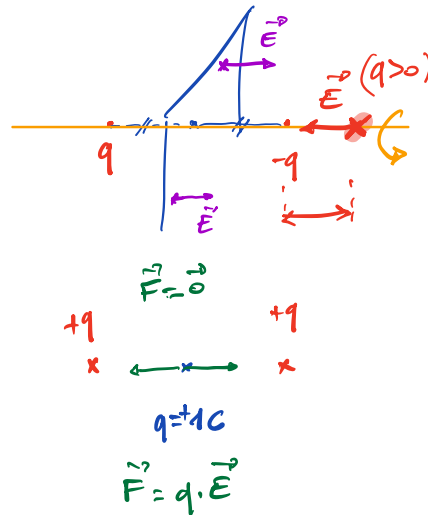
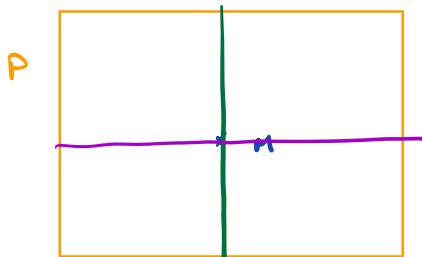


plan infini chargé uniformément :  $\sigma =$  densité surfacique de charge  $> 0$



$$d\vec{E} = k \frac{dq}{r^2} \cdot \vec{u}_n$$

$$\vec{E} = \iint_{\text{Plan}} d\vec{E}$$



$$\vec{E} = E_\rho(r, \theta, z) \cdot \vec{u}_\rho + E_\theta(r, \theta, z) \cdot \vec{u}_\theta + E_z(r, \theta, z) \cdot \vec{u}_z$$

plan  $(\vec{u}_\rho, \vec{u}_z) =$  plan de symétrie

$$\vec{E} \in (\vec{u}_\rho, \vec{u}_z)$$

$\Rightarrow \vec{E}$  n'a aucune composante selon  $\vec{u}_\theta$

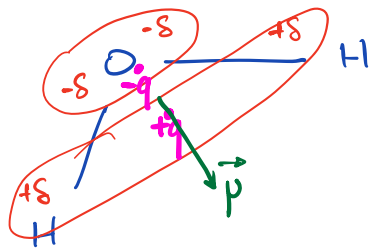
$$E_\theta(r, \theta, z) = 0$$

l'axe  $(M, \vec{u}_\rho) =$  axe de sym.  $\Rightarrow \vec{E} \in (M, \vec{u}_\rho)$

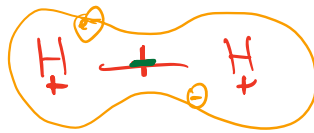
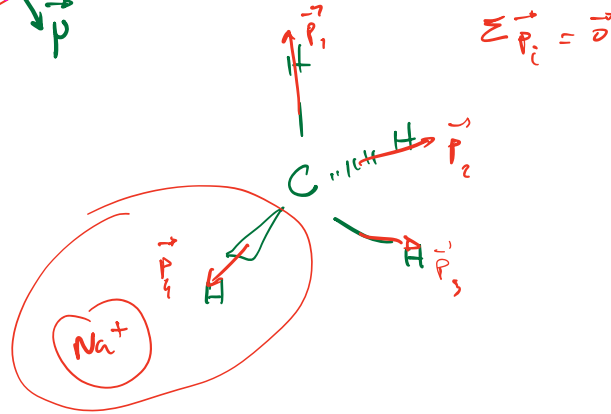
$$\vec{E} // \vec{u}_\rho$$

symétrie  $\Rightarrow \vec{E} = E(r, \theta, z) \cdot \vec{u}_\rho$

# Moment dipolaire $H_2O$



méthane



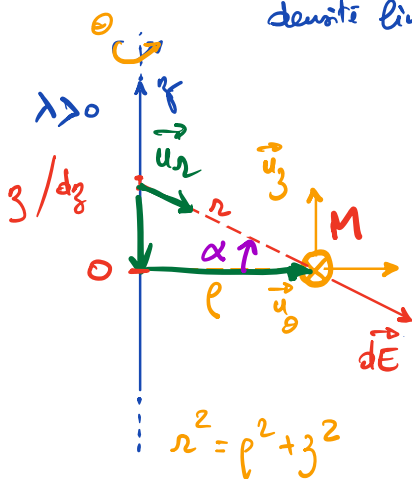
Calcul du champ électrique créé par un fil chargé uniformément et de longueur infinie

densité linéique  $\lambda$  uniforme

Coord. cylindriques

$$\vec{E}(r, \theta, z)$$

trier des directions  
 $\vec{u}_r, \vec{u}_\theta, \vec{u}_z$



$$\vec{E}(r, \theta, z) = E_r(r, \theta, z) \cdot \vec{u}_r + E_\theta(r, \theta, z) \cdot \vec{u}_\theta + E_z(r, \theta, z) \cdot \vec{u}_z$$

$$\lambda = \frac{dq}{dz}$$

charge ponctuelle  $dq = \lambda \cdot dz \Rightarrow d\vec{E} = k \cdot \frac{dq}{r^2} \cdot \vec{u}_r$

$$d\vec{E} = k \cdot \frac{\lambda \cdot dz}{r^2} \cdot \vec{u}_r$$

$$\vec{u}_r = (\cdot) \cdot \vec{u}_r + (\cdot) \cdot \vec{u}_\theta + (\cdot) \cdot \vec{u}_z$$

$$\vec{u}_r = \frac{-z \cdot \vec{u}_z + r \cdot \vec{u}_r}{\sqrt{r^2 + z^2}} \quad (+0 \cdot \vec{u}_\theta)$$

$\sqrt{z^2 + \rho^2}$

$$\vec{E} = \int_{\rho, l} \vec{dE} = \int_{\rho, l} k \frac{\lambda \cdot dz}{r^2} \cdot \frac{-z \cdot \vec{u}_z + \rho \cdot \vec{u}_\rho}{\sqrt{z^2 + \rho^2}}$$

$$= k \cdot \lambda \left[ \vec{u}_z \int_{\rho, l} \frac{-z \cdot dz}{r^2 \sqrt{z^2 + \rho^2}} + \rho \vec{u}_\rho \int_{\rho, l} \frac{dz}{r^2 \sqrt{z^2 + \rho^2}} \right]$$

$\int_{\rho, l} f(z) \cdot dz$   
 $\text{ou}$   
 $\int_{\rho, l} f(r) \cdot dr$

$r = \sqrt{\rho^2 + z^2}$   
 $\frac{\partial(r)}{\partial z} \cdot dz = \frac{\partial(\sqrt{\rho^2 + z^2})}{\partial z} \cdot dz$

variable  $z$  :  $\int_{z=-\infty}^{+\infty} f(z) \cdot dz$

variable  $r$  :  $\int_{r=+\infty}^{+\infty} f(r) \cdot dr \rightarrow 2 \cdot \int_{+\infty}^{\rho}$

$\int_{\alpha=-\frac{\pi}{2}}^{+\frac{\pi}{2}} f(\alpha) \cdot d\alpha$

Application avec le th. de Gauss

①  $M \in$  axe de symétrie  $(M, \vec{u}_\rho)$   
symétrique

$$\Rightarrow \vec{E}(\rho, \theta, z) = E(\rho, \theta, z) \cdot \vec{u}_\rho$$

② invariance:

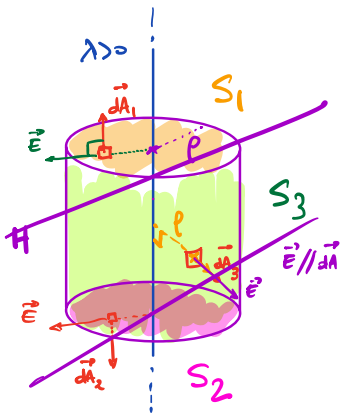
$\exists$  invariance / translation de vecteur  $\vec{u}_z$

$$\Rightarrow \vec{E}(\rho, \theta, z) = E(\rho, \theta) \cdot \vec{u}_\rho$$

∃ invariance / notation de l'angle  $\vartheta$

$$\Rightarrow \vec{E}(r, \vartheta, \varphi) = E(r) \cdot \vec{u}_\rho$$

③ R. de Gauss  $\phi = \oint_{\text{surf. fermée}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{nette}}}{\epsilon_0}$



$$\phi = \oint_{\Sigma} \vec{E} \cdot d\vec{A} = 0$$

$$E \oint_{\Sigma} dA = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 \Sigma}$$

1) Lieu des pts  $\vec{E} = cte \quad \|\vec{E}\| = cte$

$$E(r) = cte \iff \rho = cte$$

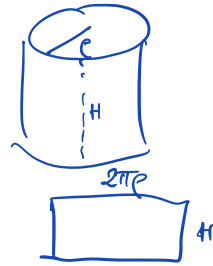
$$\phi = \oint_{\text{surf. fermée}} \vec{E} \cdot d\vec{A} = \int_{S_1} \vec{E} \cdot d\vec{A}_1 + \int_{S_2} \vec{E} \cdot d\vec{A}_2 + \int_{S_3} \vec{E} \cdot d\vec{A}_3$$

$d\vec{A}_1 // \vec{u}_z$   
 $\vec{E} // \vec{u}_\rho$   
 $\vec{E} \perp d\vec{A}_1$   
 $\vec{E} \cdot d\vec{A}_1 = 0$

$\vec{E} \perp d\vec{A}_2$

choix de la surface fermée sur laquelle

- ↳  $\vec{E} // d\vec{A}$
- $\vec{E} \perp d\vec{A}$
- $\vec{E} = cte$



$$\phi = \int_{S_3} \vec{E} \cdot d\vec{A}_3 = \int_{S_3} E \cdot dA_3 = E \left( \int_{S_3} dA_3 \right)$$

$E = \text{const}$   
sur  $S_3$

$$\phi = E \cdot 2\pi r H$$

on sait aussi

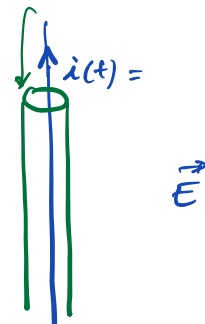
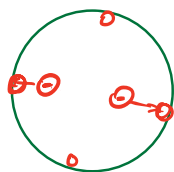
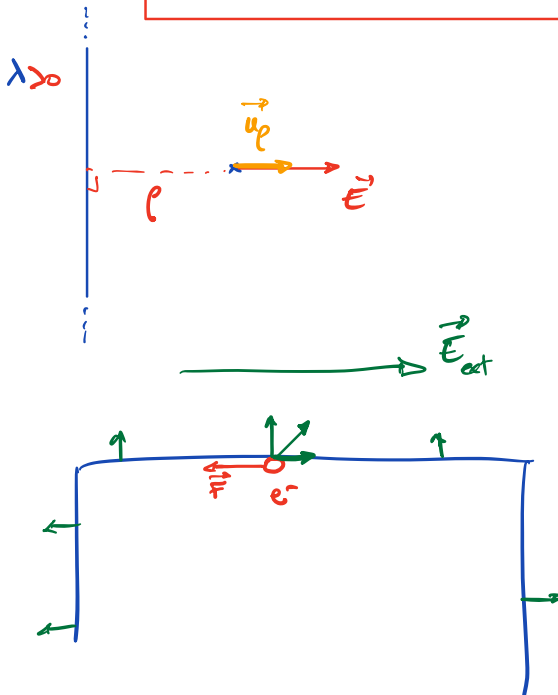
$$\phi = \frac{Q_{\text{charge}}}{\epsilon_0} = \frac{H \cdot \lambda}{\epsilon_0}$$

$$E \cdot 2\pi r H = \frac{H \lambda}{\epsilon_0}$$

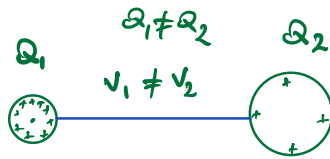
$$E = \frac{\lambda}{2\pi \epsilon_0 r} = E(r)$$

$$\vec{E}(r, \theta, z) = \frac{\lambda}{2\pi \epsilon_0 r} \cdot \vec{u}_r$$

$$k = \frac{1}{4\pi \epsilon_0}$$



# Effet du point



à l'équilibre

$$V'_1 = V'_2$$

$$V_1 = \frac{k \cdot Q_1}{R_1}$$

$$V_2 = \frac{k \cdot Q_2}{R_2}$$

$$V'_1 = k \frac{Q'_1}{R_1} = V'_2 = k \frac{Q'_2}{R_2}$$

$$\frac{Q'_1}{Q'_2} = \frac{R_1}{R_2}$$

densité surfacique

$$\sigma'_1 = \frac{Q'_1}{\pi R_1^2}$$

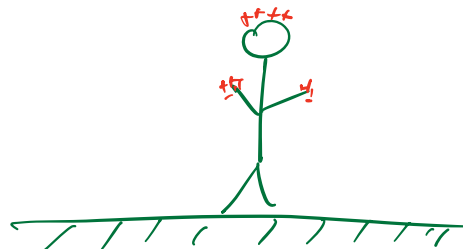
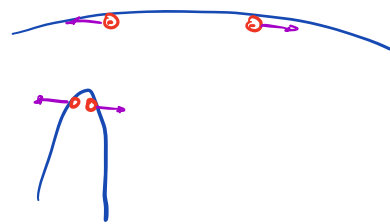
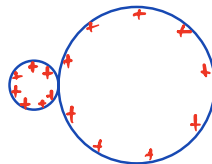
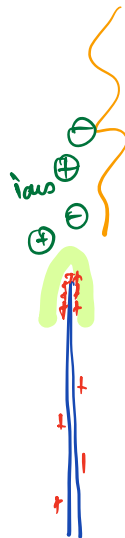
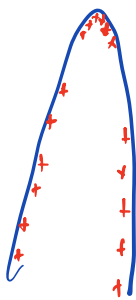
$$\sigma'_2 = \frac{Q'_2}{\pi R_2^2}$$

$$Q'_1 = \sigma'_1 \pi R_1^2$$

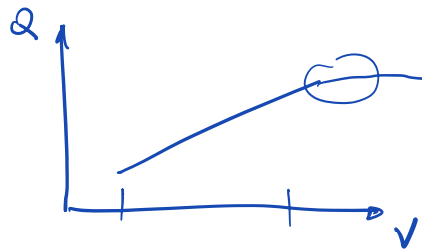
$$Q'_2 = \sigma'_2 \pi R_2^2$$

$$\frac{\sigma'_1 R_1^2}{\sigma'_2 R_2^2} = \frac{R_1}{R_2}$$

$$\Rightarrow \frac{\sigma'_1}{\sigma'_2} = \frac{R_2}{R_1}$$







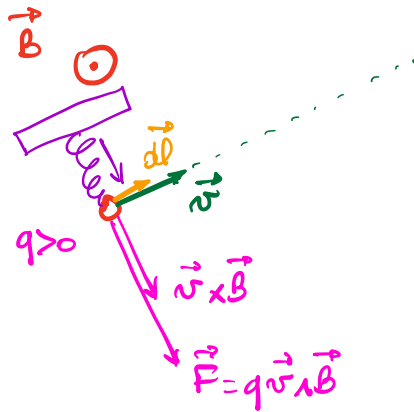
$$C = \frac{Q}{v}$$

$$Q = C \cdot v$$

$$\vec{\pi} = q \cdot \vec{v} \times \vec{B} = q \cdot \vec{v} \wedge \vec{B}$$

$$\vec{\pi} \perp \vec{v}$$

$$\vec{\pi} \perp \vec{B}$$

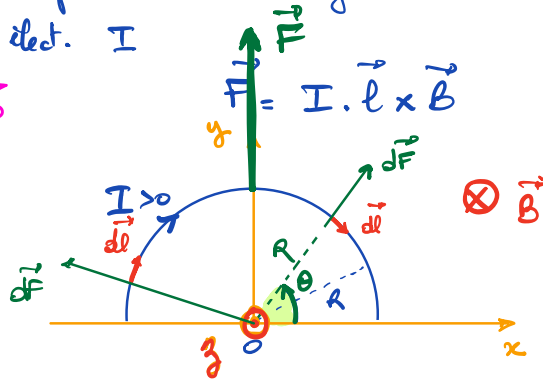
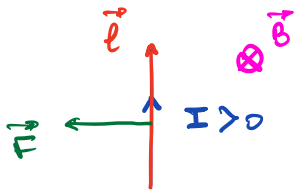


$$d\vec{l} \parallel \vec{v}$$

$$\vec{F} \perp d\vec{l}$$

$$dW = \vec{F} \cdot d\vec{l} = 0$$

Force magnétique agissant sur un segment de conducteur parcouru par l'intensité élect. I



$$\vec{F} = I \cdot \vec{l} \times \vec{B}$$

$$d\vec{F} = I \cdot d\vec{l} \times \vec{B}$$

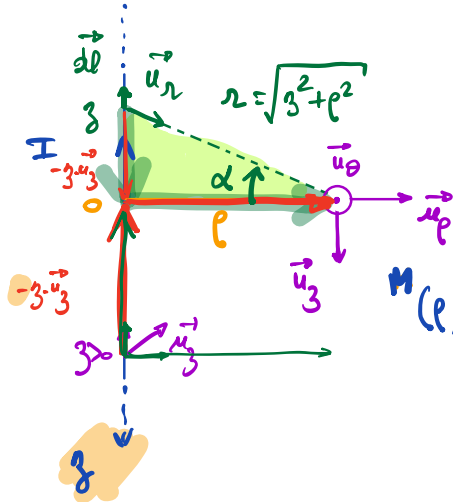
$$\vec{B} = B \cdot \vec{u}_z$$

$$d\vec{l} = \vec{u}_x + \vec{u}_y$$

$$\vec{F} = \int_{\theta=0}^{\pi} d\vec{F} = \dots = 2RIB$$

# Calcul du champ magnétique créé par un conducteur rectiligne infini

$$d\vec{B}(M) = \frac{\mu_0 I}{4\pi r^2} \cdot d\vec{l} \wedge \vec{u}_r$$



$$\vec{B}(M) = \int_{\text{fil}} d\vec{B}(M)$$

$$\vec{B}(M) = \int_{\text{fil}} \frac{\mu_0 I}{4\pi r^2} \cdot d\vec{l} \times \vec{u}_r$$

$$= \frac{\mu_0 I}{4\pi} \int_{z=-\infty}^{+\infty} \frac{d\vec{l} \wedge \vec{u}_r}{z^2 + \rho^2}$$

$$d\vec{l} = dz \cdot \vec{u}_z$$

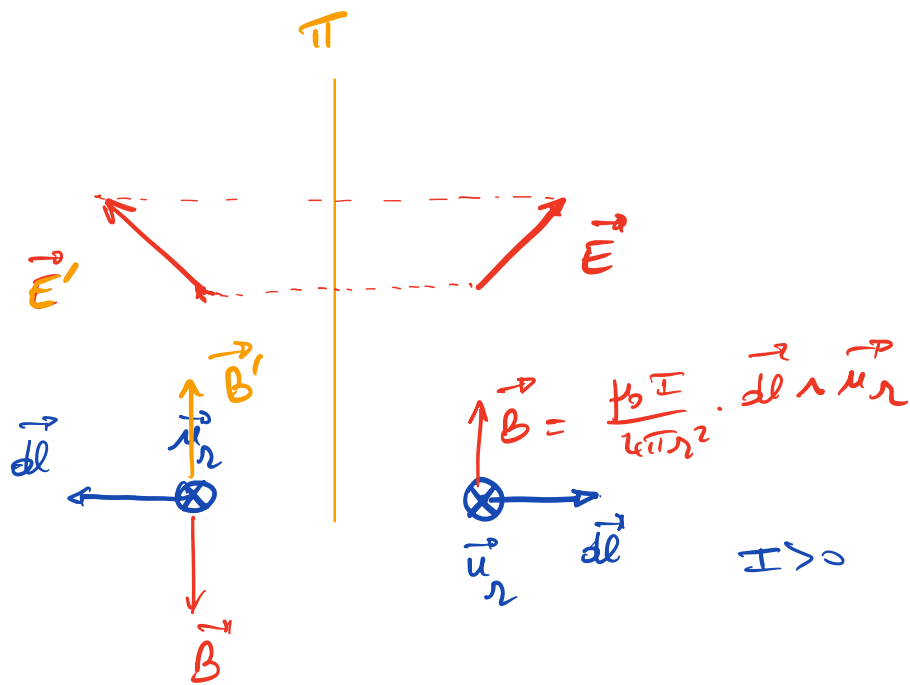
$$\vec{u}_r = \left( \rho \cdot \vec{u}_\rho + (-z) \cdot \vec{u}_z \right) \cdot \frac{1}{\sqrt{\rho^2 + (-z)^2}}$$

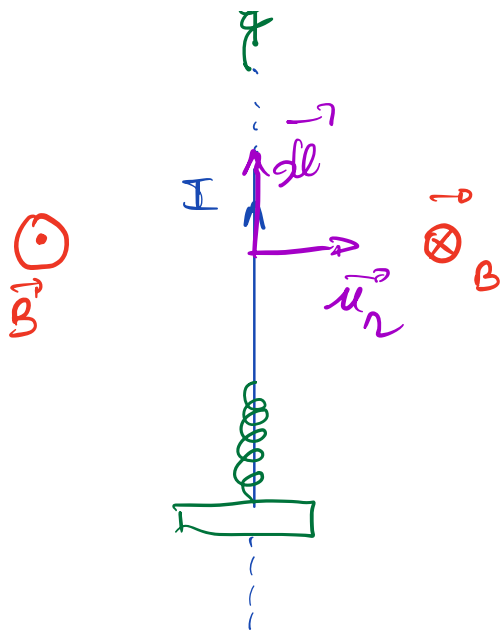
$$d\vec{l} \wedge \vec{u}_r = (dz \cdot \vec{u}_z) \times \left( \frac{\rho \cdot \vec{u}_\rho - z \cdot \vec{u}_z}{\sqrt{\rho^2 + z^2}} \right)$$

$$= \frac{\rho dz}{\sqrt{\rho^2 + z^2}} \underbrace{\vec{u}_z \wedge \vec{u}_\rho}_{\vec{u}_\theta} - \frac{z \cdot dz}{\sqrt{\rho^2 + z^2}} \cdot \underbrace{\vec{u}_z \wedge \vec{u}_z}_{\vec{0}}$$

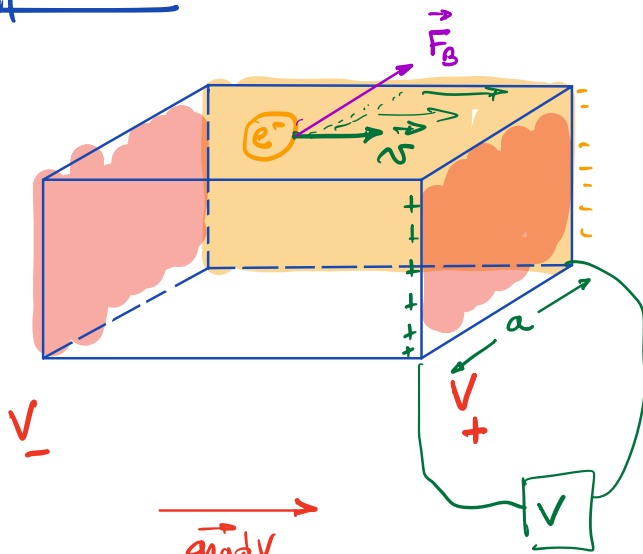
$$d\vec{l} \wedge \vec{u}_r = \frac{\rho dz}{\sqrt{\rho^2 + z^2}} \cdot \vec{u}_\theta$$

$$\begin{aligned}
 \vec{B}(M) &= \frac{\mu_0 I}{4\pi} \cdot \vec{e} \cdot \vec{u}_0 \int_{-\infty}^{+\infty} \frac{dz}{\sqrt{\rho^2+z^2}} \cdot \frac{1}{(\rho^2+z^2)} \\
 &= \frac{\mu_0 I \vec{e} \cdot \vec{u}_0}{4\pi} \underbrace{\int_{-\infty}^{+\infty} \frac{dz}{(\rho^2+z^2)^{3/2}}}_{\frac{2}{\rho^2}} \\
 &= \frac{\mu_0 I}{2\pi \rho} \cdot \vec{u}_0
 \end{aligned}$$





Effet Hall



$\vec{B}$ , uniform

$$\vec{F}_B = q \cdot \vec{v} \wedge \vec{B}$$

$$= (-e) \vec{v} \wedge \vec{B}$$

$\vec{\text{grad}} V$

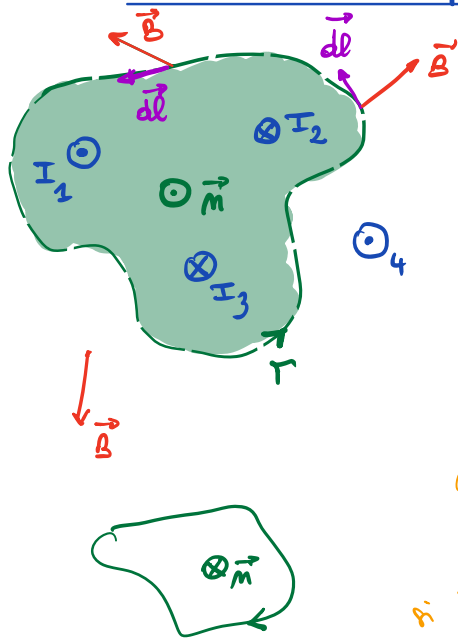
$\vec{E}$

$$\vec{F}_E = (-e) \cdot \vec{E}$$

$$\Delta V_H = \frac{B \cdot I}{n(-e) \cdot a}$$

$n =$  densité volumique d'électrons

# Théorème d'Ampère



somme des courants enlacés

$$\oint_{\Gamma} \vec{B} \cdot d\vec{l} = \mu_0 \sum_k \epsilon_k \cdot I_k$$

contour fermé  $\Gamma$

$\epsilon_k = \pm 1$   
ou 0

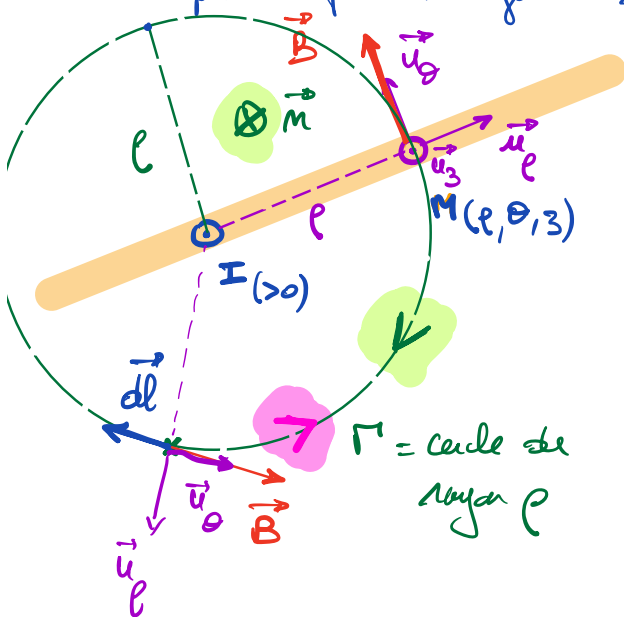
$$\oint_{\Gamma} \vec{B} \cdot d\vec{l} = \mu_0 [(+1) \cdot I_1 + (-1) \cdot I_2 + (-1) \cdot I_3]$$

si  $\vec{B} \parallel d\vec{l}$  et  $B = cte$

$$B \int dl = \mu_0 \sum \epsilon_k I_k$$

périphérie

## Exemple du fil de longueur infinie



$$\vec{B}(M) = ?$$

① Symétries et invariances

- système de coord. cylindriques

-  $M \in$  plan  $(\vec{u}_\rho, \vec{u}_\theta)$

plan d'anti-symétrie

$$\vec{B}(M) \in \text{plan } (\vec{u}_\rho, \vec{u}_\theta)$$

$$\vec{B}(M) \perp \vec{u}_3$$

$$M \in \text{plan de symétrie } (\vec{u}_\rho, \vec{u}_3) \Rightarrow \vec{B} \perp \text{plan } (\vec{u}_\rho, \vec{u}_3)$$

$$\Rightarrow \vec{B}(M) \parallel \vec{u}_\theta$$

$$\Rightarrow \vec{B}(e, \theta, z) = B(e, \theta, z) \cdot \vec{u}_\theta$$

- invariances:  $\exists$  invariance par translation de vecteur  $\vec{u}_z$
- $\exists$  " " rotation d'angle  $\theta$

$$\vec{B}(e, \theta, z) = B(e) \cdot \vec{u}_\theta$$

Th. d'Ampère: 
$$\oint_{\Gamma} \vec{B} \cdot d\vec{l} = \mu_0 \cdot \sum_k \epsilon_k \cdot I_k$$

Question: quel est le lieu des pts tels que  $B = \text{cte}$

Les cercles de rayon  $\rho$   
altitude  $z$

$$B(e) = \text{cte}$$

$$\rho = \text{cte}$$

$$d\vec{l} = -dl \cdot \vec{u}_\theta$$

$$\vec{B} = B(e) \cdot \vec{u}_\theta$$

$$\begin{aligned} \oint_{\Gamma} \vec{B} \cdot d\vec{l} &= \oint -B(e) \cdot dl \cdot \vec{u}_\theta \cdot \vec{u}_\theta = - \oint B(e) \cdot dl \\ &= -B(e) \oint dl \\ &= -B(e) \cdot 2\pi\rho \end{aligned}$$

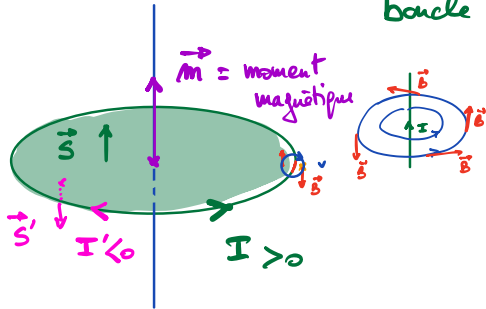
$$-B(e) \cdot 2\pi\rho = \mu_0 [-I]$$

$$B(e) = \frac{\mu_0 I}{2\pi\rho}$$

$$\vec{B}(r, \theta, \phi) = \frac{\mu_0 I}{2\pi r} \cdot \vec{u}_\theta$$

## Dipôle magnétique

boucle de courant

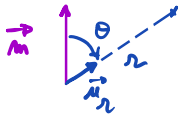


moment magnétique dipolaire

$$\vec{m} = I \cdot \vec{S} \quad (= I' \cdot \vec{S}')$$

$$\vec{m} = I' \cdot \vec{S}'$$

Coord. sphériques



$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} (2 \cos \theta \cdot \vec{u}_r + \sin \theta \cdot \vec{u}_\theta)$$

champ magnétique créé par un dipôle  $\vec{m}$

$$\vec{B} = \begin{cases} \frac{\mu_0 m}{4\pi r^3} \cdot 2 \cos \theta \\ \frac{\mu_0 m}{4\pi r^3} \cdot \sin \theta \\ 0 \end{cases}$$

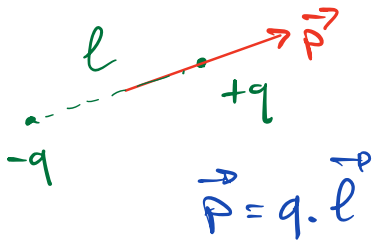
$$\vec{E} = \begin{cases} \frac{P}{4\pi \epsilon_0} \cdot 2 \cos \theta \\ \frac{P}{4\pi \epsilon_0} \cdot \sin \theta \\ 0 \end{cases}$$

$$\frac{\mu_0 m}{4\pi} \leftrightarrow \frac{P}{4\pi \epsilon_0}$$

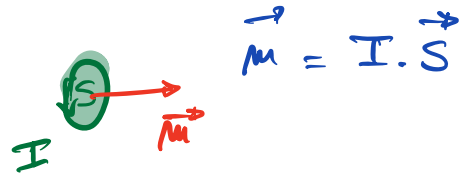
principe de correspondance

$$\begin{aligned} \vec{m} &\leftrightarrow \vec{p} \\ \frac{\mu_0}{4\pi} &\leftrightarrow \frac{1}{4\pi \epsilon_0} \\ \vec{B} &\leftrightarrow \vec{E} \end{aligned}$$

Dipôle électrique



Dipôle magnétique



approx. dipolaire

$$\vec{E} = \begin{pmatrix} \frac{p \cdot 2 \cos \theta}{4\pi \epsilon_0 r^3} \\ \frac{p \cdot \sin \theta}{4\pi \epsilon_0 r^3} \\ 0 \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} \frac{\mu_0 m \cdot 2 \cos \theta}{4\pi r^3} \\ \frac{\mu_0 m \cdot \sin \theta}{4\pi r^3} \\ 0 \end{pmatrix}$$

$\vec{p}$  dans un champ  $\vec{E}$

$$\vec{F} = -(\vec{p} \cdot \text{grad}) \cdot \vec{E}$$

$\vec{m}$  dans un champ  $\vec{B}$

$$\vec{F} = -(\vec{m} \cdot \text{grad}) \cdot \vec{B}$$

Energie potentielle

$$E_p = -\vec{p} \cdot \vec{E}$$

$E_p$

$$E_p = -\vec{m} \cdot \vec{B}$$

Couple

$$\vec{\Gamma} = \vec{p} \times \vec{E}$$

couple

$$\vec{\Gamma} = \vec{m} \times \vec{B}$$



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