

Novel approaches to multiscale modelling in bio-based materials

Talk title: A novel analytical model for predicting the effective thermal conductivity tensor of lime-hemp concrete and hemp insulation

Anh Dung Tran Le

Laboratoire des Technologies Innovantes, EA 3899

Université de Picardie Jules Verne, Amiens France

anh.dung.tran.le@u-picardie.fr

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A novel anisotropic analytical model for effective thermal conductivity tensor of dry lime-hemp concrete with preferred spatial distributions



Anh Dung Tran-Le^{a,*}, Sy-Tuan Nguyen^b, Thierry Langlet^a

Contexte et généralités

Propriétés microscopiques

Méthode d'homogénéisation

Validation

Application

Conclusion

PLAN

Hygrothermal comfort

Indoor air quality (IAQ)

Acoustical and visual comforts

One passive way by using
hygroscopic materials
(release or adsorb water vapor
depending on the surrounding air
conditions)

Use of vegetable particles (hemp shives, flax shives, straw bales, etc.) as building material aggregates:

- *bio-based materials*
- *low embodied energy*



Novel approaches to multiscale modelling in bio-based materials: the case of lime hemp concrete



Colza

Disponibilité nationale (France)



Lin

*Disponibilité régionale
(Hauts-de-France)*

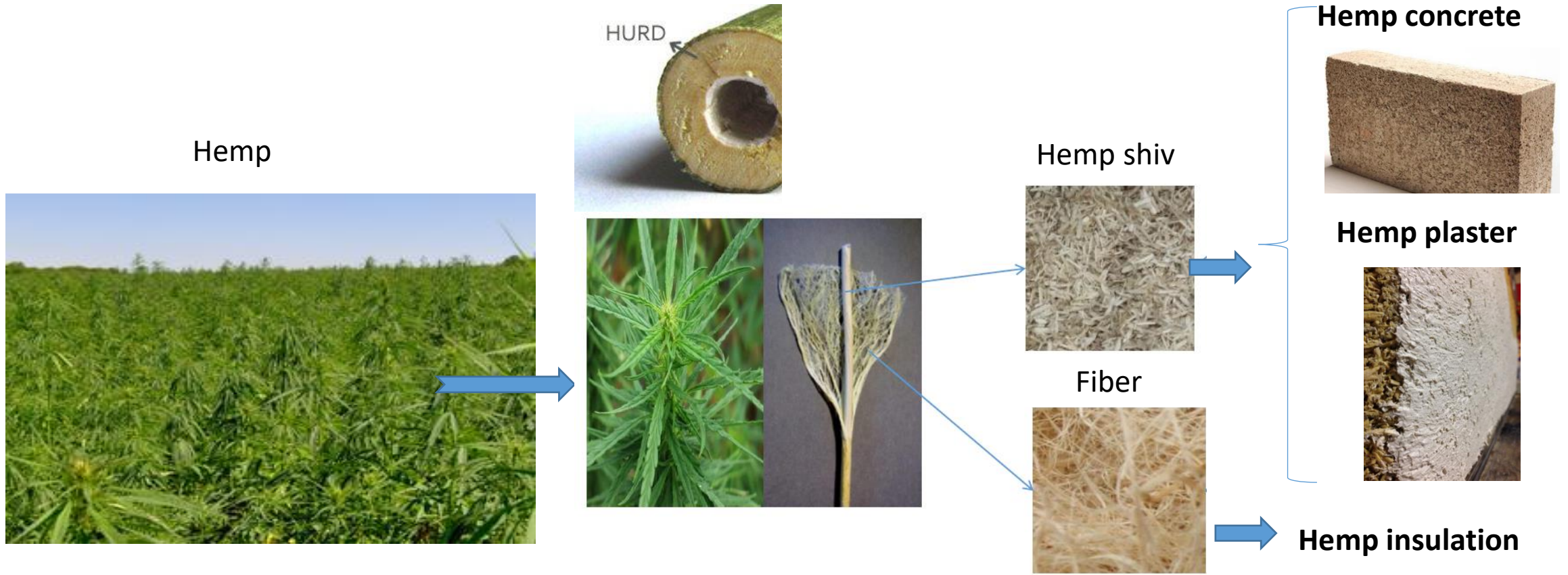
Bio-based materials



Chanvre (matériau considéré comme référence)
Disponibilité nationale (France)

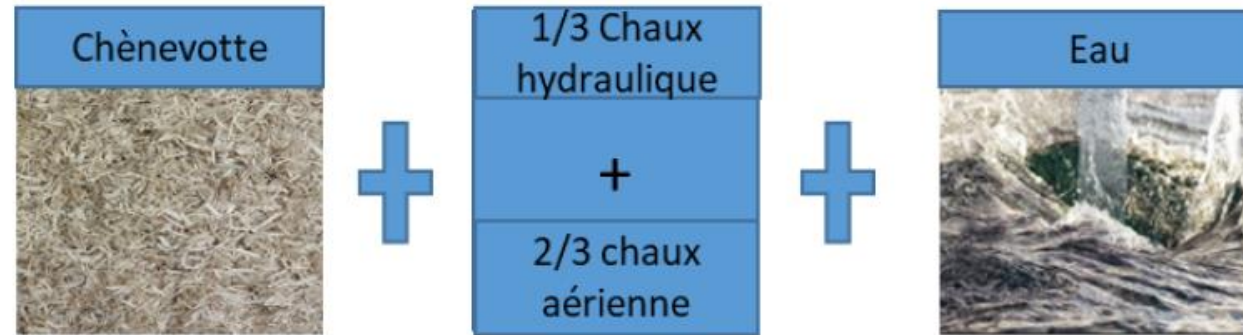
**France qui est le premier pays
producteur européen.**

Valorization of Hemp in building construction



Novel approaches to multiscale modelling in bio-based materials: the case of lime hemp concrete

Components of hemp concrete



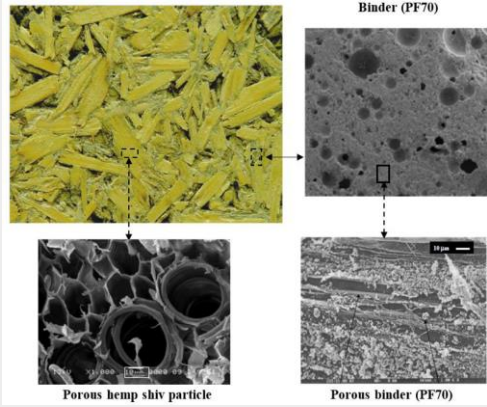
Manufacturing process of hemp concrete



Novel approaches to multiscale modelling in
bio-based materials: the case of lime hemp concrete

Démarche multi-échelle (micro au local)

Échelle microscopique



Propriétés microscopiques
des composantes

Échelle du matériau



Propriétés
Thermique
Physique
Hygrique

Échelle de la paroi



Comportement
hygrothermique

Étude expérimentale
et **validation**

Échelle du local



Simulation du comportement
hygrothermique pour prédire
la QAI, le confort et
consommation

Méthode d'homogénéisation
Et validation

Intégration des données

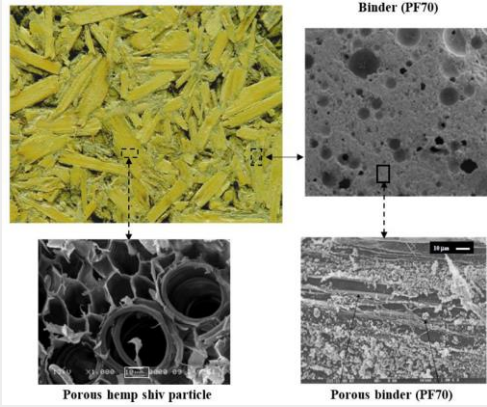
Modèle complet HAM*
paroi

Modèle complet HAM*
local

*HAM: Combined Heat, air and
moisture transport model

Démarche multi-échelle (microscopique au local)

Échelle microscopique



Propriétés microscopiques
des composantes

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Propriétés

Thermique
Physique
Hygrique

It is the « Talk title »

Méthode d'homogénéisation

Intégration des données

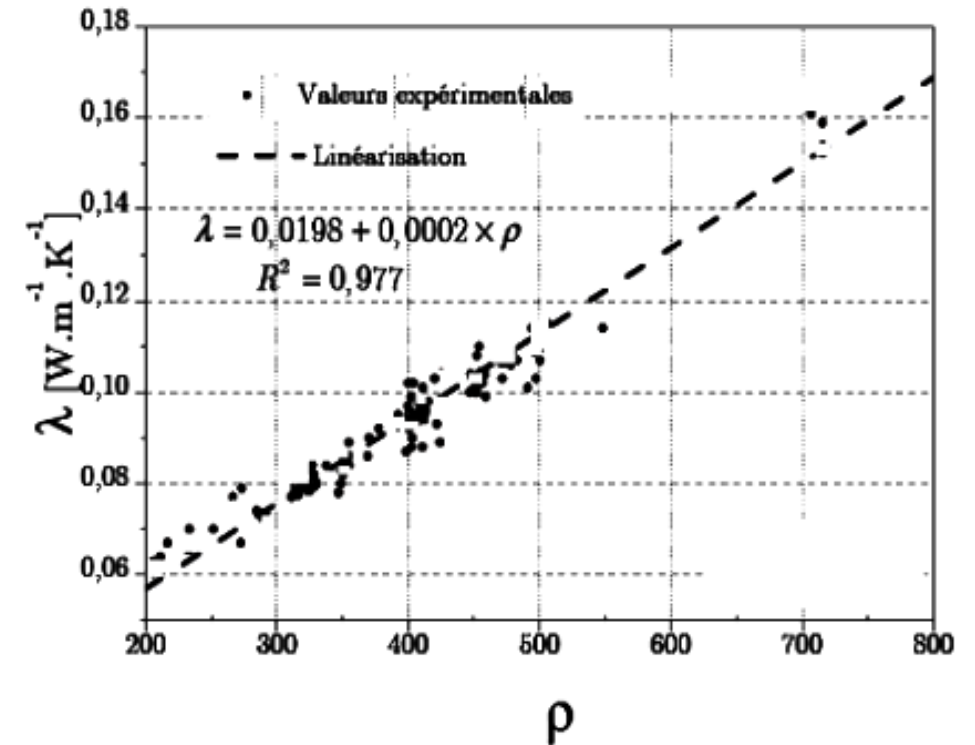
Modèle complet HAM*
paroi

Modèle complet HAM*
local

*HAM: Combined Heat, air and
moisture transport model

A novel analytical model for predicting the effective thermal conductivity tensor of lime-hemp concrete (LHC)

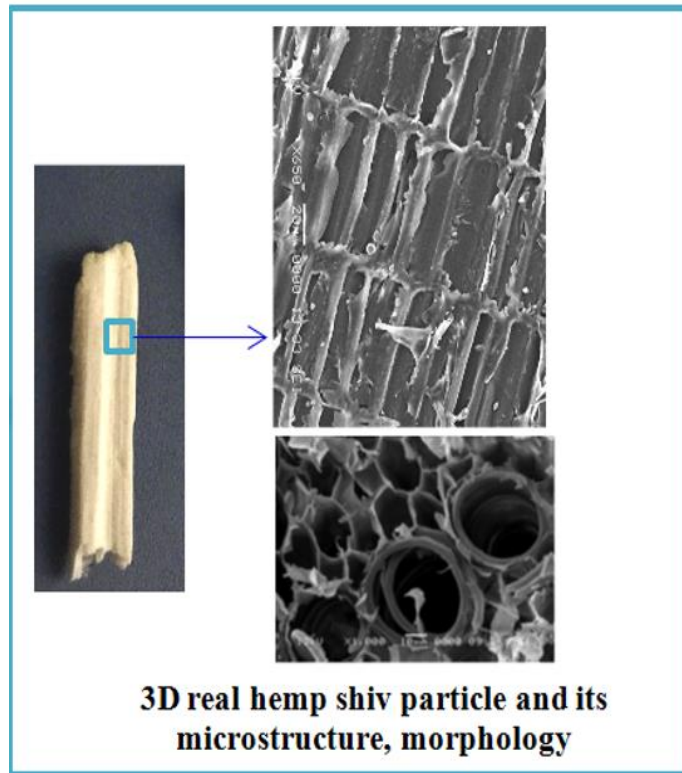
- Thermal conductivity of hemp concrete can be *directly measured in laboratory*.
- Up to date, the thermal conductivity of LHC is generally expressed *as a scalar*.
- Hemp concrete is generally either cast or sprayed, its thermal conductivity is direction dependent: *the arrangements of hemp particles are different in different directions*.



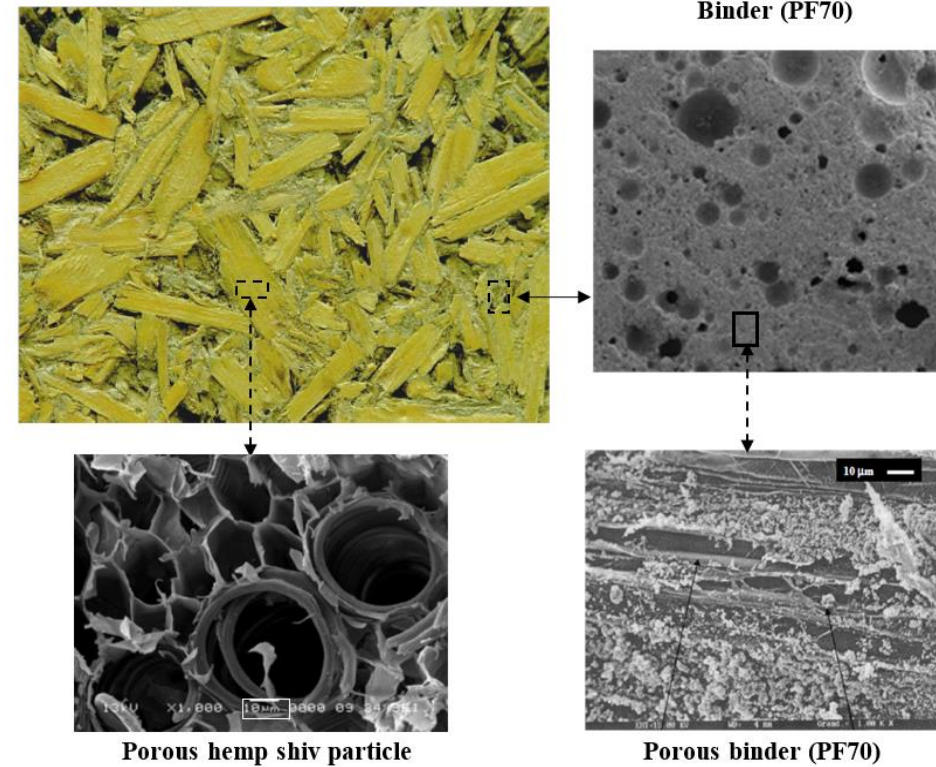
La conductivité thermique du béton de chanvre sec en fonction de la masse volumique (Cerezo, 2008).

A novel analytical model for predicting the effective
thermal conductivity tensor of lime-hemp concrete

Microscopy of hemp shiv and binder



Hemp shiv

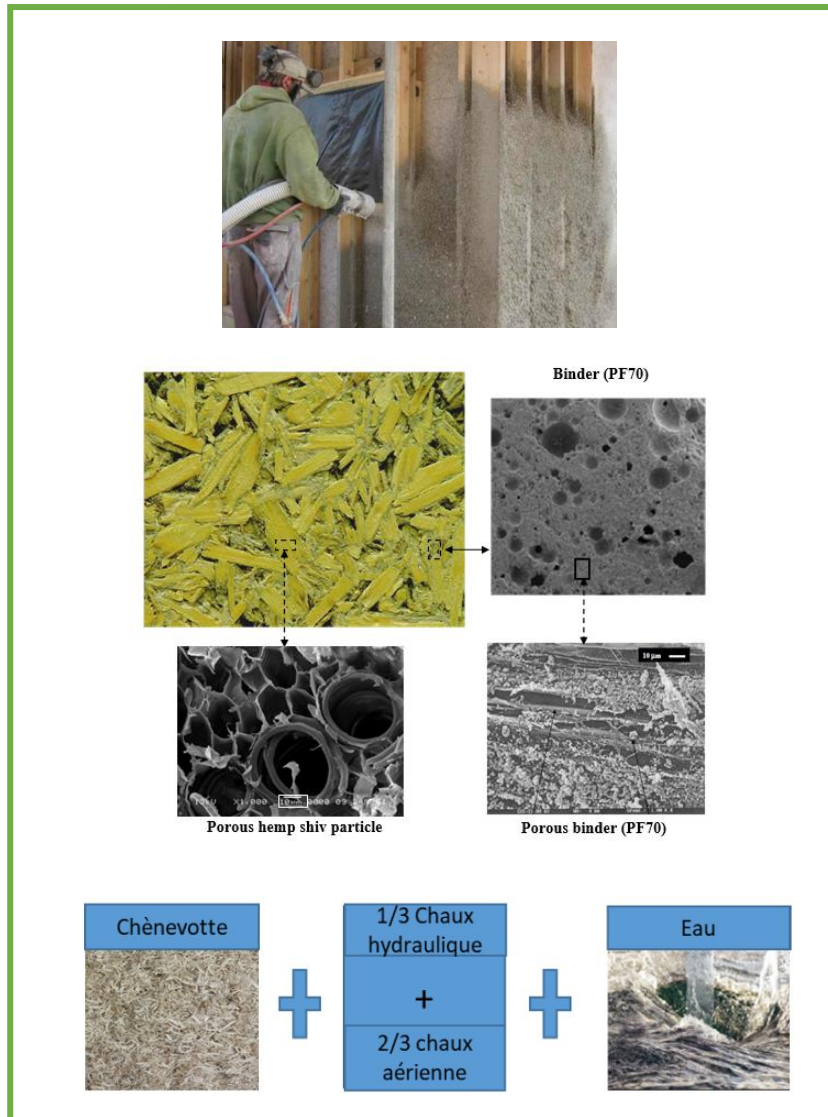


Hemp concrete

A novel analytical model for predicting the effective thermal conductivity tensor of lime-hemp concrete

Developed model takes into account:

- Hemp shiv particles
- Binder type
- Formulation of hemp concrete
- Manufacturing process
- Durability and surface treatment



Predicted thermal properties of hemp concrete



Nomenclature

| Notations | Physical meanings | Units |
|------------------------|--|-----------|
| λ | Thermal conductivity | $W/(m.K)$ |
| $\boldsymbol{\lambda}$ | Thermal conductivity tensor | $W/(m.K)$ |
| X | Aspect ratio of the inclusion | — |
| ν, Q | Anisotropic parameters of the inclusion | — |
| α | Interface parameter | — |
| f | Volume fraction of a phase | — |
| φ | Porosity | — |
| ρ | Density | kg/m^3 |
| δ | Kronecker delta | — |
| A | Second order localization tensor | — |
| P | Second order Hill's tensor | $m.K/W$ |
| n | Number of phases in a composite material | — |
| \underline{e}_3 | Unit vector in the axial direction of a spheroidal inclusion | — |
| m | Distribution parameter | — |
| W | Distribution function | — |
| Γ | Root mean square difference of the thermal conductivity | $W/(m.K)$ |

Basis formulations of the homogenization method

The overall conductivity of a heterogeneous medium can be obtained by considering the relationship between the local and the macroscopic behavior of a Representative Elementary Volume (REV)

$$\underline{q}(\underline{z}) = -\underline{\lambda}(\underline{z})\underline{\nabla}T(\underline{z}) \quad (1)$$

$$\underline{Q} = -\underline{\lambda}^{hom}\underline{E} \quad (2)$$

$$\underline{Q} = \frac{1}{|\Omega|} \int_{\Omega} \underline{q}(\underline{z})d\Omega \quad (3)$$

$$\underline{E} = \frac{1}{|\Omega|} \int_{\Omega} \underline{\nabla}T(\underline{z})d\Omega \quad (4)$$

where $\underline{q}(\underline{z})$, $\underline{\nabla}T(\underline{z})$ and $\underline{\lambda}(\underline{z})$ are the local heat flux vector, the local thermal gradient field and the local conductivity tensor at a point \underline{z} inside the REV, respectively;

\underline{Q} , \underline{E} and $\underline{\lambda}^{hom}$ are the overall heat flux vector, thermal gradient field and conductivity tensor of the REV, respectively; $|\Omega|$ the volume of the REV

Basis formulations of the homogenization method

The local and the average thermal gradient field tensors are related by the following linear equation:

$$\underline{\nabla T}(\underline{z}) = \mathbf{A}(\underline{z})\underline{E} \quad (5)$$

The combination of equations (1) to (5) yields the following equation to calculate the overall conductivity tensor of the REV:

$$\lambda^{hom} = \frac{1}{|\Omega|} \int_{\Omega} \lambda(\underline{z})\mathbf{A}(\underline{z})d\Omega \quad (6)$$

In equation (6),

- the local **conductivity tensor** $\lambda(\underline{z})$ is assumed **to be known**
- the main question is to determine the **localization tensor** $\mathbf{A}(\underline{z})$.

Basis formulations of the homogenization method

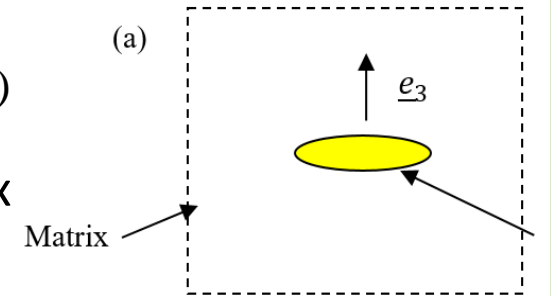
The extension of the Eshelby's solution (Eshelby, 1957) (single *ellipsoidal inclusion* in an infinite homogeneous matrix (Figure 1 a), for *transport property*) provides an analytical solution of \mathbf{A} , noted by \mathbf{A}^*

$$\mathbf{A}^* = \left(\mathbf{I} + \mathbf{P}(\boldsymbol{\lambda}_{inc} - \boldsymbol{\lambda}_m) \right)^{-1} \quad (7)$$

$\boldsymbol{\lambda}_{inc}$ and $\boldsymbol{\lambda}_m$ are the conductivity tensors of the inclusion and of the matrix reference,

\mathbf{I} the second order unit tensor.

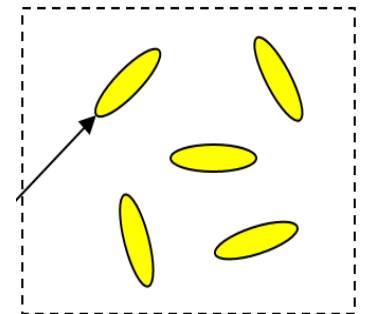
The second order tensor \mathbf{P} is the Hill tensor (Hill, 1965), that depends both on the shape of the inclusion and the conductivity of the reference matrix.



For the general case of **multi-inclusions** (Figure 1 b), (*Dormieux et al., 2006*)

$$\mathbf{A} = \mathbf{A}^* \overline{\mathbf{A}^*}^{-1} \quad (8)$$

\bar{a} : average over the whole domain of a value a : $\bar{a} = \frac{1}{|\Omega|} \int_{\Omega} a d\Omega$.



Basis formulations of the homogenization method

The combination of equations (6) to (8) yields:

$$\boldsymbol{\lambda}^{hom} = \overline{\boldsymbol{\lambda}(\mathbf{I} + \mathbf{P}(\boldsymbol{\lambda} - \boldsymbol{\lambda}_m))^{-1}(\mathbf{I} + \mathbf{P}(\boldsymbol{\lambda} - \boldsymbol{\lambda}_m))^{-1}}^{-1} \quad (9)$$

Equation (9) can be expressed by following well-known formula

$$\boldsymbol{\lambda}_{hom} = \left(\sum_{i=1}^n f_i \boldsymbol{\lambda}_i \mathbf{A}_i \right) \left(\sum_{i=1}^n f_i \mathbf{A}_i \right)^{-1} \quad (10)$$

n is the number of the inhomogeneities;

f_i and $\boldsymbol{\lambda}_i$ the volume fraction and the thermal conductivity of an inhomogeneity i , respectively;

\mathbf{A}_i the localization tensor that is function of the conductivity and shape of the phase i and the conductivity of the surrounding matrix

Basis formulations of the homogenization method

P_i the Hill tensor depends on both the **shape and the conductivity of the matrix**.

For the case of an **oblate or prolate spheroidal inclusion in an isotropic matrix** of which the conductivity tensor has a simple form $\lambda_0 = \lambda_0 \delta$, the Hill tensor is expressed by (Giraud et al., 2007):

$$P = \frac{Q}{\lambda_0} (\delta - \underline{e}_3 \otimes \underline{e}_3) + \frac{1 - 2Q}{\lambda_0} \underline{e}_3 \otimes \underline{e}_3$$

\underline{e}_3 is the unit vector in the revolution direction of the ellipsoidal inclusion

Basis formulations of the homogenization method

Parameter Q is a geometrical parameter that depends on:

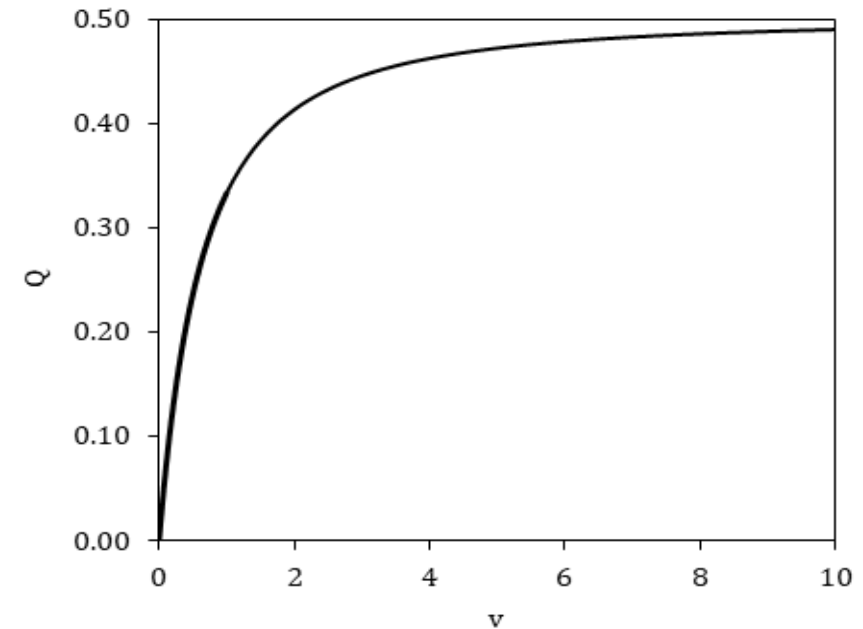
$$v = X \sqrt{\lambda^T / \lambda^N} \quad X^{hp} = \frac{l_{mean}}{\sqrt{w_{mean} e}}$$

with X , λ^T and λ^N are the aspect ratio and the conductivities of inclusion in transversal and normal directions

and for $v > 1$:

$$Q = \frac{1}{2} + \frac{2\sqrt{v^2 - 1} - v \ln(-1 + 2v^2 + 2v\sqrt{v^2 - 1})}{4(v^2 - 1)^{3/2}}$$

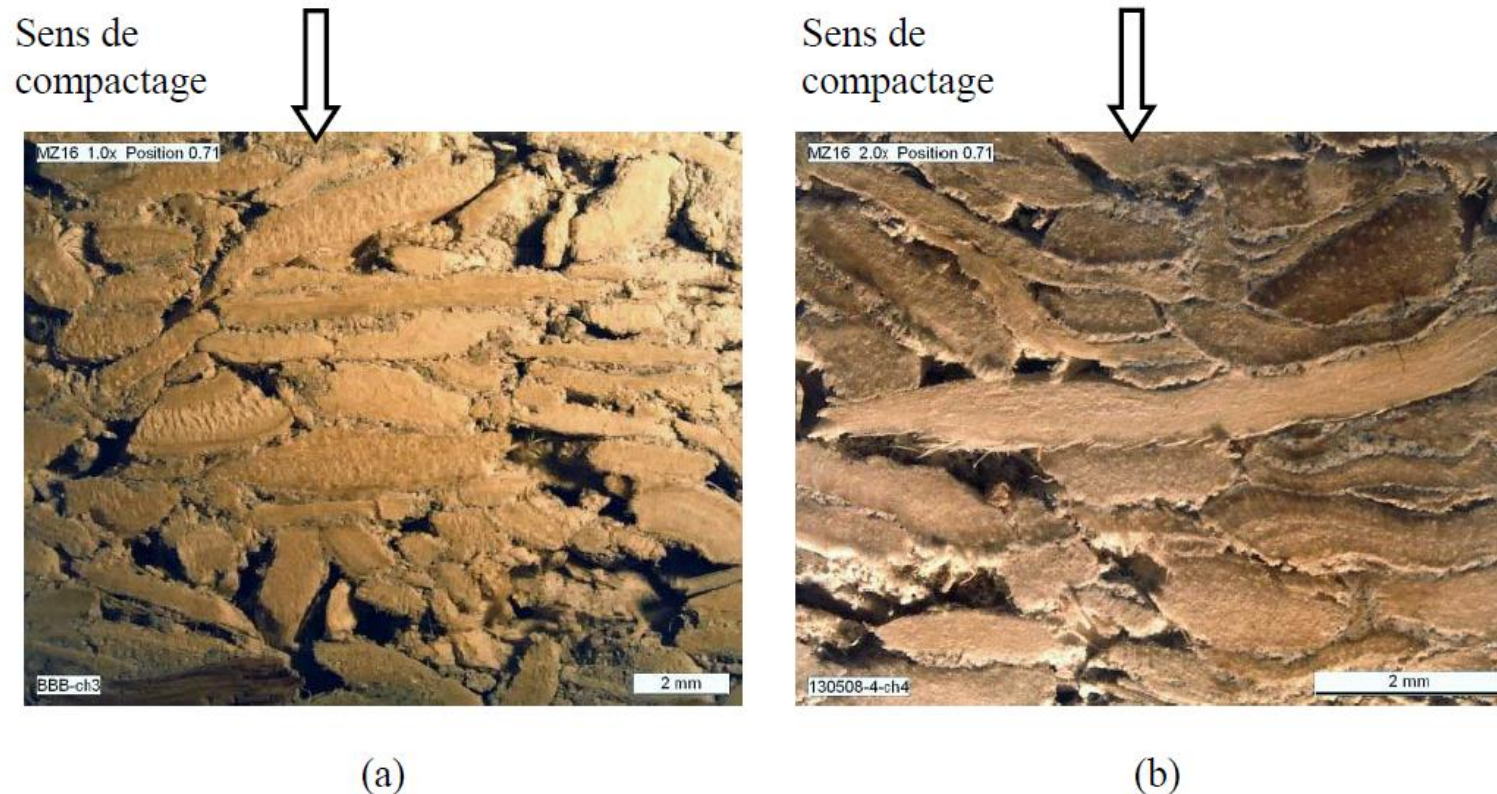
Example: for isotropic spherical inclusion: $v = X = 1$ and $Q = 1/3$.



The parameter Q as function of the parameter v

Particle orientation distribution

Lime hemp concrete has an oriented internal structure that is determined by the direction of any compacting force and the degree of orientation of hemp particles depends on the production method.



Observation des coupes verticales du béton de chanvre (Thèse NGUYEN Tai Thu, 2010)

Particle orientation distribution

Distribution function $W(\theta, \varphi)$ depends on the θ angle only and this function can take a form:

$$W(\theta, m) = (2m + 1) \cos^{2m} \theta$$

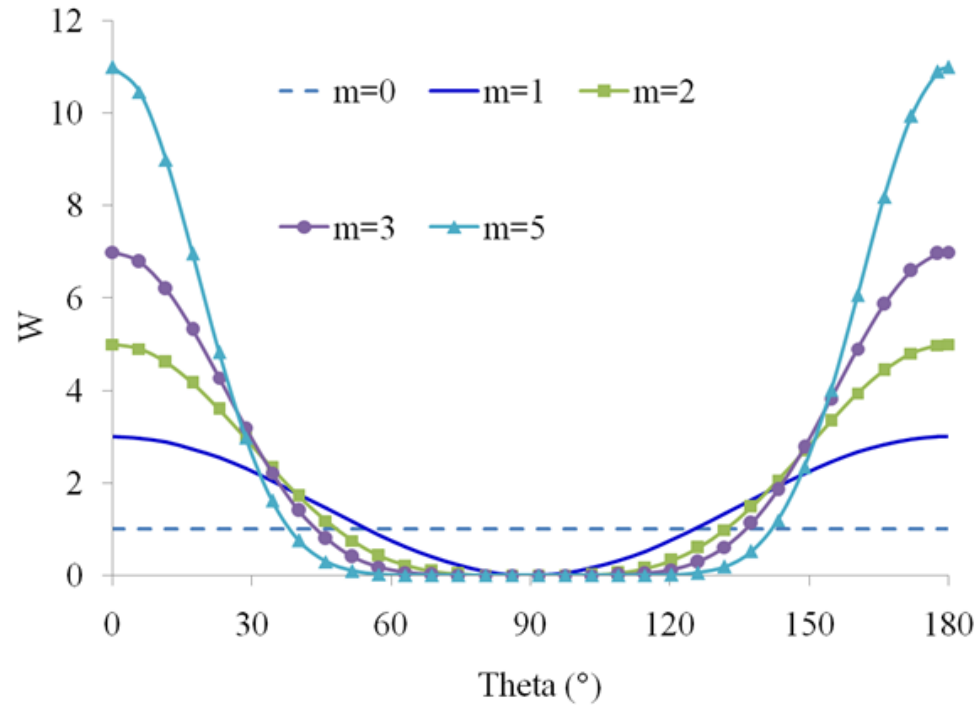
That satisfies the condition:

$$\frac{1}{2} \int_{\theta=0}^{\pi} W(\theta, m) \sin \theta d\theta = 1$$

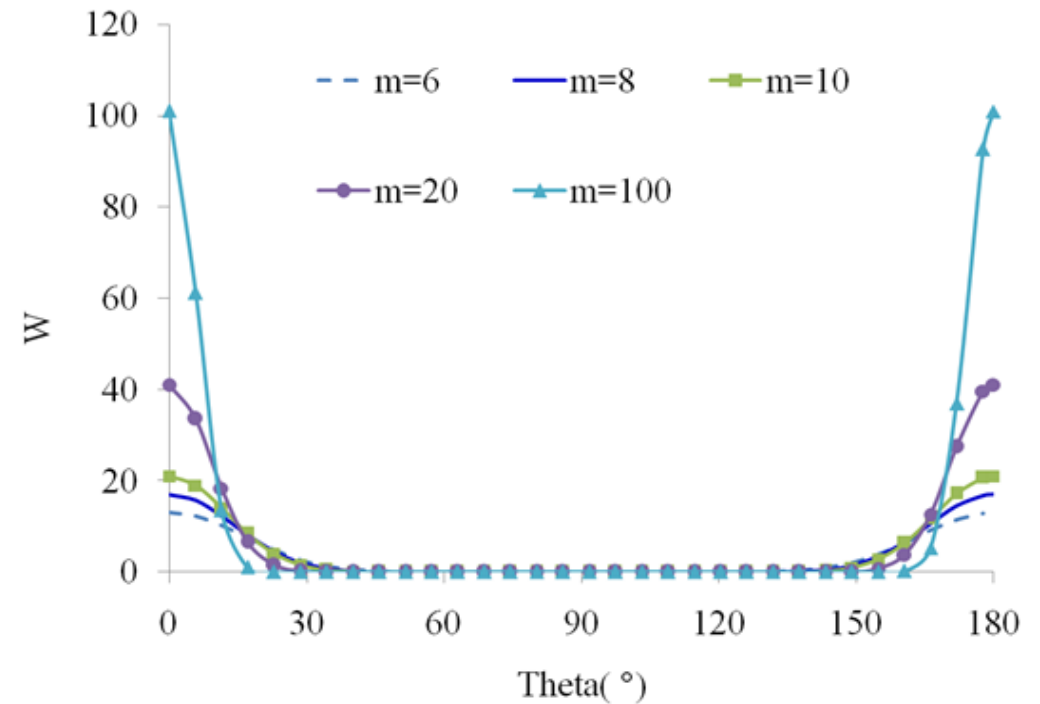
m is a distribution parameter that characterizes the degree of the preferred alignment distribution of the particles

θ is the angle between the axis of the pore and the global frame of the material.

Particle orientation distribution



Orientation distribution function
for $m=0, 1, 2, 3$ and 5



Orientation distribution function
for $m=6, 8, 10, 20$ and 100

Particle size distribution

Width

$g(w)$ the probability density function and $G(w)$ the probability that the width of a hemp shiv is equal or less than w

$$g(w) = a_w b_w w^{b_w - 1} \exp(-a_w w^{b_w})$$

$$G(w) = 1 - \exp(-a_w w^{b_w})$$

$$w_{mean} = \int_0^{\infty} w g(w) dw$$

To study the effect of the particle size of hemp particle, we define *two probability density functions for the fiber length and fiber width distributions*.



$$f(l) = a_l b_l l^{b_l - 1} \exp(-a_l l^{b_l})$$

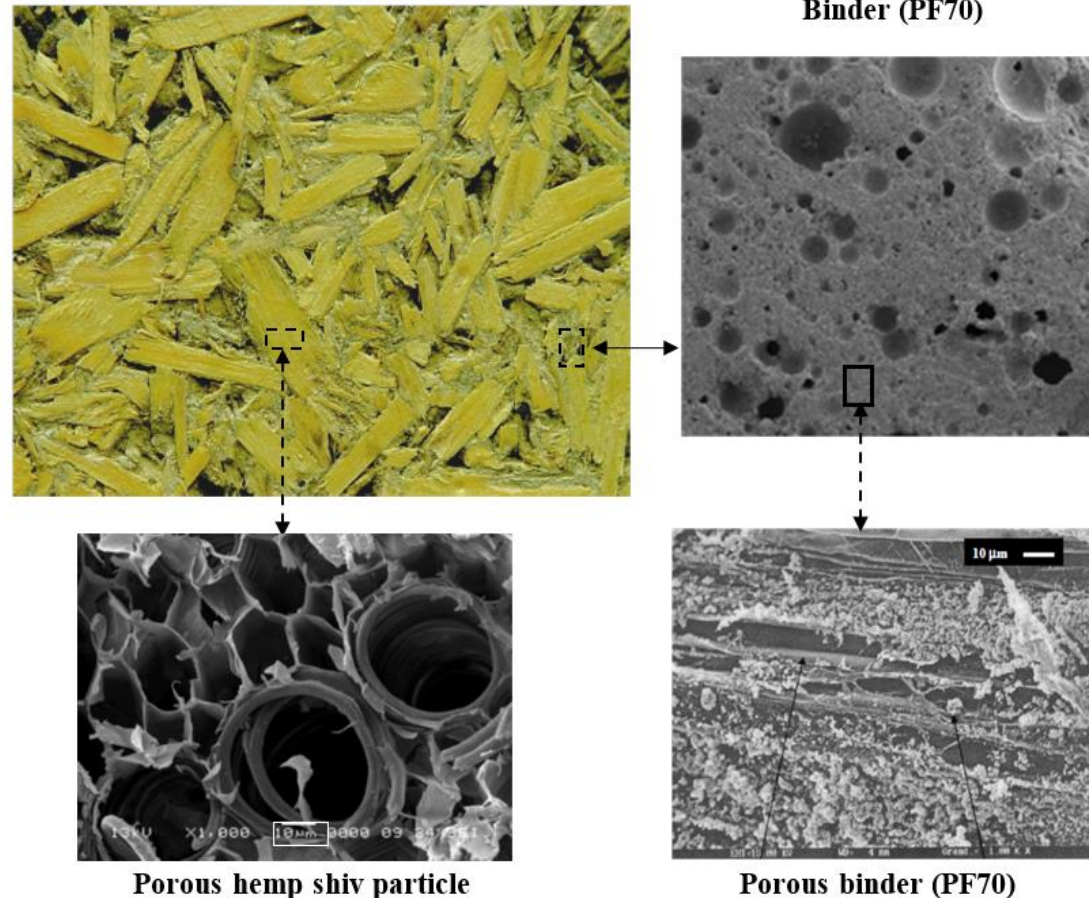
Length

$$l_{mean} = \int_0^{\infty} l f(l) dl$$

Two-scale homogenization approach

First scale: both the *hemp particle* and the *binding phase* are modeled as porous media.

Second scale (hemp concrete): a *three phase mixture* is considered: binding matrix, hemp particles (with and without imperfect interface) and macro pores



*A two-scale homogenization approach based on
microstructure and mesostructure of hemp concrete*

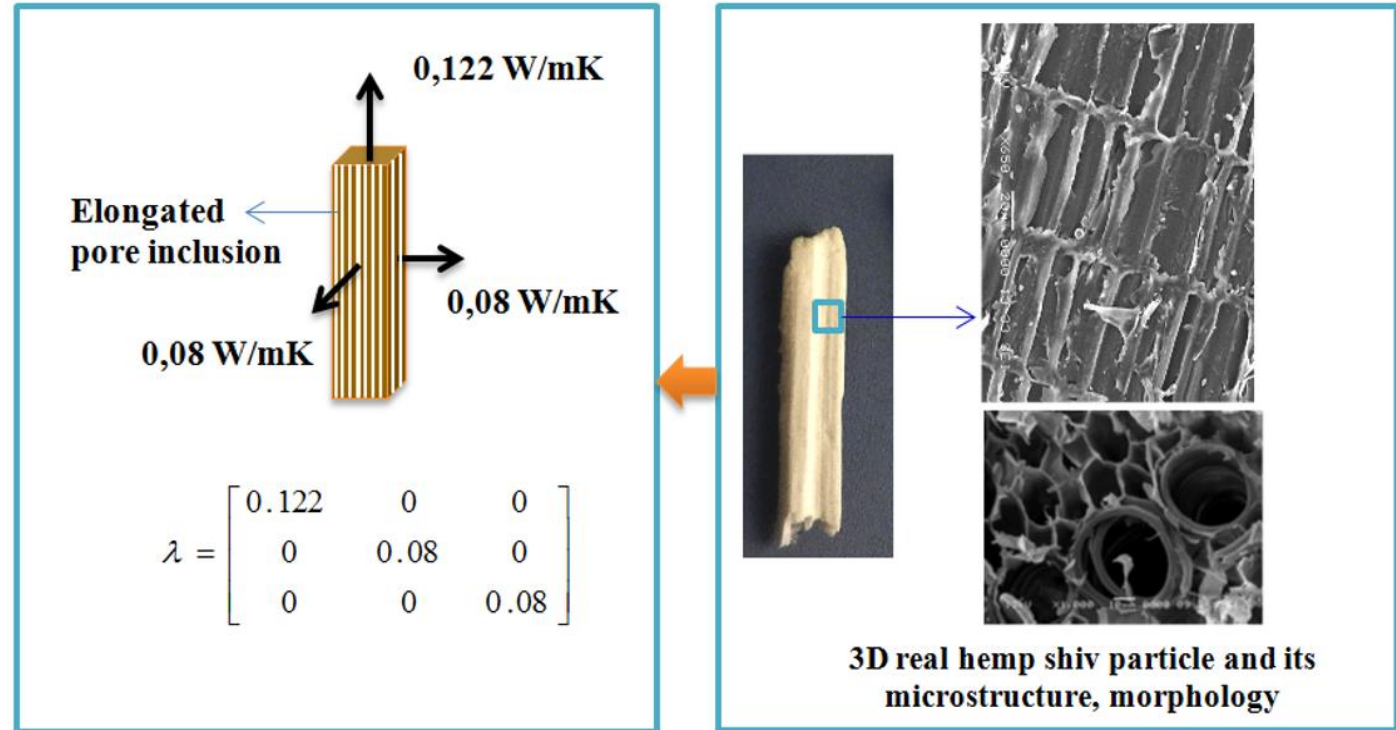
Effective thermal conductivity tensor of hemp shiv particle

$$\lambda_{hp}^N = \varphi_{hp} \lambda_a + (1 - \varphi_{hp}) \lambda_s$$

$$\lambda_{hp}^T = \left(\frac{\varphi_{hp}}{\lambda_a + \lambda_s} + \frac{1 - \varphi_{hp}}{2\lambda_s} \right)^{-1} - \lambda_s$$

λ_a and λ_s the conductivity of air and the solid phases, respectively

φ_{hp} the porosity of hemp particle.



3D, microstructure and morphology of hemp shiv particle (right) and anisotropy of thermal conductivity tensor of a dry hemp shiv particle

Effective thermal conductivity tensor of dry binder

$$\lambda_b = \left(\frac{\varphi_b}{\lambda_a + 2\lambda_{bs}} + \frac{1 - \varphi_b}{\lambda_{bs} + 2\lambda_{bs}} \right)^{-1} - 2\lambda_{bs}$$

λ_b and λ_{bs} are the overall conductivity and the conductivity of the solid phase of binder, respectively;

φ_b is the porosity of binder.

| φ_b | λ_a (W/(m.K)) | λ_b (W/(m.K)) | λ_{bs} (W/(m.K)) |
|-------------|-----------------------|-----------------------|--------------------------|
| 0.506 | 0.0255 | 0.24 | 0.56 |

Conductivity of binder

Modeling imperfect thermal contact between hemp particles and surrounding binder

The imperfect interface contact between shiv and the binder has been highlighted and observed by scanning electron microscopy

The equivalent conductivity of a particle and its interface, noted by $\tilde{\lambda}_{hp}$: thermal jump across the interface to the macroscopic thermal gradient

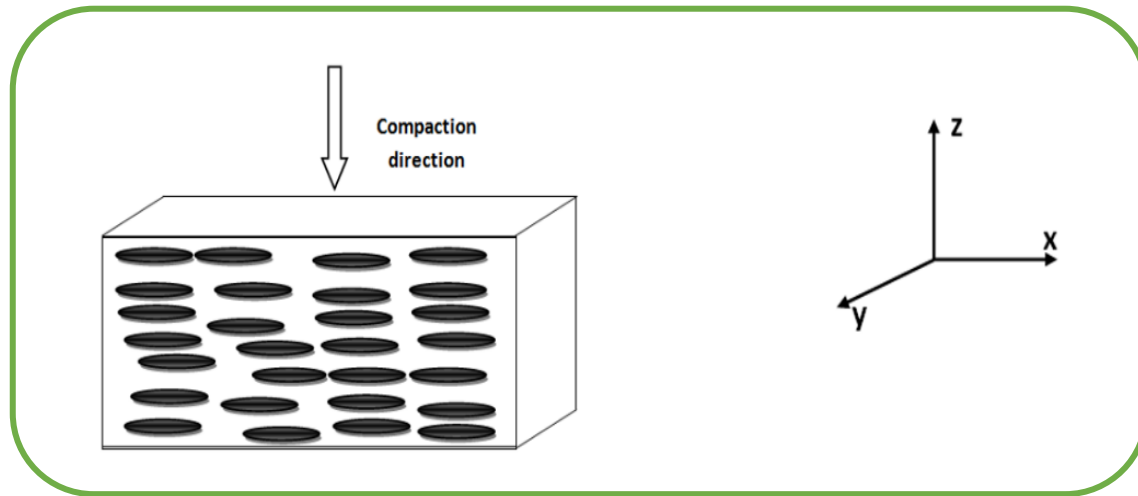
$$\tilde{\lambda}_{hp}^T = \frac{\lambda_{hp}^T}{1 + \alpha \lambda_{hp}^T / \lambda_b}; \quad \tilde{\lambda}_{hp}^N = \lambda_{hp}^N$$

λ_b the thermal conductivity of the surrounding binding matrix that is assumed to be isotropic;

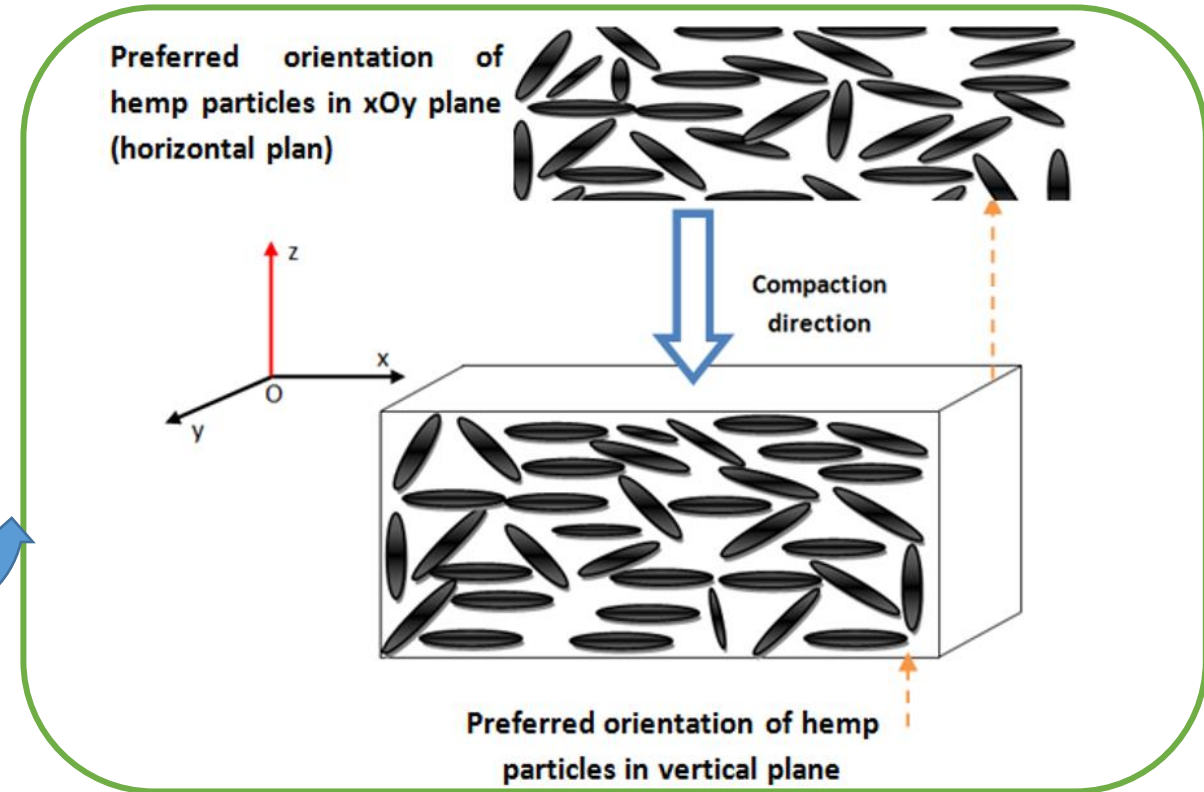
α interface parameter that characterizes the thickness, the thermal conductivity of the interface zone.

α is considered as an intrinsic parameter that will be calibrated by an inverse analysis using experimental data.

Effective thermal conductivity tensor of dry hemp concrete



*Perfect alignment of hemp particles in the
horizontal plan*

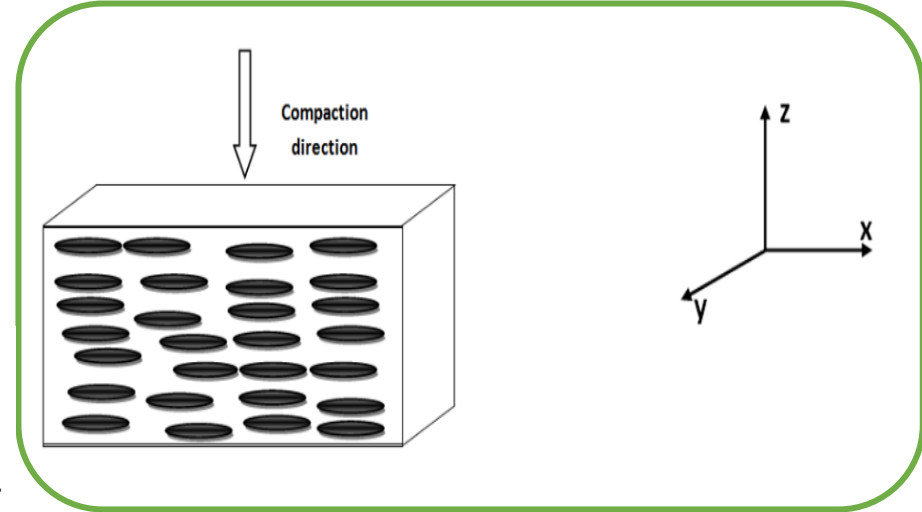


*Preferred spatial orientations in different planes
(horizontal and vertical planes) for developing model*

Effective thermal conductivity tensor of dry hemp concrete: *Perfect alignment model*

$$\lambda_{hom}^T = \frac{(1 - f_a - f_{hp})\lambda_b + f_a\lambda_a \frac{3\lambda_b}{\lambda_a + 2\lambda_b} + f_{hp} \frac{\tilde{\lambda}_{hp}^T \lambda_b}{Q(\tilde{\lambda}_{hp}^T - \lambda_b) + \lambda_b}}{(1 - f_a - f_{hp}) + f_a \frac{3\lambda_b}{\lambda_a + 2\lambda_b} + f_{hp} \frac{\lambda_b}{Q(\tilde{\lambda}_{hp}^T - \lambda_b) + \lambda_b}}$$

$$\lambda_{hom}^N = \frac{(1 - f_a - f_{hp})\lambda_b + f_a\lambda_a \frac{3\lambda_b}{\lambda_a + 2\lambda_b} + f_{hp} \frac{\tilde{\lambda}_{hp}^N \lambda_b}{(1-2Q)(\tilde{\lambda}_{hp}^N - \lambda_b) + \lambda_b}}{(1 - f_a - f_{hp}) + f_a \frac{3\lambda_b}{\lambda_a + 2\lambda_b} + f_{hp} \frac{\lambda_b}{(1-2Q)(\tilde{\lambda}_{hp}^N - \lambda_b) + \lambda_b}}$$



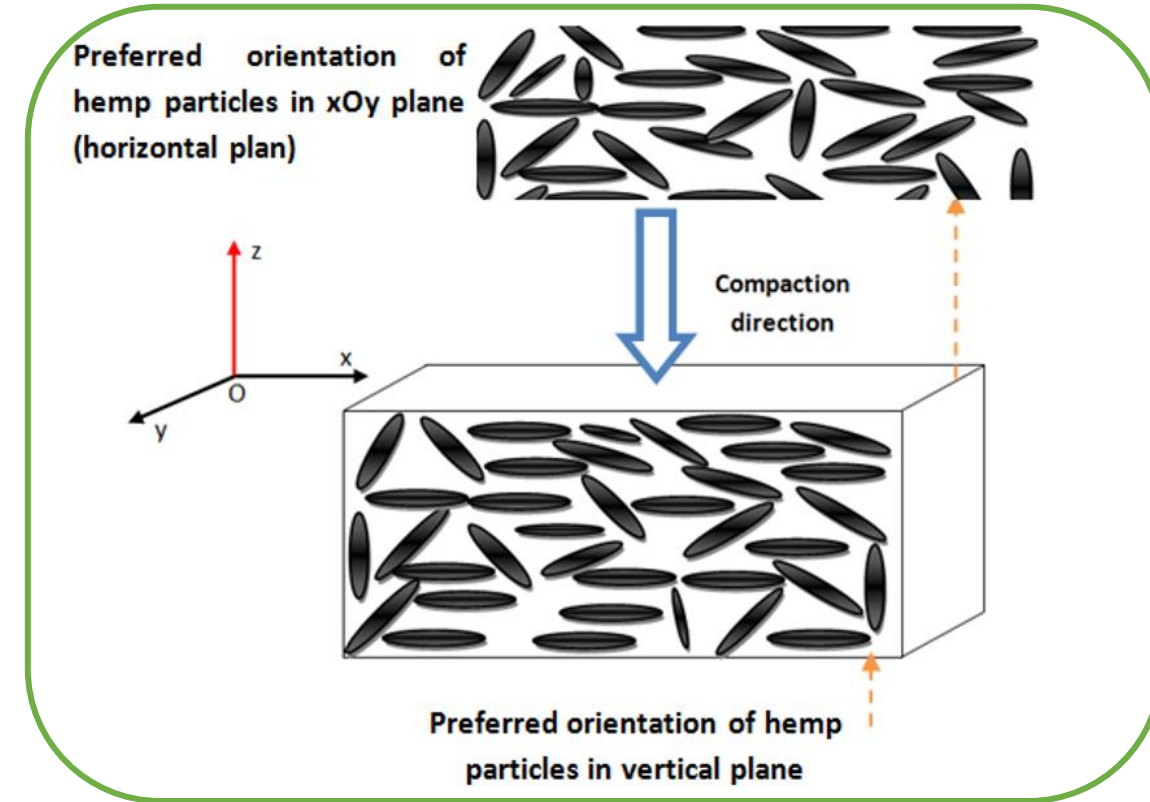
*Perfect alignment of hemp particles in the
horizontal plan*

$$\lambda_x = \begin{bmatrix} \lambda_{xx} & 0 & 0 \\ 0 & \lambda_{yy} & 0 \\ 0 & 0 & \lambda_{zz} \end{bmatrix} = \begin{bmatrix} \lambda_{al}^N & 0 & 0 \\ 0 & \lambda_{al}^T & 0 \\ 0 & 0 & \lambda_{al}^T \end{bmatrix}$$

Effective thermal conductivity tensor of dry hemp concrete: *preferred spatial orientations model*

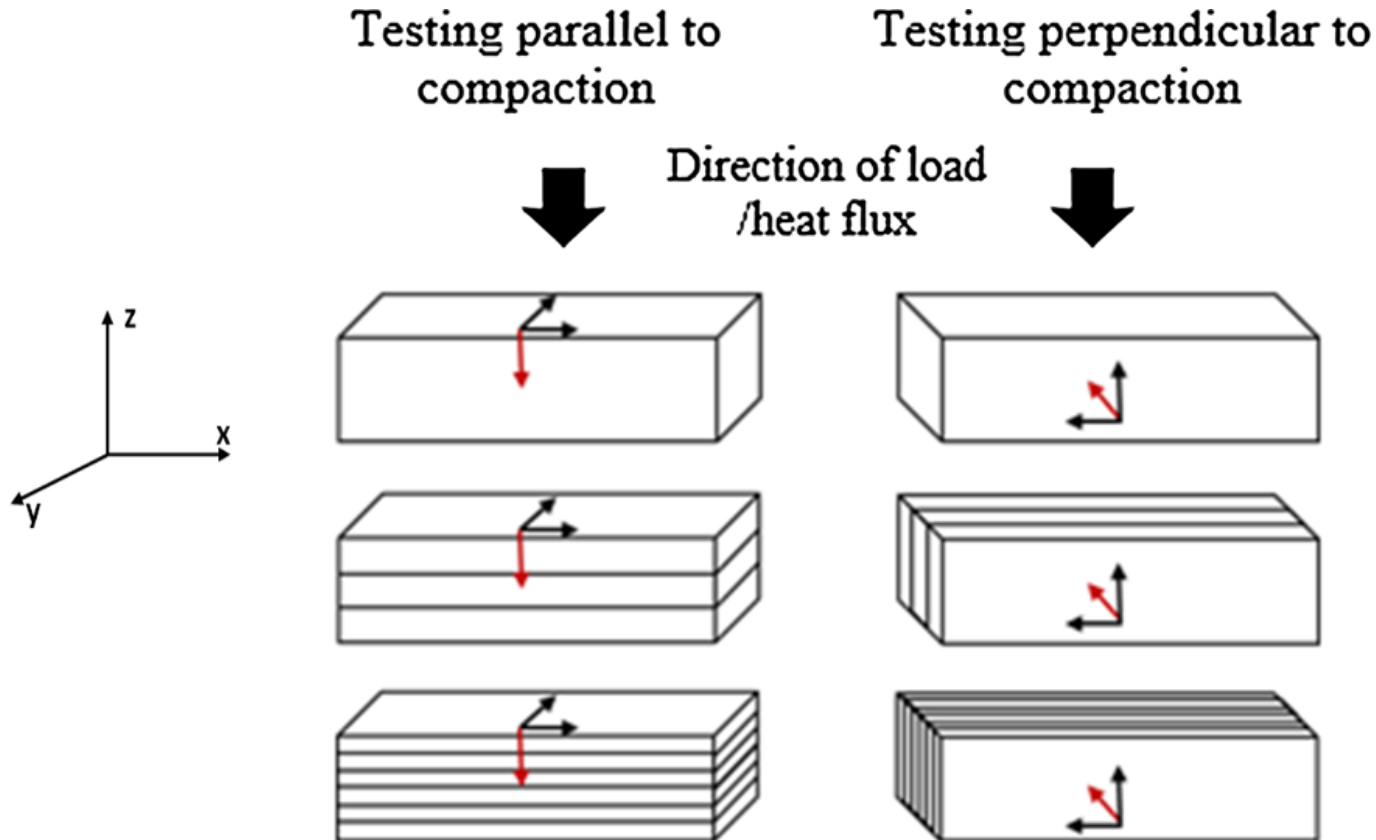
$$\langle \lambda^{\varphi\theta}_{hom} \rangle = \begin{bmatrix} \frac{2m+1}{2(2m+3)} \lambda_{al}^N + \frac{2m+5}{2(2m+3)} \lambda_{al}^T & & & & & & \\ & \frac{1}{2} (\lambda_{al}^N + \lambda_{al}^T) & & & & & \\ & & \frac{1}{2m+3} \lambda_{al}^N + \frac{2m+2}{2m+3} \lambda_{al}^T & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ & & & & & & 0 \end{bmatrix} = \begin{bmatrix} \lambda_{xx\ hom}^{\varphi\theta} \\ \lambda_{yy\ hom}^{\varphi\theta} \\ \lambda_{zz\ hom}^{\varphi\theta} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Random orientation distribution: $m = 0$
isotropic average tensor $\langle \lambda \rangle$.
- Aligned distribution, $m \rightarrow \infty$, $\langle \lambda \rangle = \lambda$.



Preferred spatial orientations in different planes (horizontal and vertical planes) for developing model

Model validation



Testing direction arrangements for parallel and perpendicular loading with **direction of compaction** and **preferred plane** of orientation of particles indicated by the **red** and **black axis** respectively

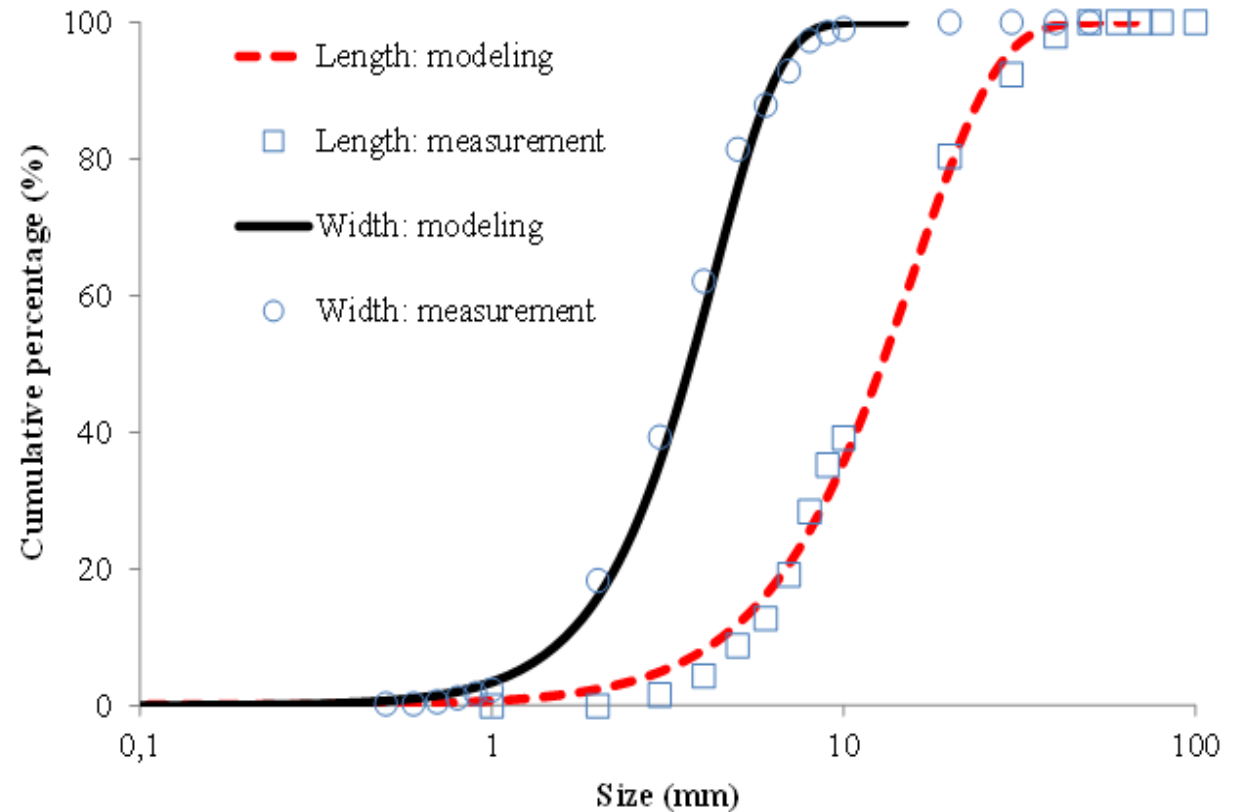
(source: Williams et al., 2017).

Model validation

Size distribution of hemp shiv particles

| a_w | b_w | w_{mean} (cm) |
|-------|-------|------------------------|
| 0.035 | 2.3 | 0.38 |

| a_l | b_l | l_{mean} (cm) |
|-------|-------|------------------------|
| 0.07 | 1.8 | 1.4 |



Computed and measured values of cumulative distributions of lengths and widths of hemp shiv particles

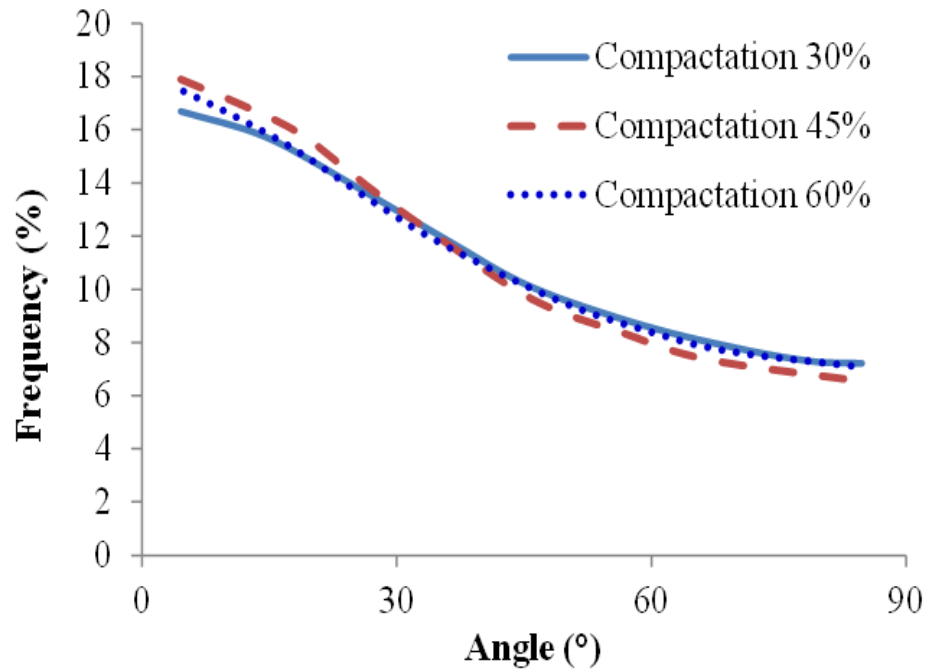
Model validation

Oriented distribution of hemp particles

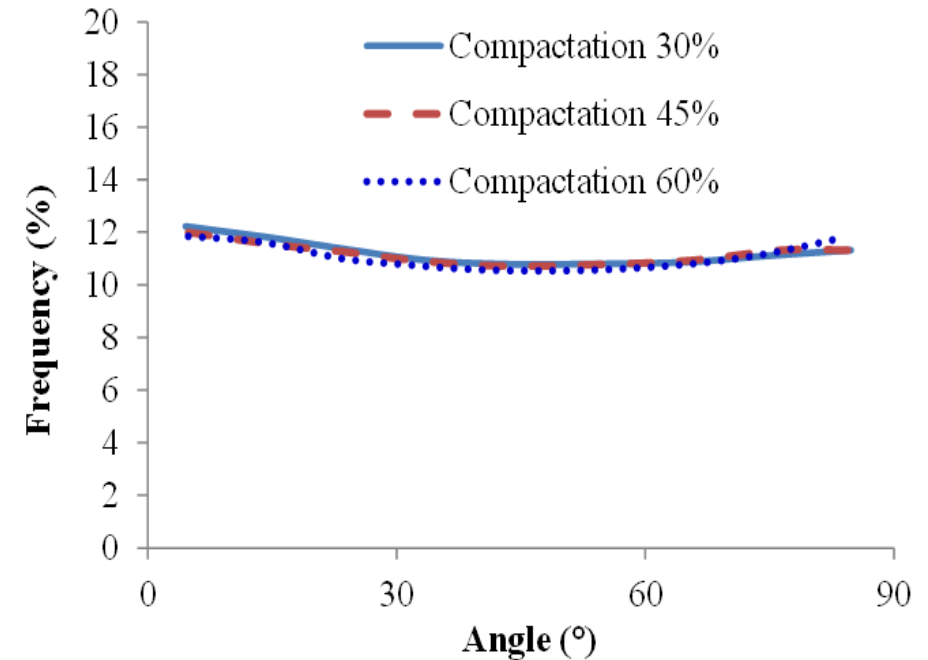
Internal structure : A novel application of image analysis methodology developed by *Williams et al. (2017)*

- Frequency distributions of particle orientation
- Three compaction levels: 30, 45 and 60% volumetric decrease from the uncompact state

Distribution parameter (m value)=1



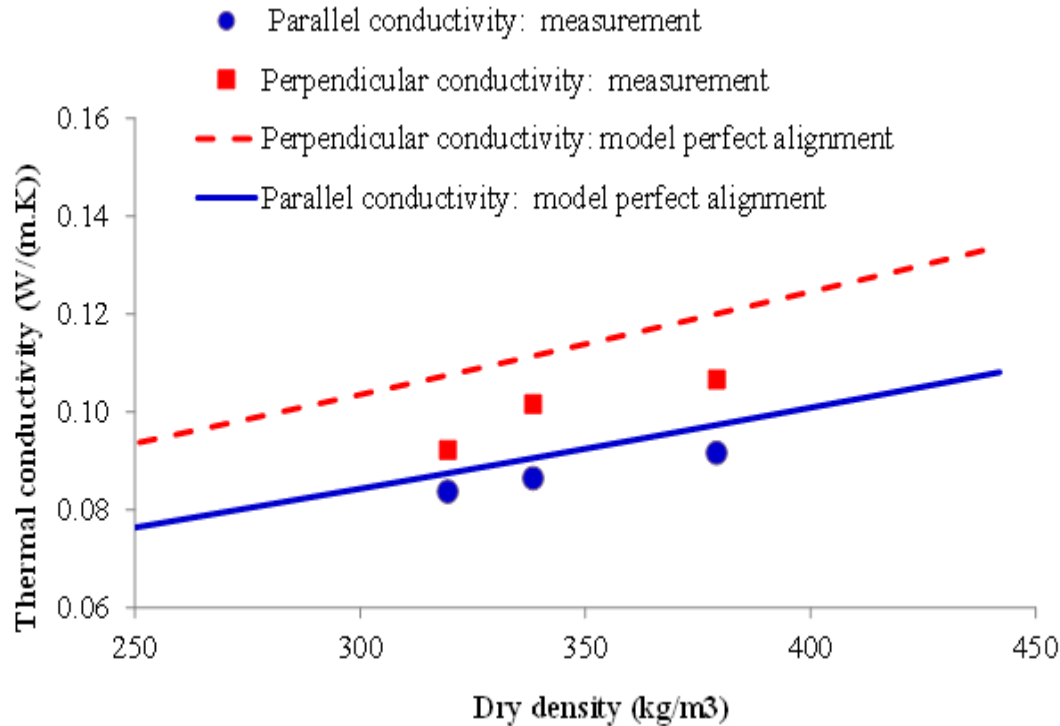
Perspective perpendicular to compaction



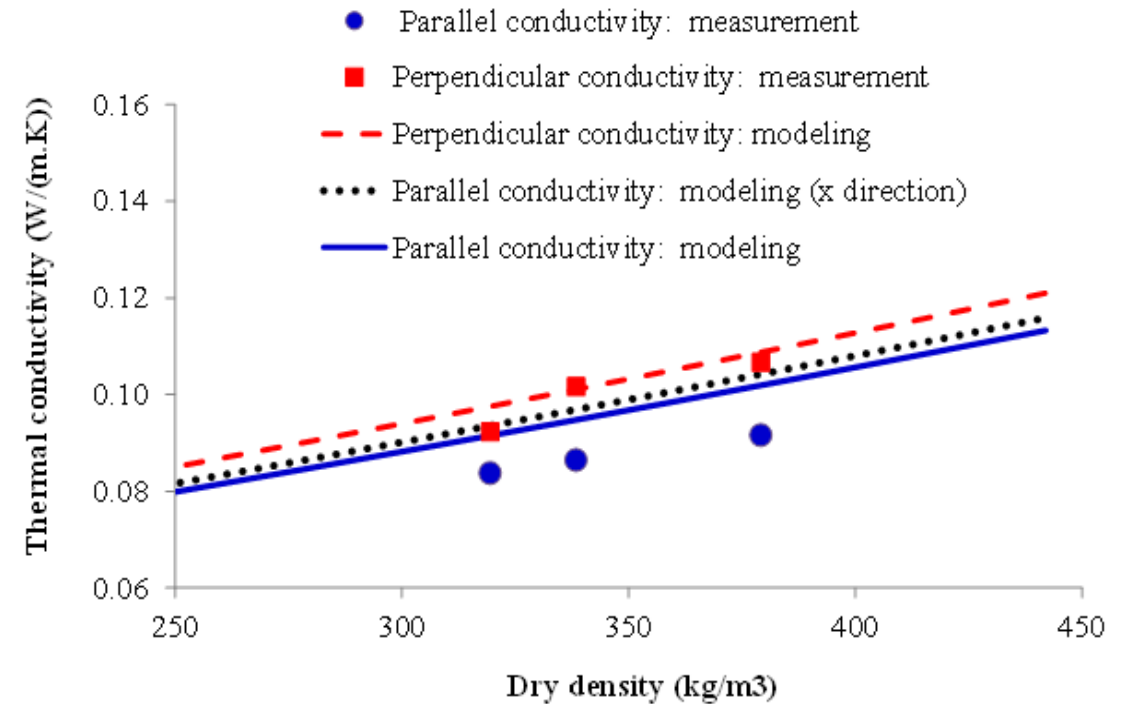
Parallel to compaction

Model validation

Perfect alignment vs Preferred spatial orientations models



Perfect alignment model



Preferred spatial orientations model

Model validation

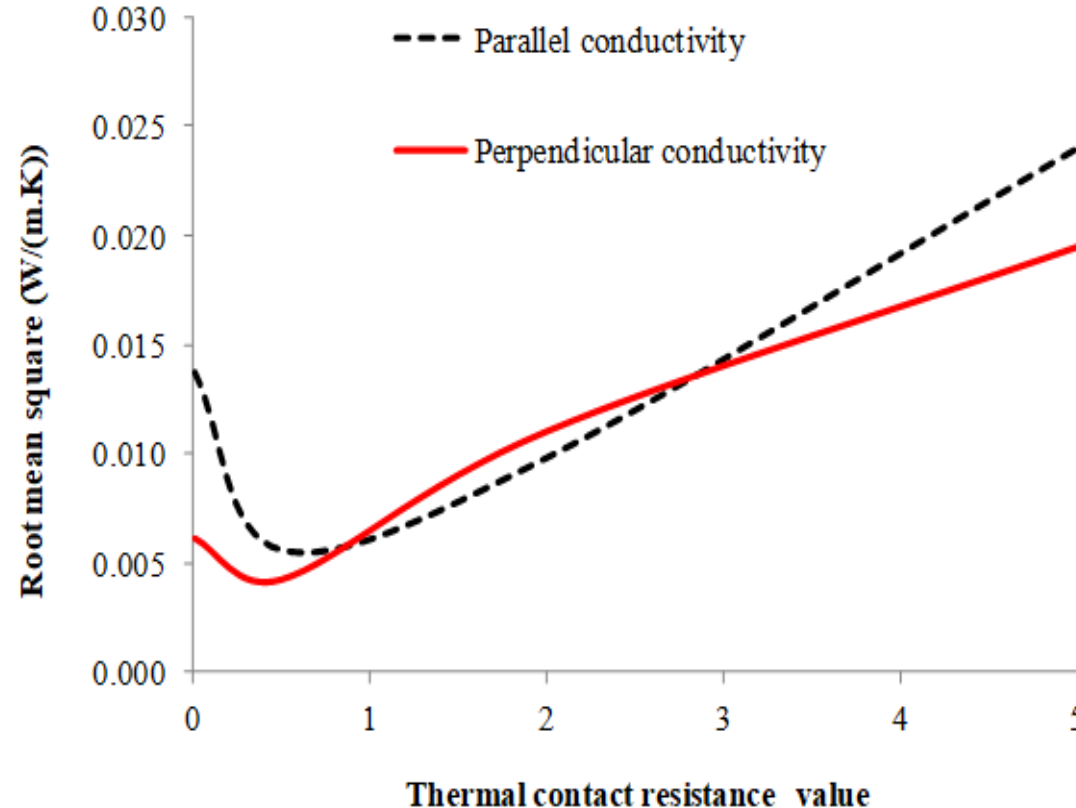
Impact of imperfect thermal interface

Root mean square difference (*gamma*)

$$\Gamma_{RMS} = \sqrt{\frac{\sum_{n=1}^N (\Delta\lambda)^2}{N}}$$

$\Delta\lambda$: difference thermal conductivity between the measured and predicted values,

N: number of values in the data set



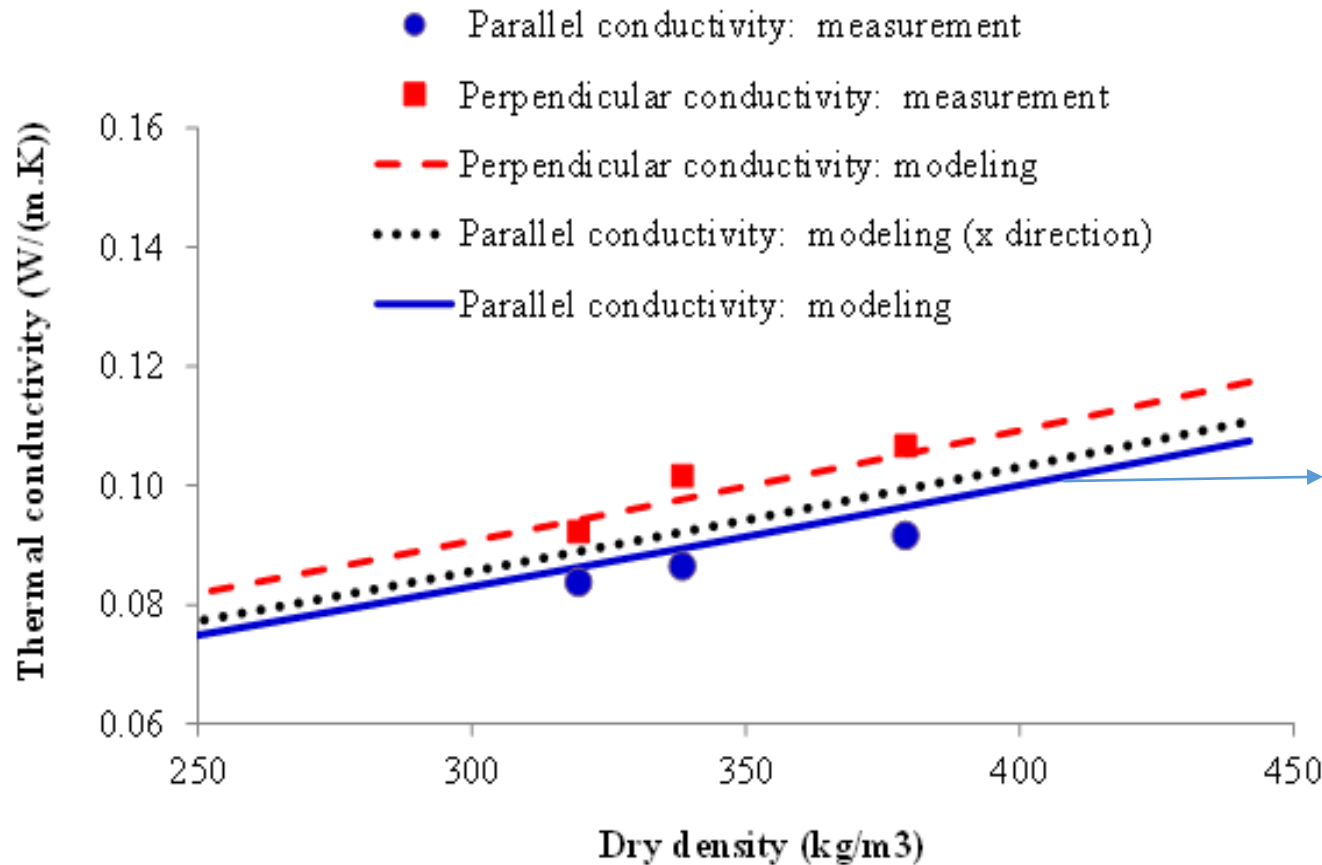
$\alpha=0.5$: smallest values of Γ_{RMS} values of both conductivity components

Effect of thermal interface contact resistance (α) on the root mean square difference

A novel analytical model for predicting the effective thermal conductivity tensor of lime-hemp concrete

Model validation

***Preferred spatial orientations model
with imperfect thermal interface ($\alpha=0.5$)***

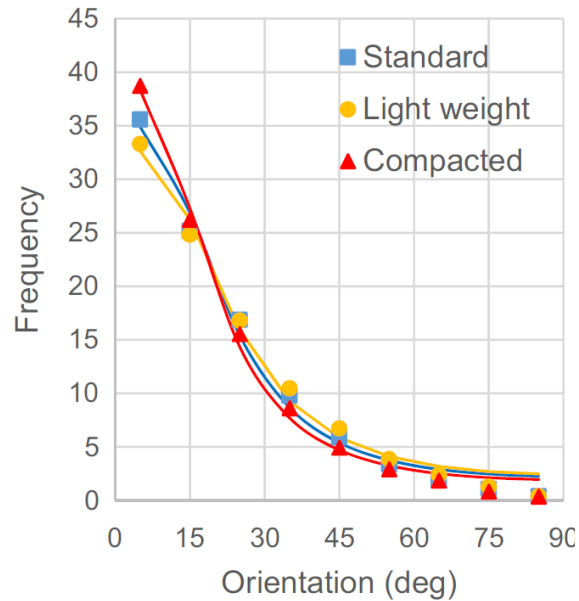


the modelling consistently over-predicts for parallel direction

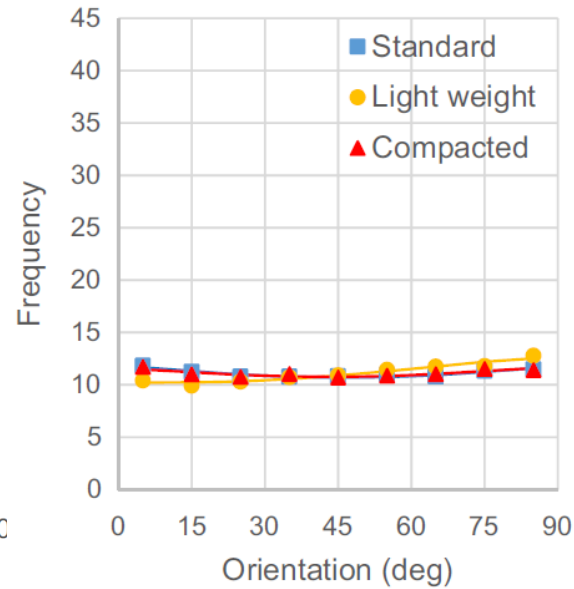


Model validation

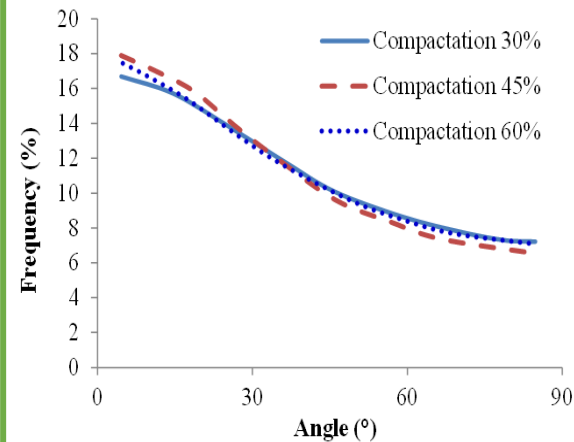
Two methods to determine the oriented distribution of hemp particles*



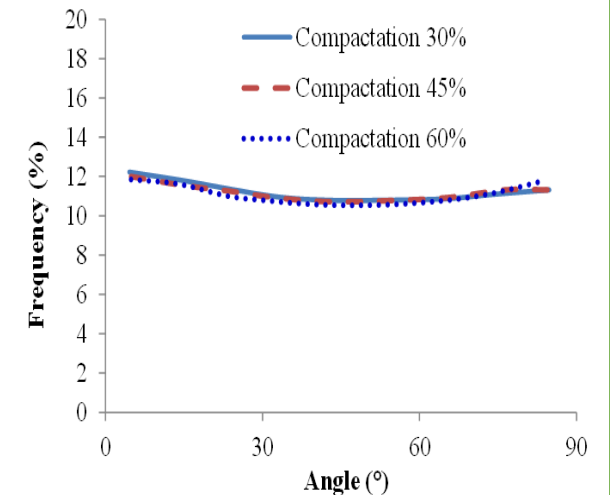
Perpendicular to compaction



Parallel to compaction



Perpendicular to compaction



Parallel to compaction

Computer tomography scanning (CT scanning)

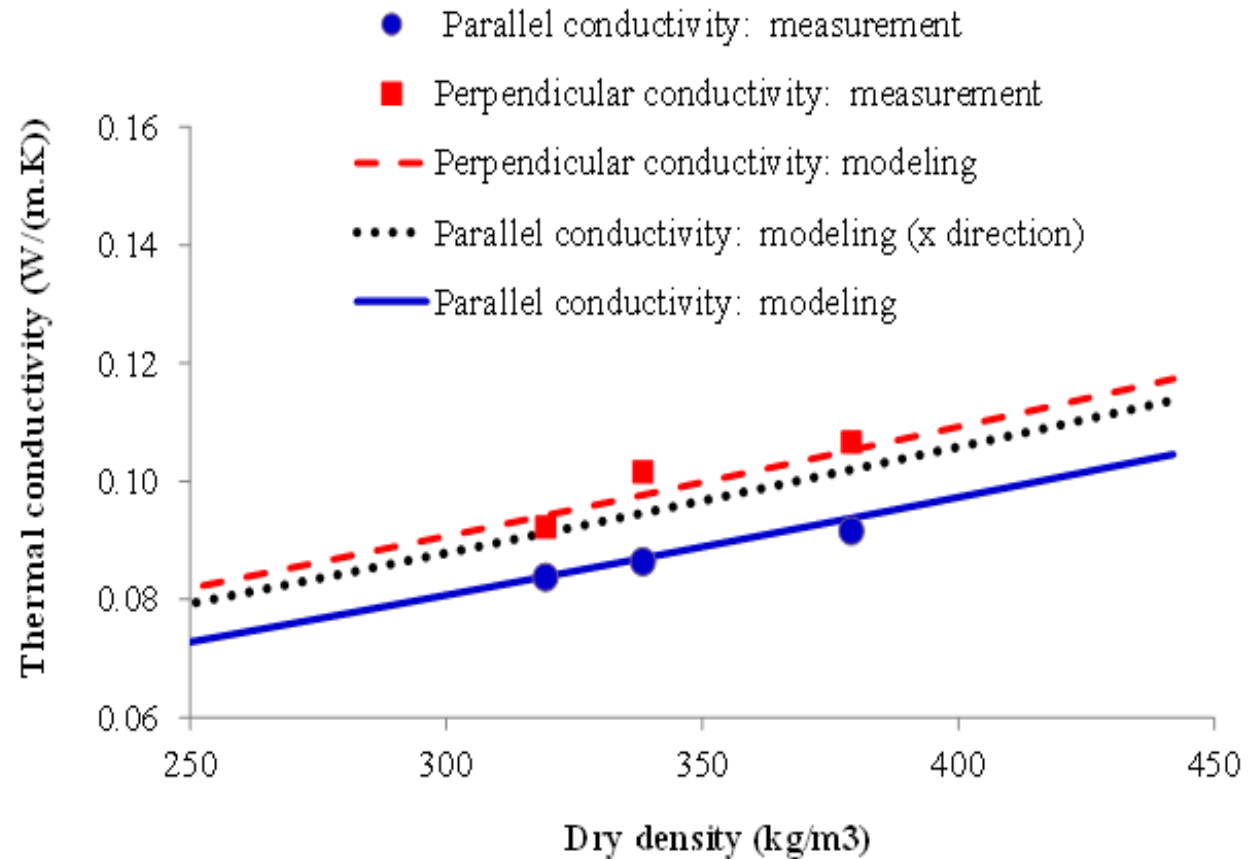
Distribution parameter (m value)=3

Visual digital imaging method

Distribution parameter (m value)=1

Model validation

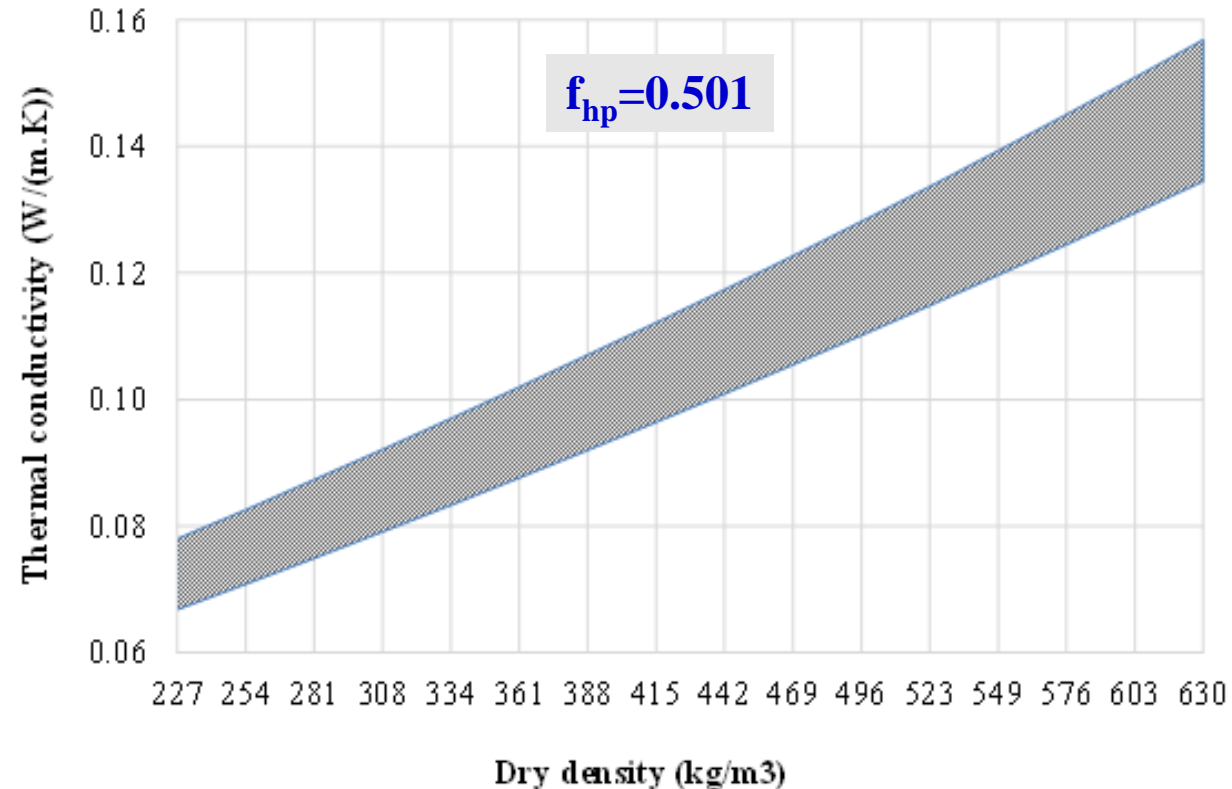
Distribution parameter (m value)=3



Computed values and measured thermal conductivities with $m=3$ based on the results obtained from CT scanning method (Williams et al., 2016).

Application example

The range of thermal conductivity of hemp concrete in different direction



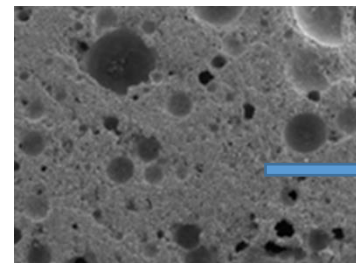
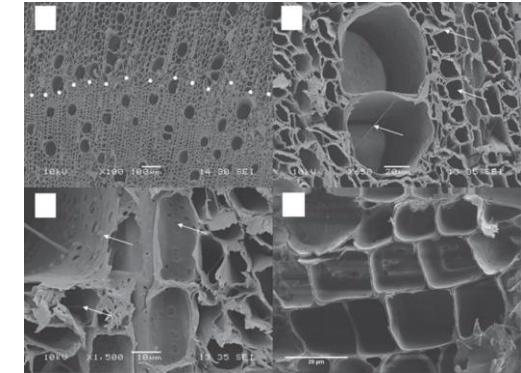
The range of the thermal conductivity of hemp concrete depending on the measured direction when m value varies between 0 and ∞ .

Use of the developed model

Many parameters have been taken into account in only one model to optimize
the effective thermal conductivity of hemp concrete

- **Hemp shiv particles:** defined by λ_s (conductivity of solid phases of hemp shiv), ϕ_{hp} (porosity of hemp particle) and the hemp shiv size distribution function.
- **Hemp shape and the anisotropy:** of the inclusion: the Q , a and b parameters introduced in the model.
- **Binder type:** it is defined by λ_{bs} (solid phase of binder) and ϕ_b (porosity of binder).

Components of hemp concrete



Porous binder (PF70)

Use of the developed model

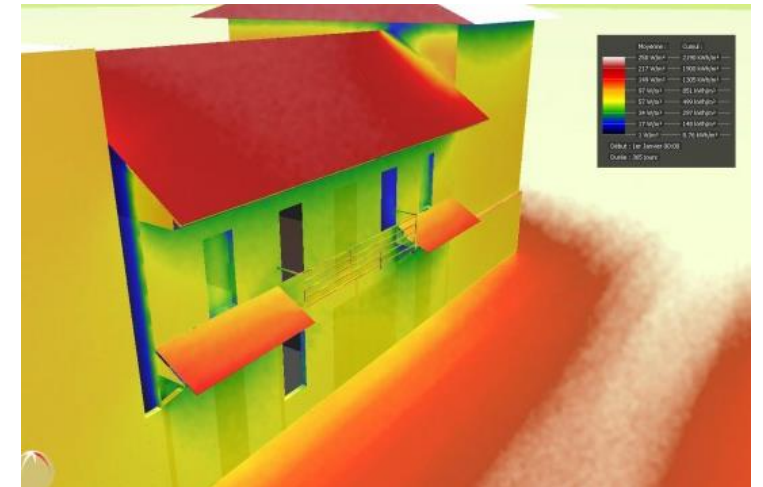
Many parameters have been taken into account in only one model to optimize the effective thermal conductivity of hemp concrete

- **Formulation of hemp concrete:** f_{hp} (volume fraction of hemp particles) and f_b (volume fraction of binder).
- **Manufacturing process:** the distribution parameter m determined based *on the image analysis technique* (CT imaging and VD imaging methods as presented in *Williams et al., 2016; 2017*).
- **Durability and surface treatment:** an interface parameter α , imperfect thermal contact between hemp particles and surrounding binder (can appear during drying period, surface treatment or the aging).



Use of the developed model

- Very good analytical tool to investigate the variability of the thermal conductivity of hemp concrete.
- Effect of one parameter on the overall thermal conductivity of hemp concrete can be quickly assessed and predicted (when it is not possible to quantify experimentally due to the technical problems such as measurement precision errors of the testing apparatus).
- Presented analytical solutions offer potential opportunities for using 3D thermal simulation tools to optimize hemp-building performance.



Conclusion

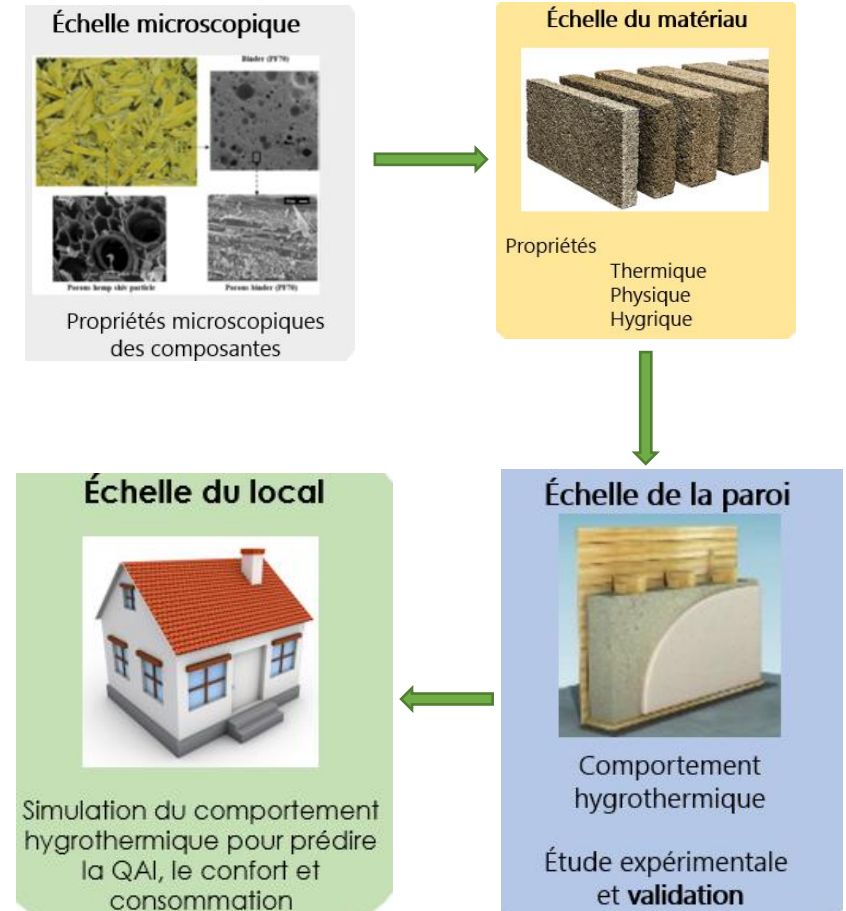
Anisotropy of the thermal conductivity of hemp concrete: the thermal conductivity components are different in different direction

Anisotropy can be strongly modified by the *manufacturing process*.

The CT scanning method is a suitable method for the assessment of hemp concrete.

The novel mathematical model developed is very useful to **optimize the thermal performance of bio-based materials** such as hemp concrete and building design.

For future works, the developed model will be extended and developed for **multi-scale study** focusing on **hygrothermal behavior of hemp concrete** from **micro scale to higher scale levels**: such as materials, building envelope and buildings.



For future works