# Learning Activity 4 - Homework
## Quasi-phase matching

N. Dubreuil

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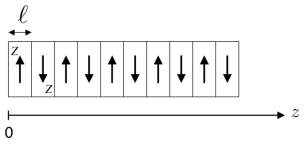
## Second harmonic generation

One seeks to double the frequency of a beam of a Ti:Sa laser working in CW regime around 852 nm and emitting a power of 800 mW. For that purpose, one uses a 2nd order nonlinear material selected for its strong nonlinear susceptibility coefficient $\chi^{(2)}_{ZZZ}$. It is made of a succession of piles of thickness $\ell$, whose crystal orientation $Z$ is inverted periodically as a function of position (see the figure below). It forms a periodically poled material.




1. Which is the wavelength associated with the second harmonic beam ?
2. In order to maximize the quantity of generated light at $2\omega$, which has to be the polarization state of the Ti:Sa laser beam ? Which will be the polarization state of the $2\omega$ wave ?
3. Write the nonlinear wave equation for the $2\omega$ wave within the first pile, $0 < z < \ell$, in terms of the effective nonlinear susceptibility $\chi^{(2)}_{eff}$. Justify your answer.

   Within the undepleted pump approximation, give the expression of the $2\omega$ wave amplitude $A_2(z = \ell)$. The boundary condition is $A_2(z = 0) = 0$.
4. Do the same for the second pile, $\ell < z < 2\ell$, and give the expression for $A_2(z = 2\ell)$.
5. Show that $A_2(z = 2\ell)$ is at maximum when $\Delta k \cdot \ell = \pi$.
6. By generalizing this result, show that the intensity at $2\omega$ for $z = 2p\ell$, whith $p$ a positive integer, is written :
$$I_{2\omega}(2p\ell) = \frac{2\omega^2(\chi^{(2)}_{eff})^2}{\epsilon_0 c^3 n_2 n_1^2 \pi^2}(2p\ell)^2 I_1^2 \tag{1}$$
7. The selected material is a crystal of PP-KTP. It has a length $L_c = 20$ mm and its nonlinear coefficient is $\chi^{(2)}_{ZZZ} = 20$ pm/V. The refractive indices $n_1$ and $n_2$ are equal to 1.8405 et 1.9419, respectively.

(a) Calculate the optimum poling period $\Lambda = 2\ell$ of the crystal.

(b) The beam from the Ti:Sa laser, correctly polarized, is focused in the PP-KTP crystal with a waist of $w = 70\,\mu$m. By neglecting diffraction effect, calculate the power of the generated beam at $2\omega$. It will be supposed that the two beams have the same size.