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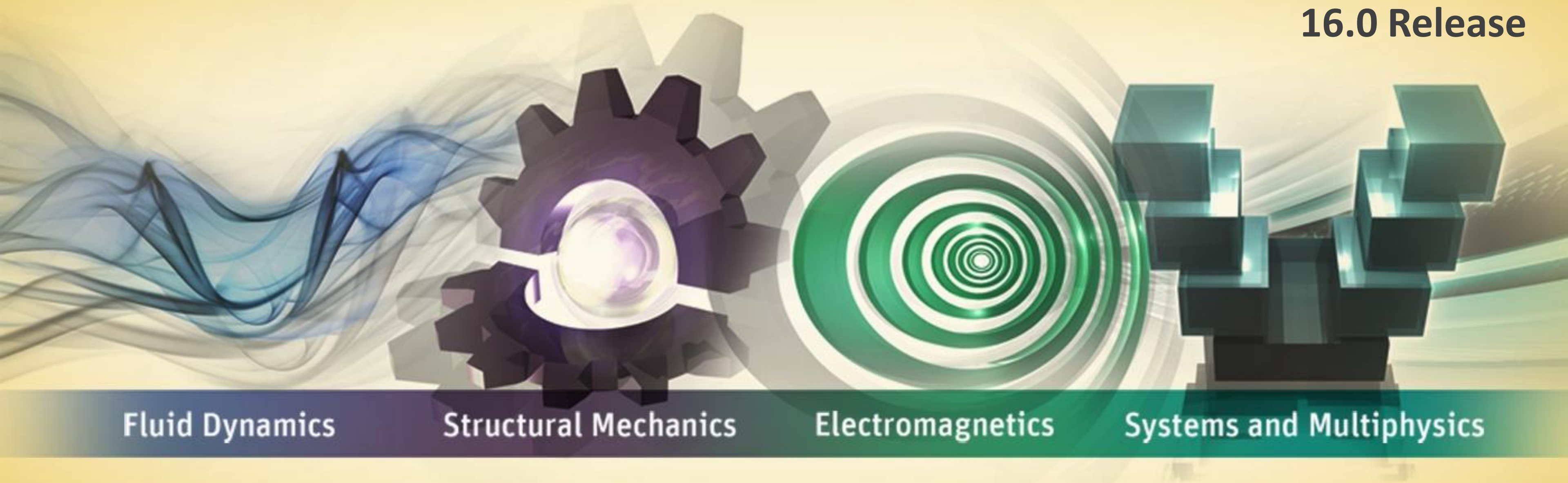
# Lecture 4:
# Rate Independent Plasticity

16.0 Release




## ANSYS Mechanical
## Introduction to Structural Nonlinearities

# Chapter Overview

The following will be covered in this Chapter:

A. Background Elasticity/Plasticity
B. Yield Criteria
C. Flow Rule
D. Hardening Rules
E. Material Data Input
F. Analysis Settings
G. Reviewing Results
H. Workshop

The capabilities described in this section are generally applicable to *ANSYS Structural* licenses and above.

# A. Metal Plasticity Overview

**Review of Elasticity:**

**Before proceeding to a discussion on plasticity, it may be useful to review elasticity of metals.**

- **In elastic response, if the induced stresses are below the material's yield strength, the material can fully recover its original shape upon unloading.**
- **From a standpoint of metals, this behavior is due to the stretching but not breaking of chemical bonds between atoms. Because elasticity is due to this stretching of atomic bonds, it is fully recoverable. Moreover, these elastic strains tend to be small.**
- **Elastic behavior of metals is most commonly described by the stress-strain relationship of Hooke's Law:**

$$\sigma = E\varepsilon$$

# ... Metal Plasticity Overview

***What is plasticity?***

**When a ductile material experiences stresses beyond the elastic limit, it will yield, acquiring large permanent deformations.**

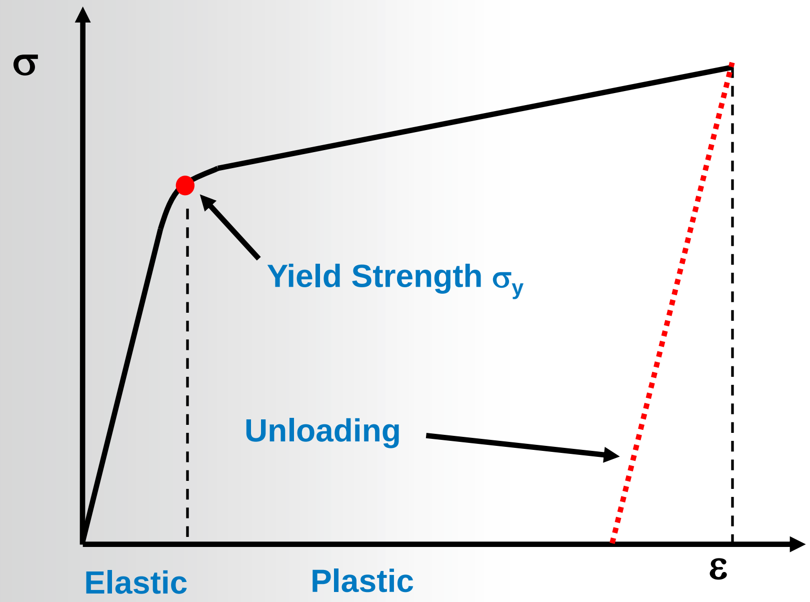
- **Plasticity refers to the material response beyond yield.**
- **Plastic response is important for metal forming operations.**
- **Plasticity is also important as an energy-absorbing mechanism for structures in service.**
  - **Materials that fail with little plastic deformation are said to be brittle.**
  - **Ductile response is safer in many respects than is brittle response.**

**This Lecture will review some basics of plasticity by defining certain terminology.**

# … Metal Plasticity Overview

**Plastic deformation results from slip between planes of atoms due to shear stresses (deviatoric stresses). This dislocation motion is essentially atoms in the crystal structure rearranging themselves to have new neighbors**

- **Results in unrecoverable strains or permanent deformation after load is removed.**
- **Slipping does not generally result in any volumetric strains (condition of incompressibility), unlike elasticity**



# … Metal Plasticity Overview

The plastic response of a structure (typically due to a multiaxial stress state) is based on the results of a uniaxial test specimen.

From the results of a uniaxial stress-strain test the following information can be derived:

- Proportional limit
- Yield point
- Strain hardening

# … Metal Plasticity Overview

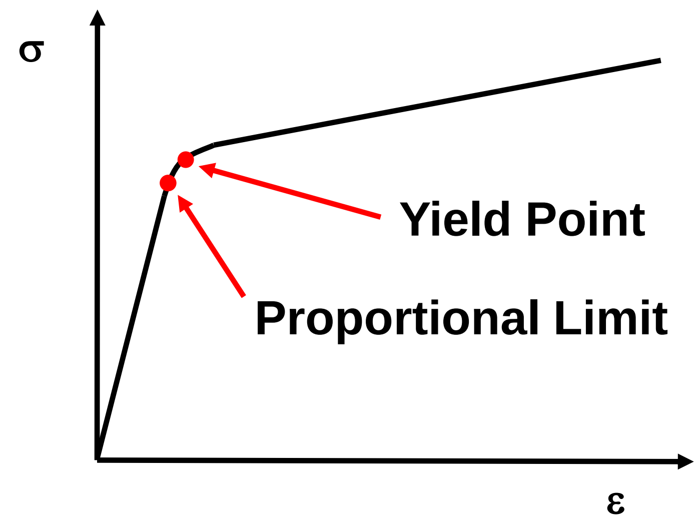
***Proportional limit and Yield point***

**Most ductile metals behave linearly below a stress level called the *proportional limit.***

- **Below the proportional limit, the stress is linearly related to the strain.**

**Additionally, below a stress level or *yield point,* the stress-strain response is elastic.**

- **Below the yield point, any straining that occurs is completely recoverable upon removal of the load.**

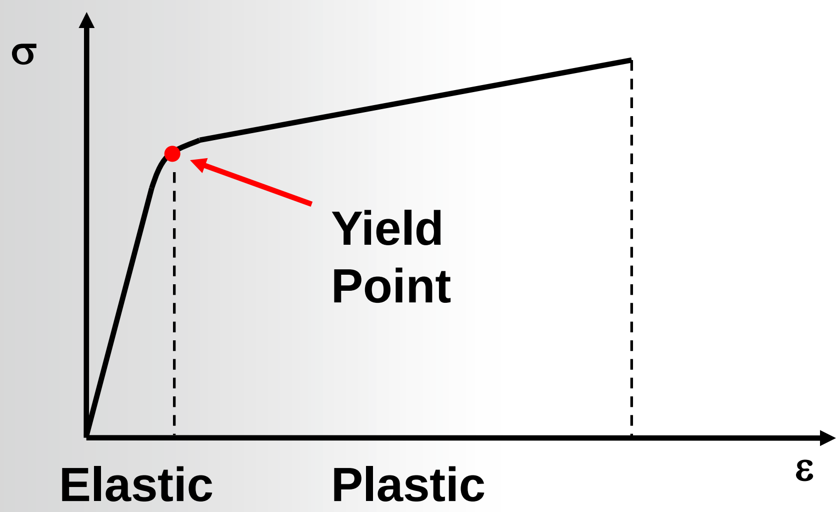


# ... Metal Plasticity Overview

***... proportional limit and yield point:***

**Because there is usually little difference between the yield point and the proportional limit, the ANSYS program always assumes them to be the same.**

- **The portion of the stress-strain curve below the yield point is called the elastic portion, and the portion above the yield point is the plastic portion.**



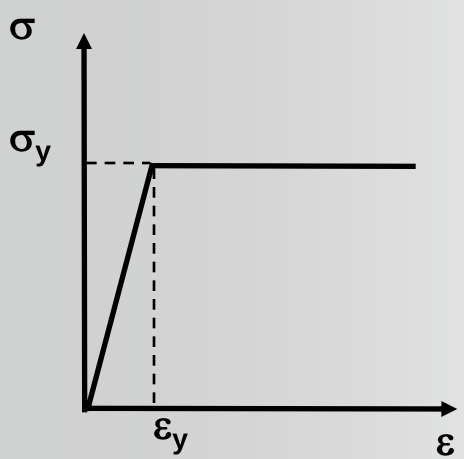

  

# … Metal Plasticity Overview

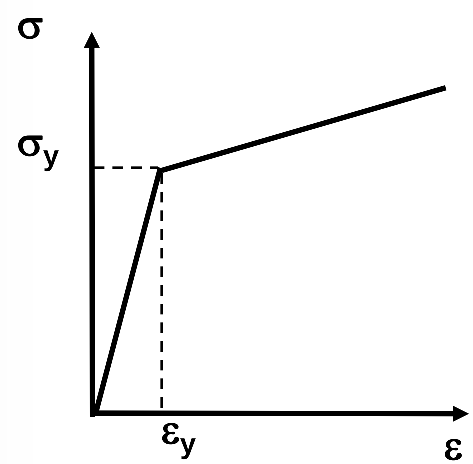
***Strain hardening***

**Post-yield behavior is typically characterized as being either *elastic – perfectly plastic* or *strain hardening* behavior.**

- ***Strain hardening*** **is a material response in which the yield stress increases with increasing strain beyond the initial yield point.**




**Elastic - Perfectly Plastic**




**Strain Hardening**

**Uniaxial Stress-Strain Curves**

# … Metal Plasticity Overview

**Rate-Independent Plasticity:**

- **If the material response is not dependent on the rate of loading or deformation, the material is said to be *rate-independent*.**
- **Most metals exhibit rate-independent behavior at low temperatures (< 1/4 or 1/3 melting temperature) and low strain rates.**

# … Metal Plasticity Overview

*Incremental plasticity theory* **provides a mathematical relationship that characterizes the increments of stress and strain ($\Delta\sigma$ and $\Delta\varepsilon$) to represent the material behavior in the plastic range.**

**There are three basic components to incremental plasticity theory:**

- **Yield criterion.**
- **Flow rule.**
- **Hardening rule.**

# B. Yield Criterion

**Yield Criterion:**

**The yield criteria is used to relate multiaxial stress state with the uniaxial case.**

- **Tensile testing on specimens provide uniaxial data, which can easily be plotted on one-dimensional stress-strain curves, such as those presented earlier in this section.**

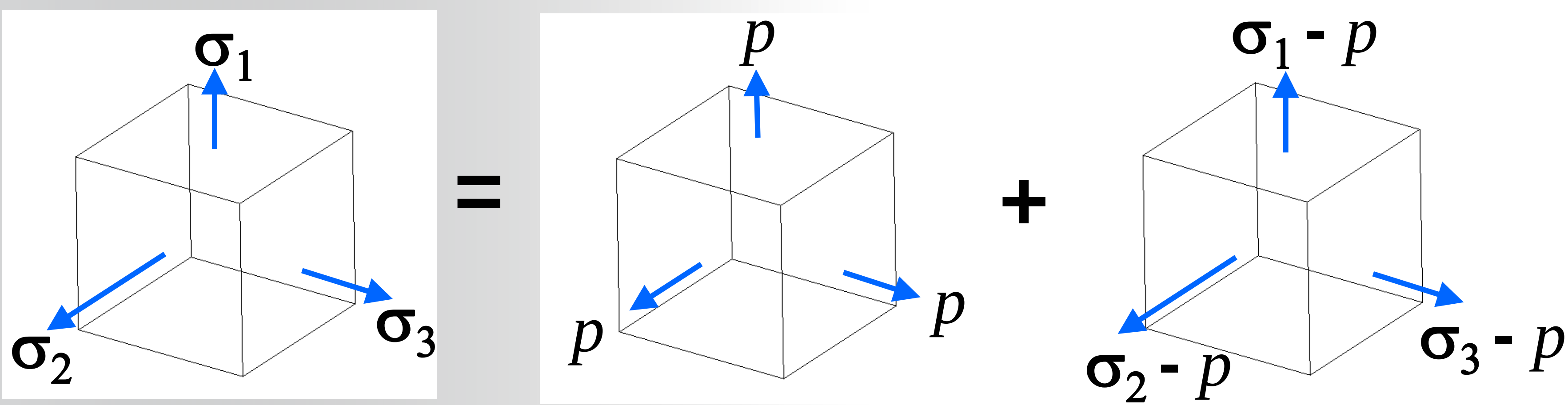
- **The actual structure usually exhibits multiaxial stress state. The yield criterion provides a scalar invariant measure of the stress state of the material which can be compared with the uniaxial case.**

## ... Yield Criterion

**In general, a stress state can be separated into two components.**

- **Hydrostatic stress - generates volume change.**
- **Deviatoric stress - generates angular distortion.**




Stress State
(Where: $\sigma_1 \neq \sigma_2 \neq \sigma_3$)

Hydrostatic stress $(p)$ causing volume change only

Deviatoric stress causing angular distortion only

# … Yield Criterion

The von Mises yield criterion predicts that yielding will occur whenever the distortion energy in a unit volume equals the distortion energy in the same volume when uniaxially stressed to the yield strength.

- From this theory, a scalar invariant (von Mises equivalent stress) is derived as:

$$\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

- When von Mises equivalent stress exceeds the uniaxial material yield strength, general yielding will occur.

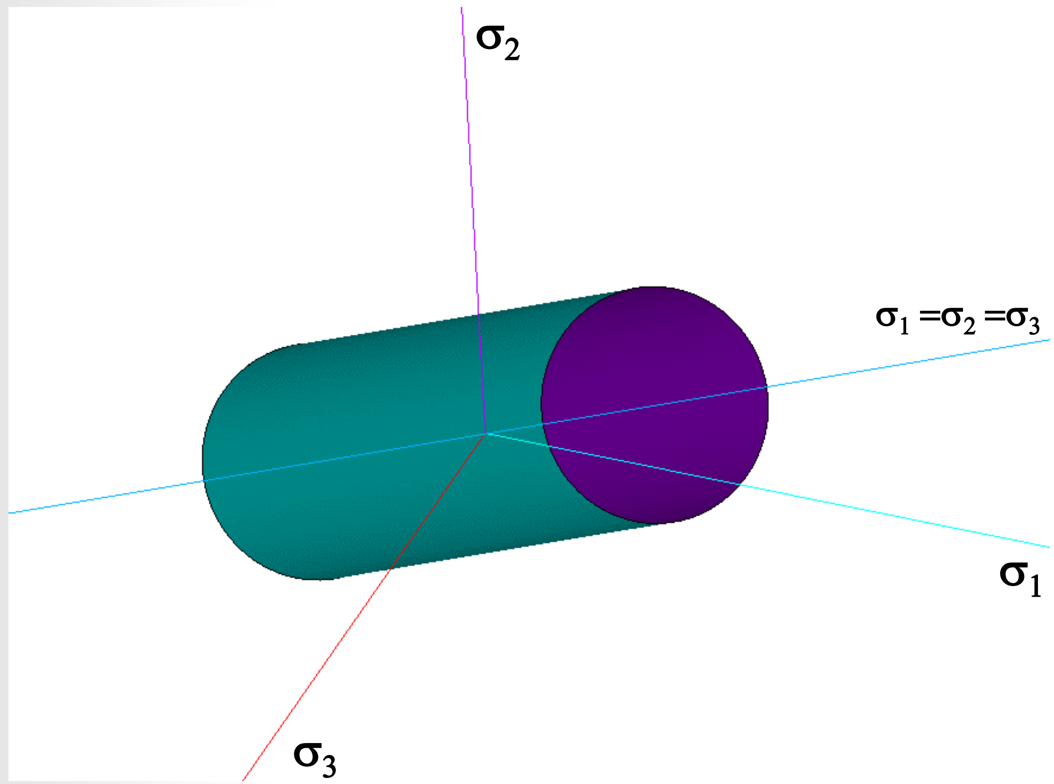
# ... Yield Criterion

**If plotted in 3D principal stress space, the von Mises yield surface is a cylinder.**

The cylinder is aligned with the axis $\sigma_1=\sigma_2=\sigma_3$.

Note that if the stress state is inside the cylinder, no yielding occurs. This means that if the material is under hydrostatic pressure ($\sigma_1=\sigma_2=\sigma_3$), no amount of hydrostatic pressure will cause yielding.

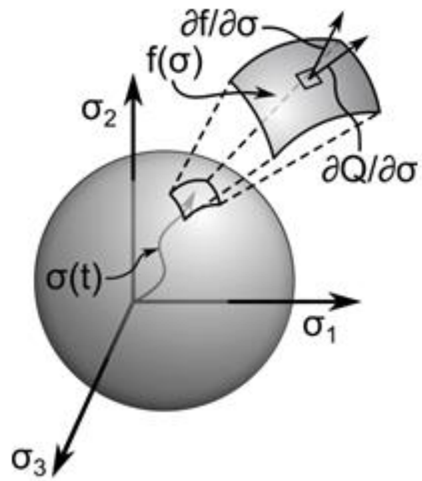
Another way to view this is that stresses which deviate from the axis ($\sigma_1=\sigma_2=\sigma_3$) contribute to the von Mises stress calculation.



## C. Flow Rules

**The evolution of plastic strain is determined by the flow rule:**

$$d\varepsilon^{pl} = d\lambda \frac{\delta Q}{\delta \sigma}$$




**Where:** ***$d\lambda$*** **is magnitude of plastic strain increment**

***Q*** **is the plastic potential**

**The flow rule defines how the individual plastic strain components ($\varepsilon_x^{pl}$, $\varepsilon_y^{pl}$, etc.) develop with yielding.**

# ... Flow Rules

**Flow rules which are derived from the yield criterion typically imply that the plastic strains develop in a direction *normal* to the yield surface.**

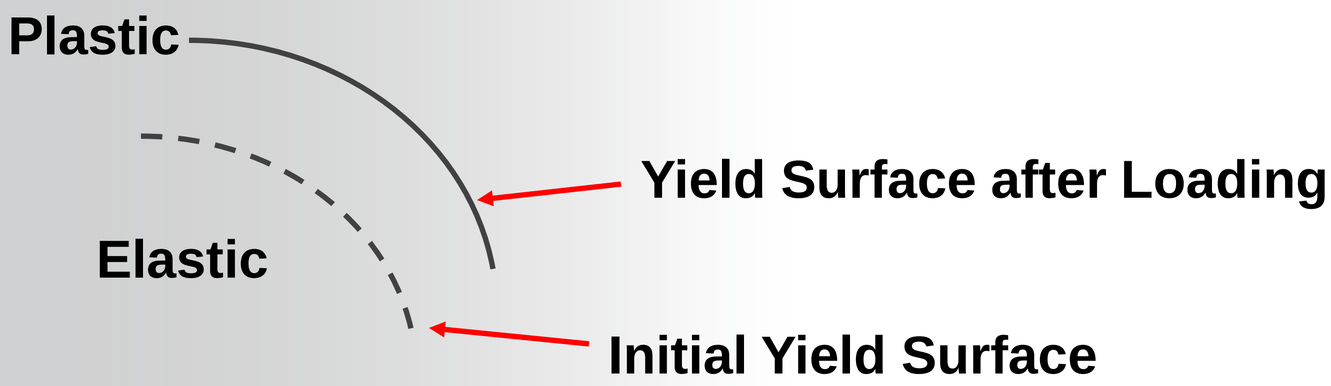
- **Such a flow rule is termed *associative*.**
- **The metal plasticity models exposed in WB-Engineering Data all have associative flow rules**

**Flow rules which are derived from a function that is different from the yield criteria are called *non-associative.***

- **Some advanced plasticity models not directly exposed in WB-Engineering Data (I.e. Drucker-Prager, Cast Iron) have non-associative flow rules.**

# C. Hardening Rules

- The hardening rule describes how the yield surface changes (size, center,shape) as the result of plastic deformation.
- The hardening rule determines when the material will yield again if the loading is continued or reversed.
- This is in contrast to elastic-perfectly-plastic materials which exhibit no hardening -- i.e., the yield surface remains fixed.



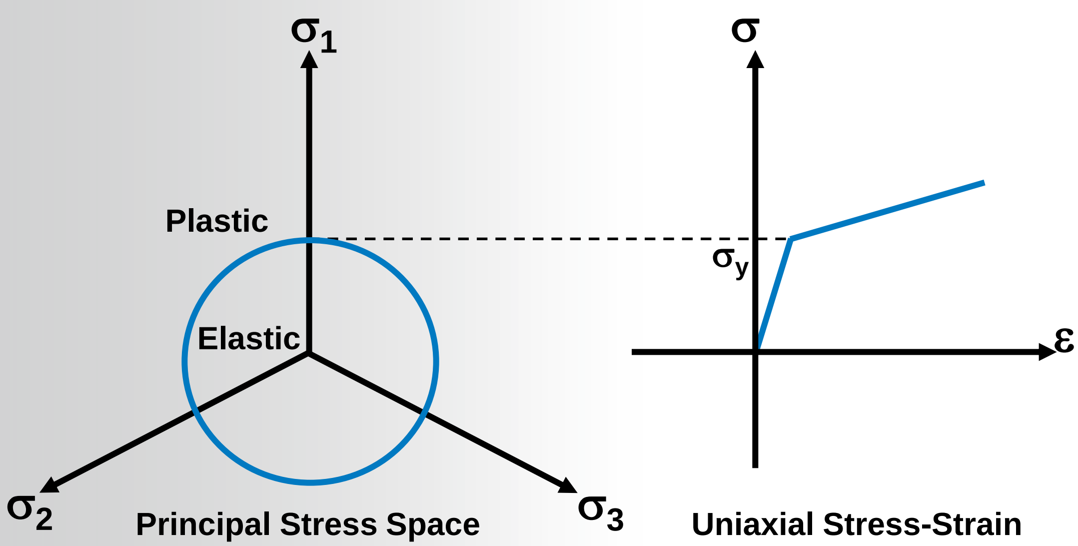

  

# … Hardening Rules

**Recall, at the edge of the cylinder (circle), yielding will occur.**

**No stress state can exist *outside* of the cylinder.**

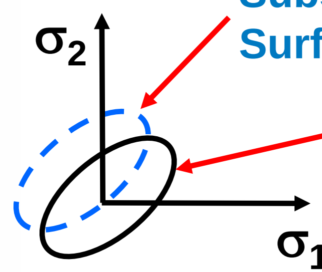
**Instead, hardening rules will describe how the cylinder changes with respect to yielding.**



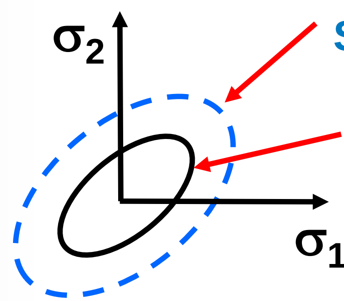
# … Hardening Rules

- There are two basic hardening rules to prescribe the modification of the yield surface:

  – *Kinematic* hardening.
    - The yield surface remains constant in size and translates in the direction of yielding.



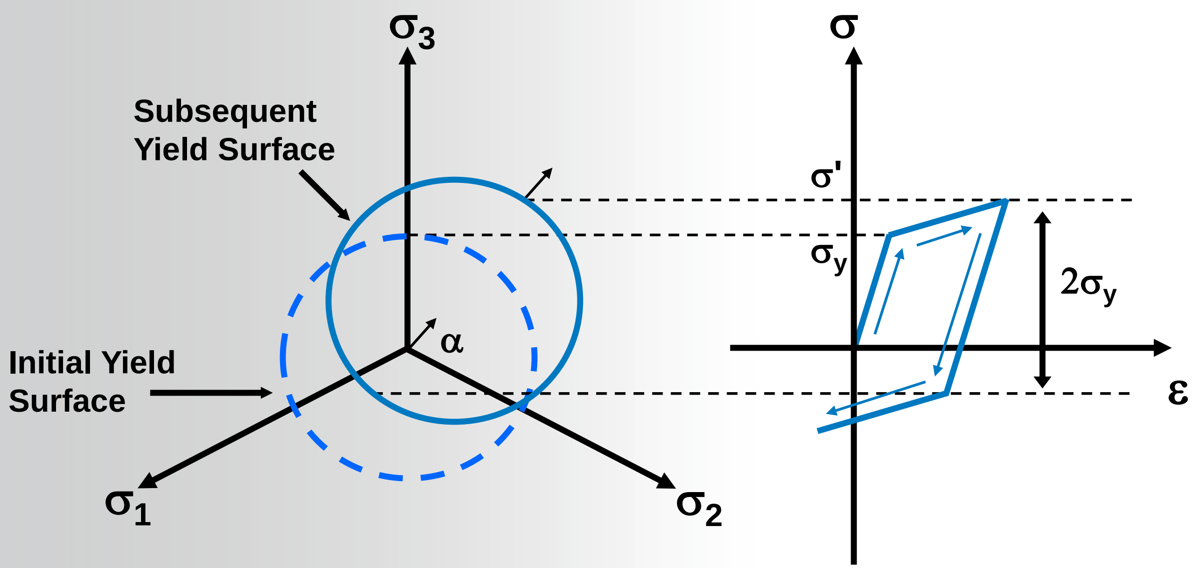

  – *Isotropic* hardening.
    - The yield surface expands uniformly in all directions with plastic flow.




- Most metals exhibit kinematic hardening behavior for small strain cyclic loading.

# … Kinematic Hardening

- The stress-strain behavior for linear kinematic hardening is illustrated below:

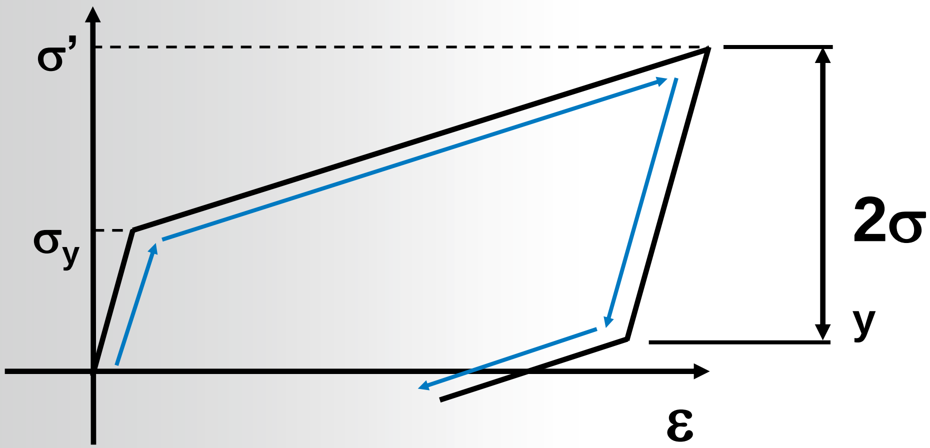



- Subsequent yield in compression is decreased by the amount that the yield stress in tension increased, so that a $2\sigma_y$ difference between the yields is always maintained. (This is known as the *Bauschinger effect.*)

# … Kinematic Hardening

- An initially isotropic material is no longer isotropic after it yields and experiences kinematic hardening.
- For very large strain simulations, the linear kinematic hardening model can become inappropriate because of the Bauschinger effect.

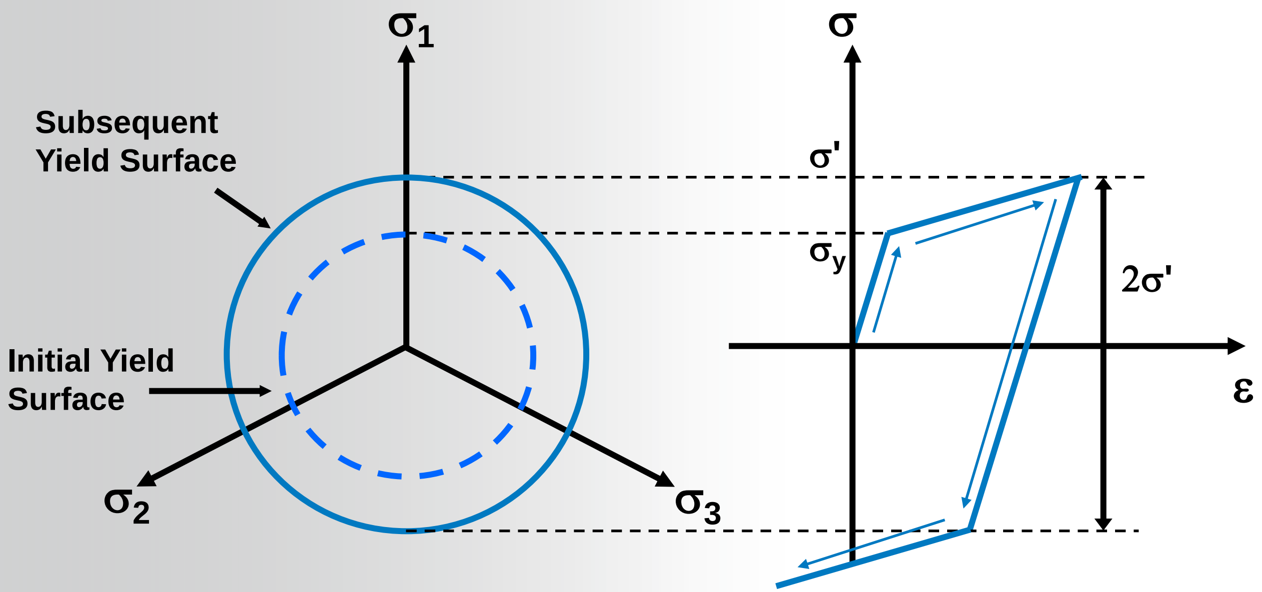



- Kinematic hardening is generally used for small strain, cyclic loading applications.

# … Isotropic Hardening

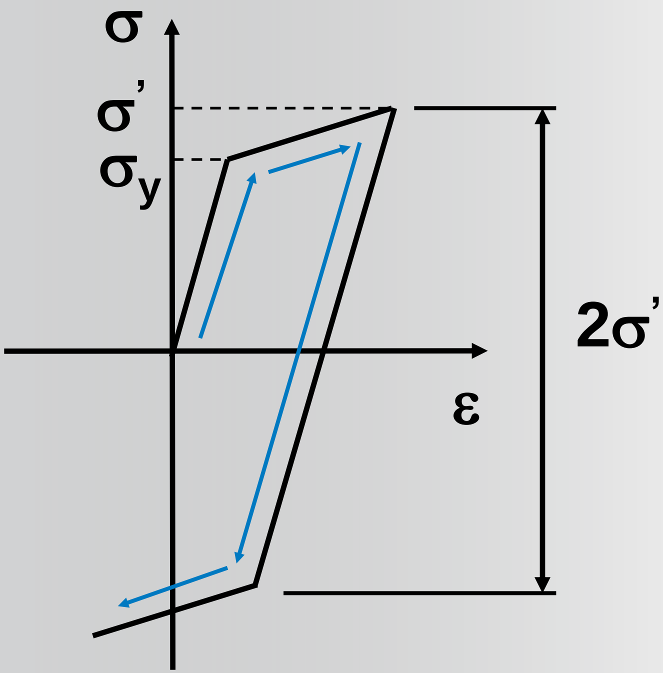
- Isotropic hardening states that the yield surface expands uniformly during plastic flow. The term 'isotropic' refers to the uniform dilatation of the yield surface and is different from an 'isotropic' yield criterion (i.e., material orientation).



# … Isotropic Hardening

- **Plotting the stress-strain curve enables an understanding of what occurs during a loading and reverse loading cycle:**




**Note that the subsequent yield in compression is equal to the highest stress attained during the tensile phase.**

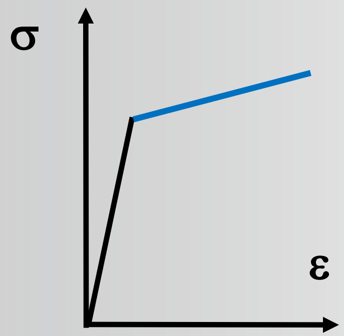
**Isotropic hardening is often used for large strain or proportional loading simulations. It is usually not applicable for cyclic loading.**

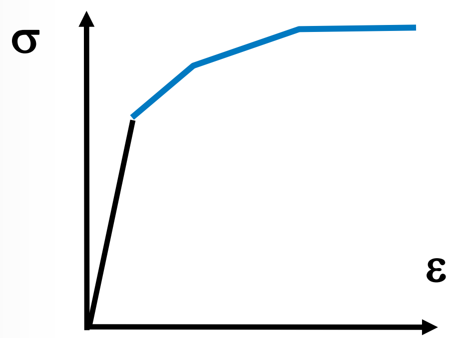
# D. Material Data Input

***Curve shapes***

**Two different type of stress-strain curve representations are possible:**




Bilinear



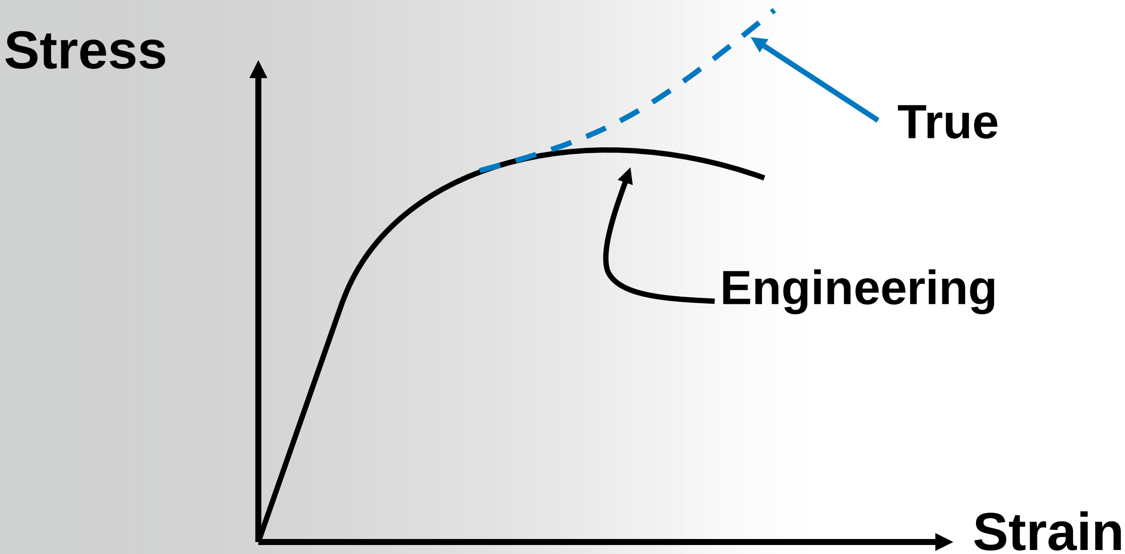

Multilinear

# ... Material Data Input

## Engineering vs. True Stress-Strain:

- While engineering stress-strain can be used for small-strain analyses, true stress-strain must be used for plasticity, as they are more representative measures of the state of the material.



# ... Material Data Input

**Engineering vs. True Stress-Strain (cont'd):**

- If presented with engineering stress-strain data, one can convert these values to true stress-strain with the following approximations:
  - Up until twice the strain at which yielding occurs:

$$\sigma = \sigma_{eng} \qquad \varepsilon = \varepsilon_{eng}$$

- Up until the point at which necking occurs:

$$\sigma = \sigma_{eng}\left(1+\varepsilon_{eng}\right) \qquad \varepsilon = \ln\left(1+\varepsilon_{eng}\right)$$

- Note that, only for stress conversion, the following is assumed:
  - Material is incompressible (acceptable approximation for large strains)
  - Stress distribution across cross-section of specimen is assumed to be uniform.
- Beyond necking:
  - There is no conversion equation relating engineering to true stress-strain at necking. The instantaneous cross-section must be measured.

# ... Material Data Input

**Linear elastic material properties must be supplied**

- **The same requirements exist for linear static structural analyses, namely that Young's Modulus and Poisson's Ratio must be defined as a minimum.**

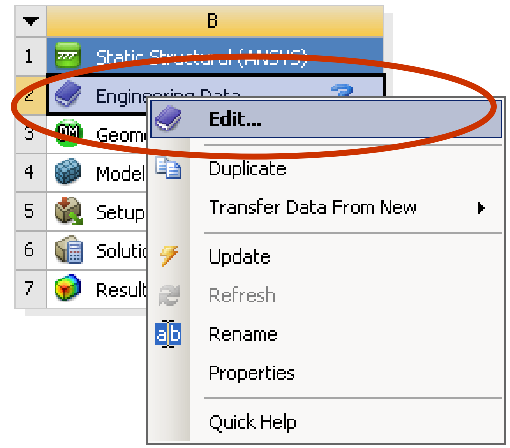
**Metal plasticity is available as a nonlinear material model.**

- **Note that only *ANSYS Professional* NLS licenses and above support nonlinear material laws.**

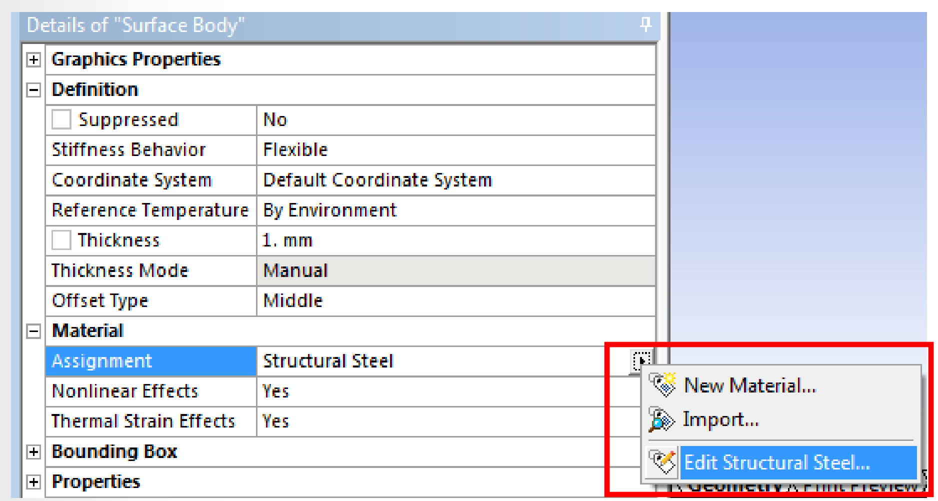
# ... Material Data Input

There are two ways to introduce metal plasticity:

- From the project schematic, highlight the Engineering Data branch, double click or RMB and click on Edit…



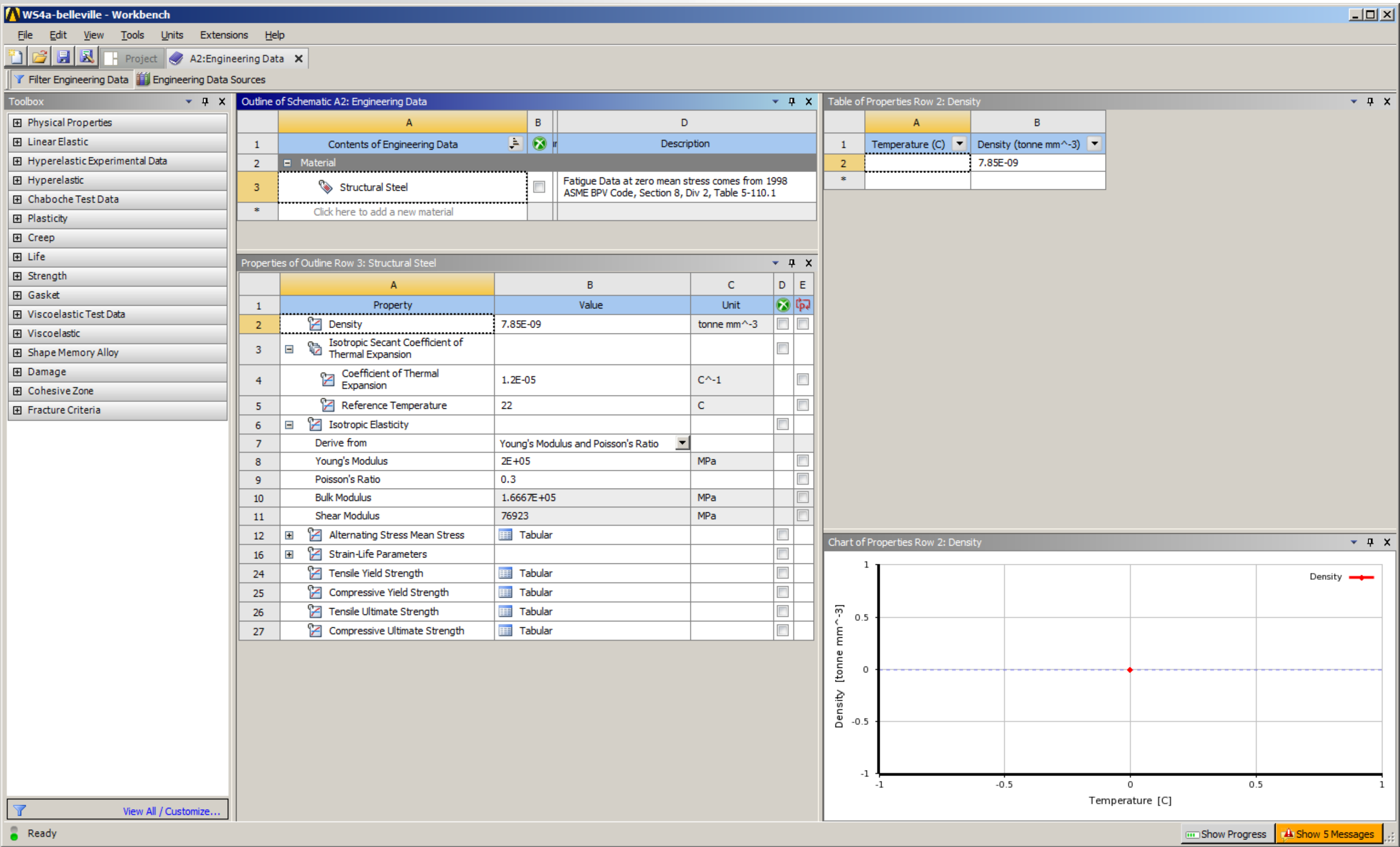

- From the Mechanical GUI, within the Details Window of the body to be modified, highlight the current material assignment and RMB to choose one of three options:
  - New Material …
  - Import …
  - Edit …



# ... Material Data Input

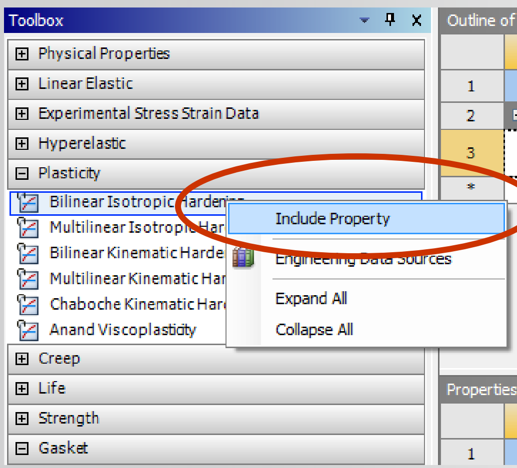
**This opens the Engineering Data dialogue box for adding and editing various material properties related to the active project(s).**

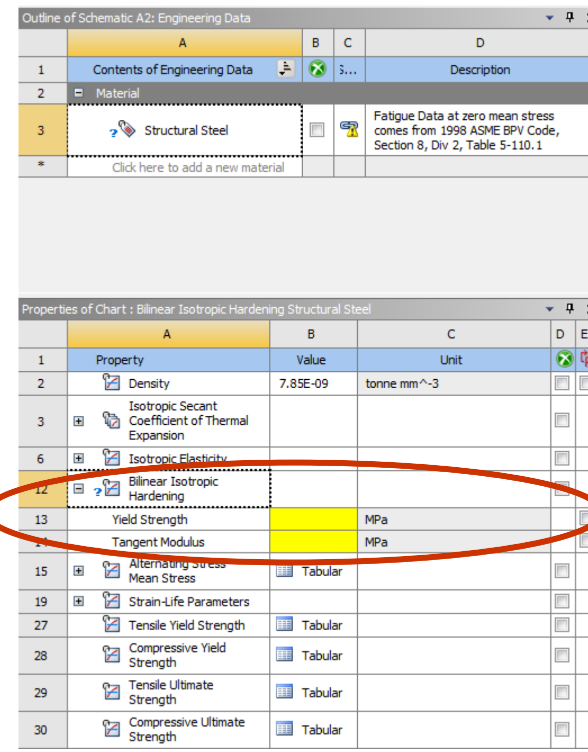


# ... Material Data Input

**From the Toolbox, open the plasticity folder:**

- **Highlight the metal plasticity model of interests (in the example below, Bilinear Isotropic is selected)**
- **RMB on the material model and click on "Include Property"**




- **The Bilinear Isotropic Hardening model will then appear in the Properties Dialogue box.**
- **The yellow blank boxes are now available for user to define yield strength and tangent modulus.**

# ... Material Data Input

**After defining the yield strength and tangent modulus, the data will automatically be plotted graphically for inspection:**

Outline of Schematic A2: Engineering Data

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Contents of Engineering Data | | S... | Description |
| 2 | Material | | | |
| 3 | Structural Steel | | | Fatigue Data at zero mean stress comes from 1998 ASME BPV Code, Section 8, Div 2, Table 5-110.1 |

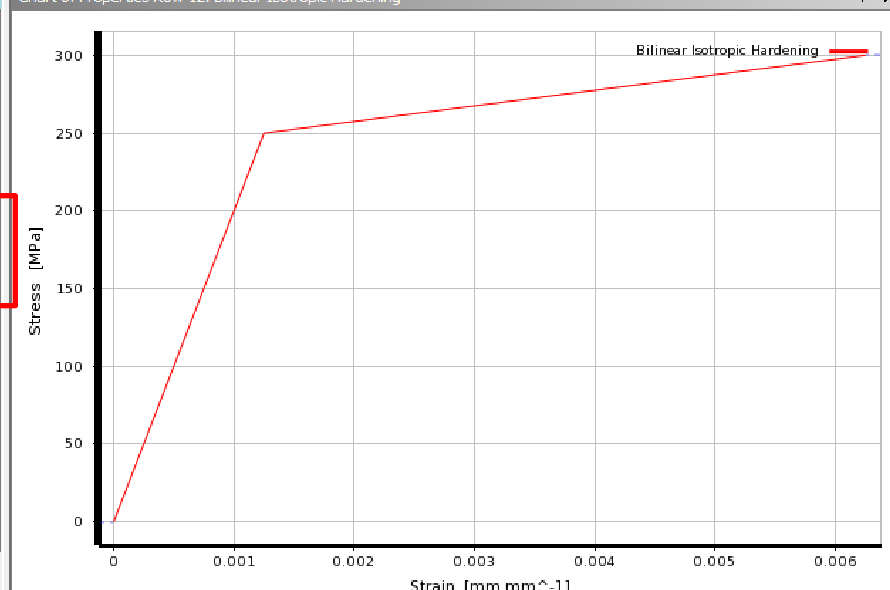
Table of Properties Row 12: Bilinear Isotropic Hardening

| | A | B | C |
|---|---|---|---|
| 1 | Temperature (C) | Yield Strength (MPa) | Tangent Modulus (MPa) |
| 2 | | 250 | 10000 |
| * | | | |

Properties of Chart : Bilinear Isotropic Hardening Structural Steel

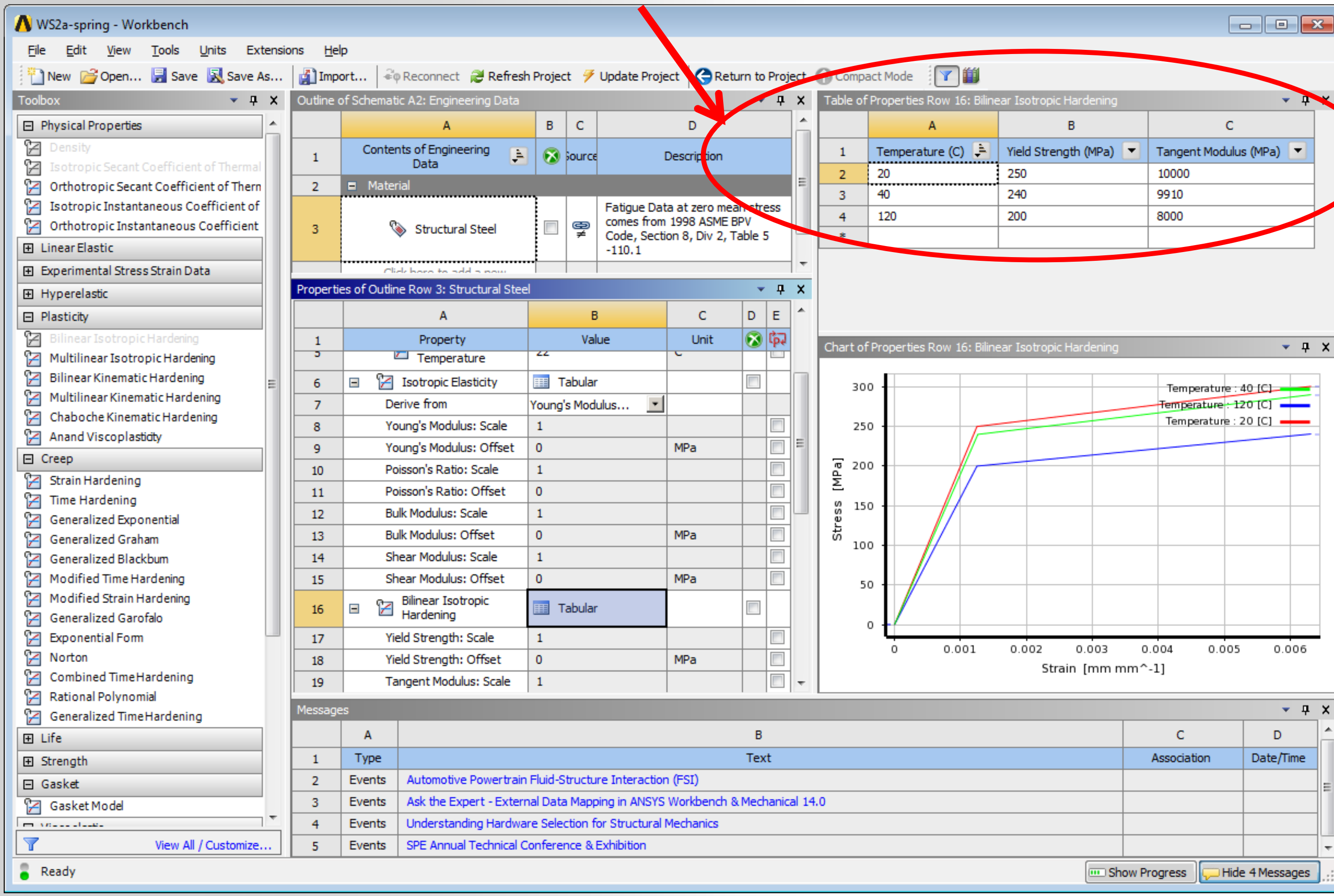
| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Property | Value | Unit | | |
| 2 | Density | 7.85E-09 | tonne mm^-3 | | |
| 3 | Isotropic Secant Coefficient of Thermal Expansion | | | | |
| 6 | Isotropic Elasticity | | | | |
| 12 | Bilinear Isotropic Hardening | | | | |
| 13 | Yield Strength | 250 | MPa | | |
| 14 | Tangent Modulus | 10000 | MPa | | |
| 15 | Alternating Stress Mean Stress | Tabular | | | |
| 19 | Strain-Life Parameters | | | | |
| 27 | Tensile Yield Strength | Tabular | | | |
| 28 | Compressive Yield Strength | Tabular | | | |
| 29 | Tensile Ultimate Strength | Tabular | | | |
| 30 | Compressive Ultimate Strength | Tabular | | | |

Chart of Properties Row 12: Bilinear Isotropic Hardening



# ... Material Data Input

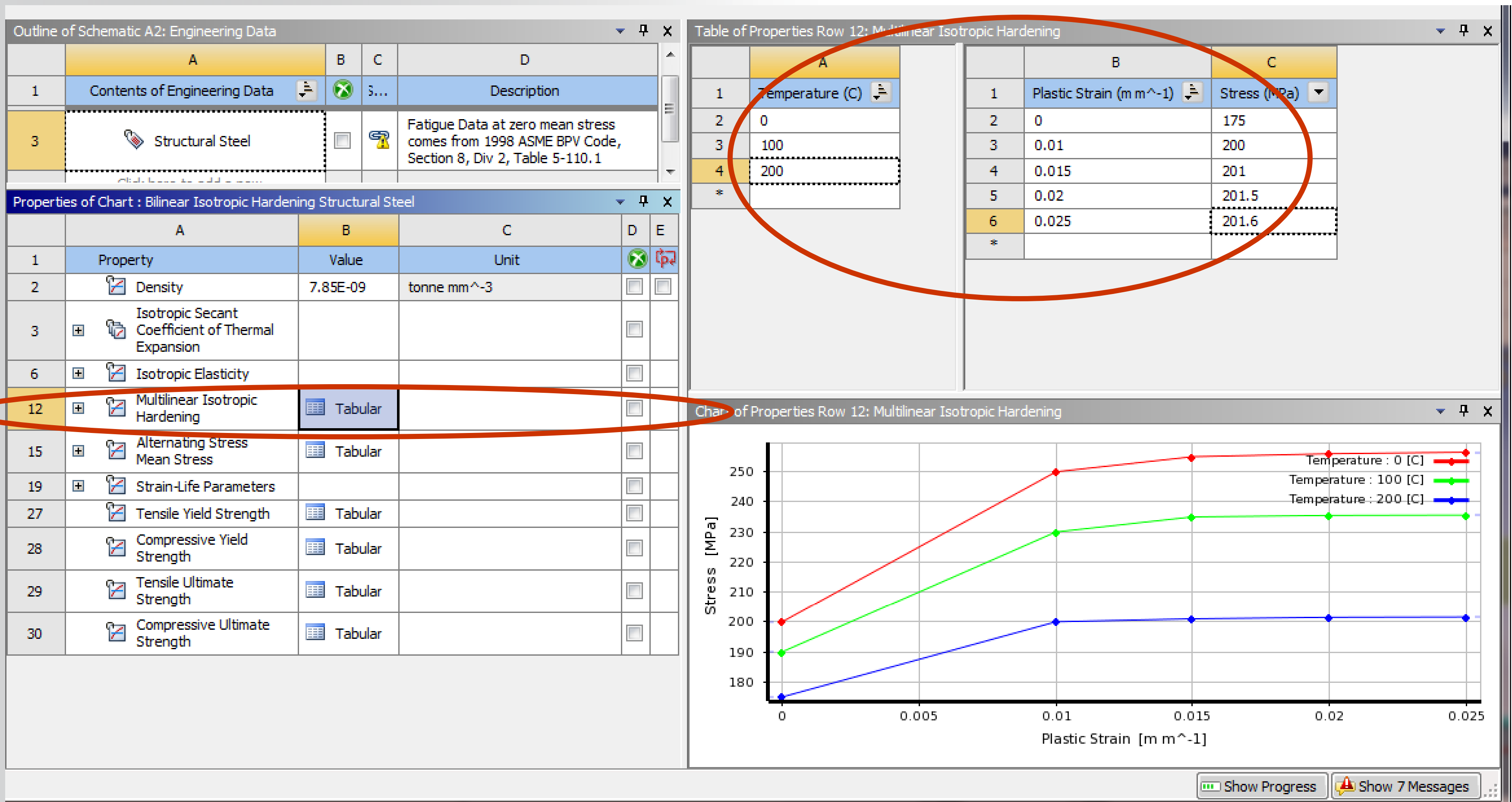
**Bilinear isotropic or kinematic hardening models also support temperature dependent properties via Tabular input.**



# … Material Data Input

**In a similar procedure, multilinear isotropic or kinematic hardening models can also be defined and verified:**



# E. Analysis Settings for Metal Plasticity

- Ensure that the substep size is adequate to capture the path dependent response accurately with minimal bisections.
- Solver will trigger a bisection automatically for plastic strains exceeding 15% in a substep
- Refer to CUTCONTROL command doc.
- Large Deflection = ON is recommended
- For large models with long run times and potential convergence trouble, consider setting up a Restart Control strategy in the event that adjustment to time step range or convergence criteria is necessary

Details of "Analysis Settings"

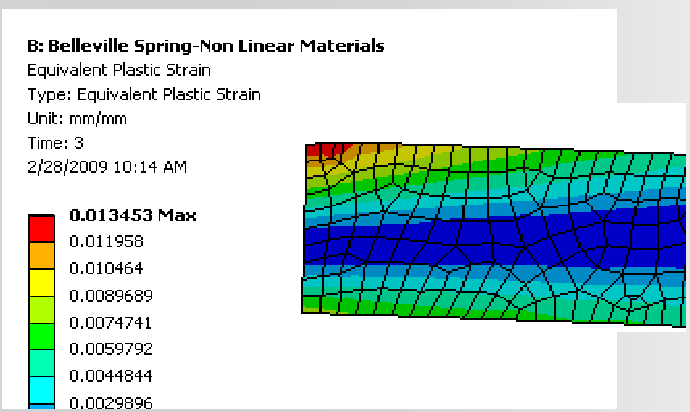
| | |
|---|---|
| **Step Controls** | |
| Number Of Steps | 1. |
| Current Step Number | 1. |
| Step End Time | 1. s |
| Auto Time Stepping | On |
| Define By | Substeps |
| Initial Substeps | 25. |
| Minimum Substeps | 1. |
| Maximum Substeps | 100. |
| **Solver Controls** | |
| Solver Type | Program Controlled |
| Weak Springs | Program Controlled |
| Large Deflection | On |
| Inertia Relief | Off |
| **Restart Controls** | |
| Generate Restart Points | Manual |
| Load Step | All |
| Substep | Specified Recurrence Rate |
| --- Value | 5 |
| Maximum Points to Save Per Step | All |
| Retain Files After Full Solve | No |
| **Nonlinear Controls** | |
| Force Convergence | Program Controlled |
| Moment Convergence | Program Controlled |

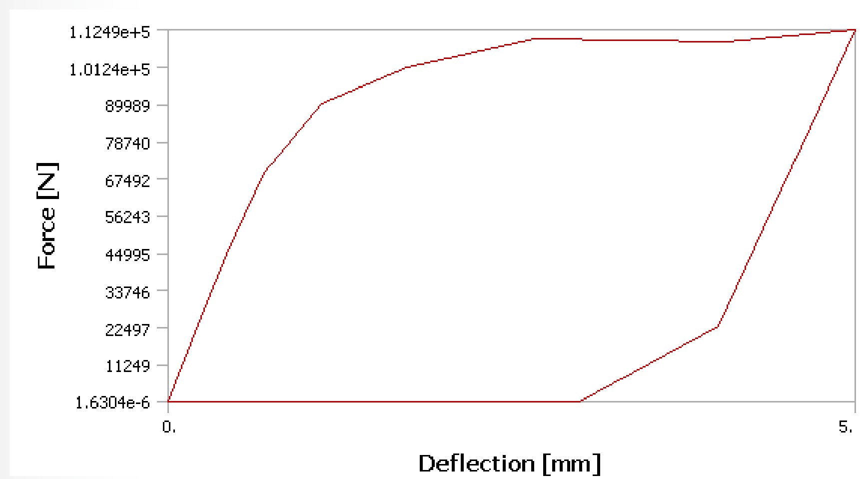
# F. Reviewing Results

- Reviewing results in a metal plasticity model is similar to a linear elastic run with the exception that there is now a path dependent plastic strain to consider.
- Review multiple results sets along the path

```
*** LOAD STEP     2   SUBSTEP     3  COMPLETED.    CUM ITER =     16
*** TIME =    1.08575          TIME INC =  0.367500E-01
*** MAX PLASTIC STRAIN STEP = 0.4841E-04   CRITERION = 0.1500
*** AUTO TIME STEP:  NEXT TIME INC = 0.55125E-01  INCREASED (FACTOR = 1.5000)
```

- Examine the nonlinear force deflection curve to better understand how the plastic strain is influencing the overall nonlinearity of the structure.



# … Summary of Plasticity in Mechanical

- **Metal plasticity deals with elastic and *inelastic* (permanent) deformation. Inelastic or plastic deformation occurs when the stress is higher than the *yield strength*. There will always be some recoverable strain (elastic strain) upon unloading.**
- **A stress-strain curve is based on *scalar* data, usually from a uniaxial test. A system may undergo a multiaxial stress state, so WB-Mechanical uses the *Mises yield criterion* to relate a multiaxial stress state with scalar test data. In this situation, *true stress vs. strain* data should be supplied.**
- **After yielding occurs, the yield point may increase due to *strain hardening*. This changes the *yield surface*, and the way in which it evolves in**
- ***Mechanical* is determined by *Isotropic or Kinematic hardening* assumption.**
- **The stress-strain curve can be represented by a *bilinear* or *multilinear* curve.**

# … Summary of Plasticity in Mechanical

| Name | TB Lab | Yield Criterion | Flow Rule | Hardening Rule | Material Response |
|---|---|---|---|---|---|
| Bilinear Isotropic Hardening | BISO | von Mises/Hill | associative | work hardening | bilinear |
| Multilinear Isotropic Hardening | MISO | von Mises/Hill | associative | work hardening | multilinear |
| Nonlinear Isotropic Hardening | NLISO | von Mises/Hill | associative | work hardening | nonlinear |
| Classical Bilinear Kinematic Hardening | BKIN | von Mises/Hill | associative (Prandtl-Reuss equations) | kinematic hardening | bilinear |
| Multilinear Kinematic Hardening | MKIN/KINH | von Mises/Hill | associative | kinematic hardening | multilinear |
| Nonlinear Kinematic Hardening | CHAB | von Mises/Hill | associative | kinematic hardening | nonlinear |
| Anisotropic | ANISO | modified von Mises | associative | work hardening | bilinear, each direction and tension and compression different |
| Drucker- Prager | DP | von Mises with dependence on hydrostatic stress | associative or non-associative | none | elastic- perfectly plastic |
| Extended Drucker-Prager | EDP | von MIses with dependence on hydrostatic stress | associative or non-associative | work hardening | multilinear |
| Cast Iron | CAST | von Mises with dependence on hydrostatic stress | non- associative | work hardening | multilinear |
| Gurson | GURS | von Mises with dependence pressure and porosity | associative | work hardening | multilinear |

**Advanced plasticity models not directly exposed in Engineering Data.**

# G. Workshop – Metal Plasticity

**Please refer to your *Workshop Supplement* for instructions on:**

- ***W4A-Metal Plasticity***