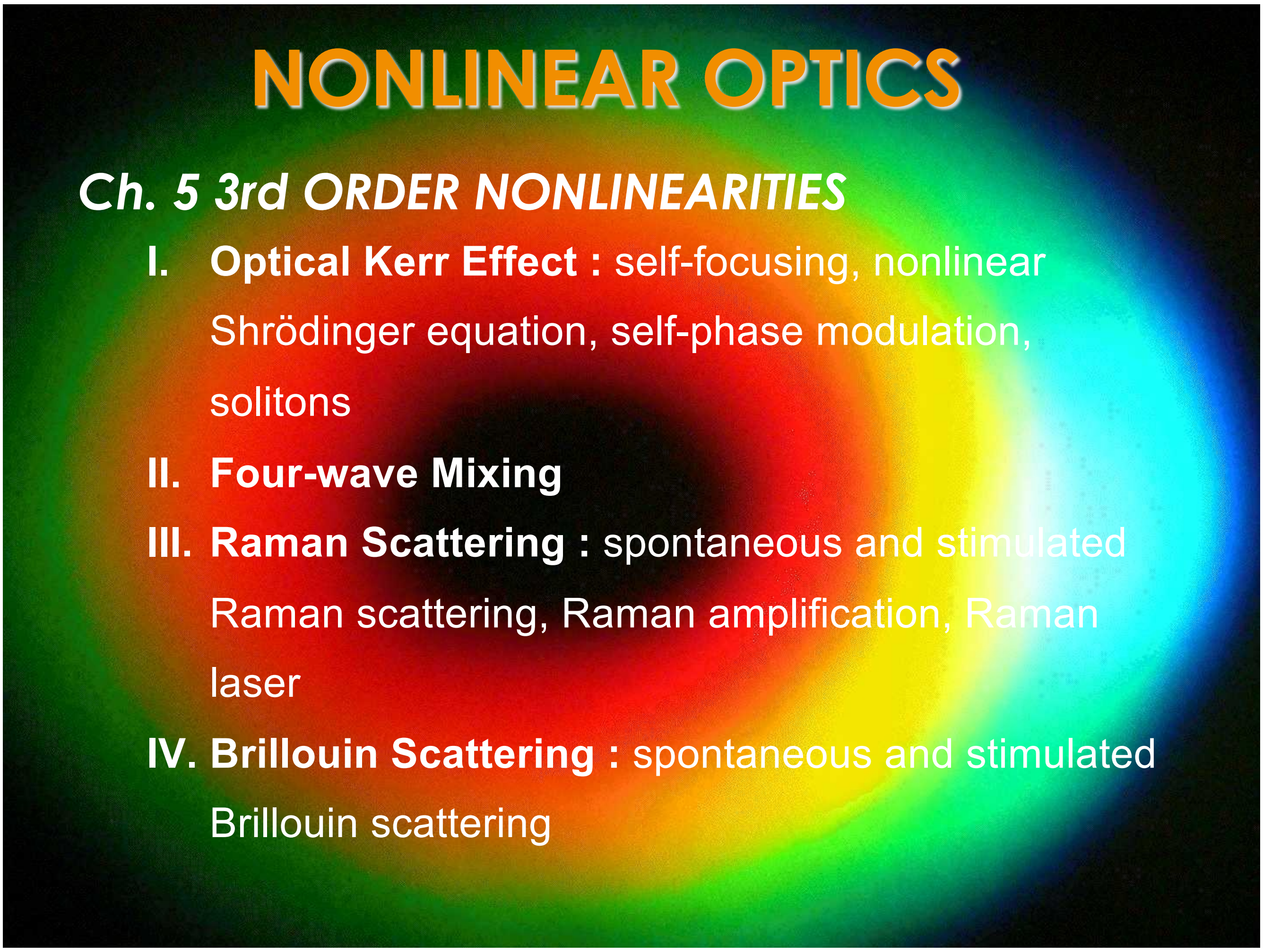


NONLINEAR OPTICS

Ch. 5 3rd ORDER NONLINEARITIES

- **I. Optical Kerr Effect :** self-focusing, nonlinear Shrödinger equation, self-phase modulation, solitons
- **II. Four-wave Mixing**
- **III. Raman Scattering :** spontaneous and stimulated Raman scattering, Raman amplification, Raman laser
- **IV. Brillouin Scattering :** spontaneous and stimulated Brillouin scattering

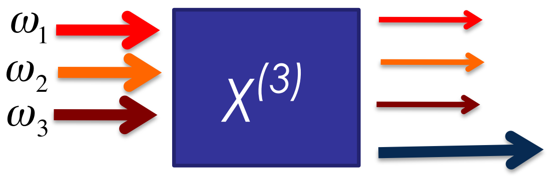
II - Four-wave Mixing

Interaction between 4 waves at $\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4$

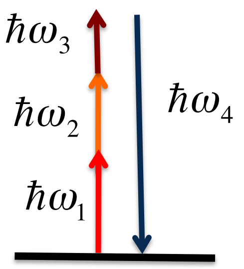
With : $\omega_4 = \omega_1 + \omega_2 + \omega_3$

$$\vec{P}_{NL}(\omega_4) = D^{(3)}\varepsilon_0 \underline{\underline{\underline{\underline{\chi}}}}^{(3)}(\omega_4;\omega_1,\omega_2,\omega_3)\vec{e}_1\vec{e}_2\vec{e}_3E(\omega_1)E(\omega_2)E(\omega_3)$$

- ***Generation of UV light source***



$\omega_4 = \omega_1 + \omega_2 + \omega_3$

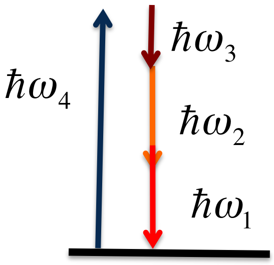


Creation of one photon at ω_4, by means of the simultaneous annihilation of one photon at ω_1, one photon at ω_2 and one photon at ω_3

Phase matching condition to be fullfiled : $\vec{k}_4 = \vec{k}_1 + \vec{k}_2 + \vec{k}_3$

II - Four-wave Mixing

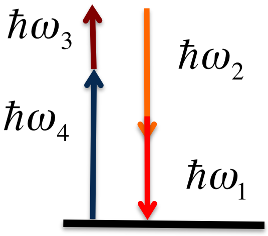
- ***Generation of IR light source***



$$\omega_4 = \omega_1 + \omega_2 + \omega_3$$

$$\vec{k}_4 = \vec{k}_1 + \vec{k}_2 + \vec{k}_3$$

Annihilation of one photon at ω_4, accompanied by the simultaneous creation of three photons, resp. at ω_1, ω_2 and ω_3



$$\omega_4 + \omega_3 = \omega_1 + \omega_2$$

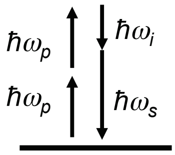
$$\vec{k}_4 + \vec{k}_3 = \vec{k}_1 + \vec{k}_2$$

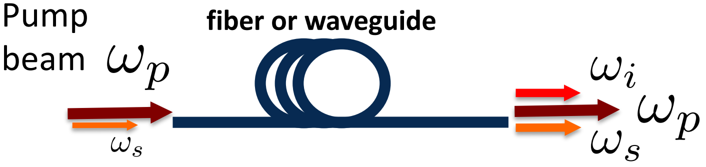
II - Four-wave Mixing

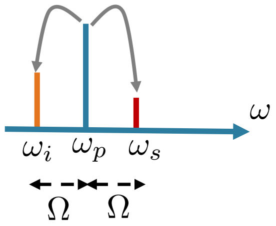
- **Degenerate FWM : parametric amplification**

Nonlinear interaction between a pump wave @ ω_p
and a weak signal @ ω_s

$$\hbar\omega_p + \hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$







II - Four-wave Mixing

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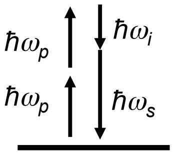
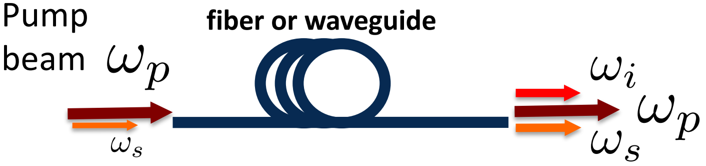
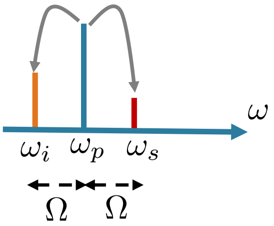


Singlemode optical fiber or waveguide





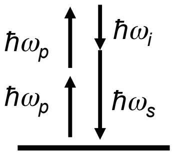
$$\begin{cases} \dfrac{dA_p}{dz} = \\ \dfrac{dA_s}{dz} = \\ \dfrac{dA_i}{dz} = \end{cases}$$

II - Four-wave Mixing

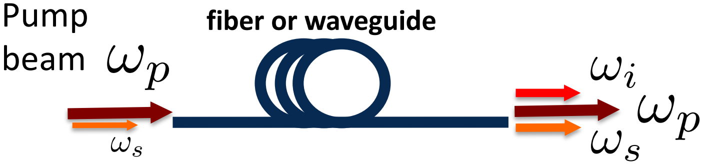
- **Degenerate FWM : parametric amplification**

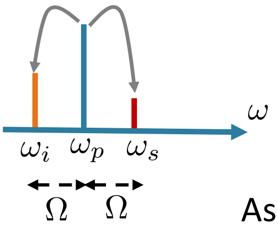
Nonlinear interaction between a pump wave @ ω_p
and a weak signal @ω_s

$$\hbar\omega_p + \hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$



Singlemode optical fiber or waveguide





$$\begin{cases} \dfrac{dA_p}{dz} = \dfrac{i\omega_p}{2n_p c} 3\chi_{eff}^{(3)} |A_p|^2 A_p \\ \dfrac{dA_s}{dz} = \\ \dfrac{dA_i}{dz} = \end{cases}$$

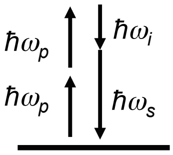
Assumption :

$$|A_p|^2 \gg |A_s|^2, |A_i|^2$$

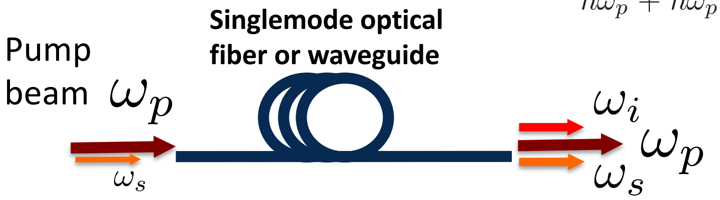
II - Four-wave Mixing

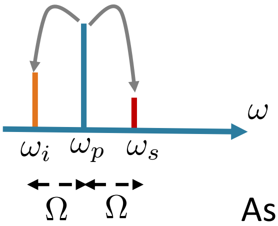
- **Degenerate FWM : parametric amplification**

Nonlinear interaction between a pump wave @ ω_p
and a weak signal @ ω_s



$$\hbar\omega_p + \hbar\omega_p \;=\; \hbar\omega_s + \hbar\omega_i$$





$$\begin{cases} \dfrac{dA_p}{dz} = \dfrac{i\omega_p}{2n_p c} 3\chi^{(3)}_{eff} |A_p|^2 A_p \\ \dfrac{dA_s}{dz} = \dfrac{i\omega_s}{2n_s c} 3\chi^{(3)}_{eff} \left[2|A_p|^2 A_s + A_p^2 A_i^* e^{i\Delta\beta z}\right] \\ \dfrac{dA_i}{dz} = \dfrac{i\omega_i}{2n_i c} 3\chi^{(3)}_{eff} \left[2|A_p|^2 A_i + A_p^2 A_s^* e^{i\Delta\beta z}\right] \end{cases}$$

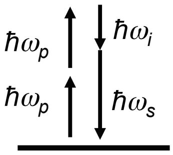
Assumption :

$$|A_p|^2 \gg |A_s|^2, |A_i|^2$$

$$\Delta\beta = 2\beta_p - \beta_s - \beta_i$$

II - Four-wave Mixing

- **Degenerate FWM : parametric amplification**



$$\boxed{\frac{dA_p}{dz} = \frac{i\omega_p}{2n_p c} 3\chi^{(3)}_{eff} |A_p|^2 A_p}$$

$$\frac{dA_p}{dz} = i\gamma P_p A_p$$

Mode effective Area (surface)

Pump Power : $P_p = 2nc\epsilon_0 |A_p|^2 A_{eff}$

Kerr coef. : $\gamma = \dfrac{3\,\omega_p}{4\epsilon_0 n_p^2 c^2 \, A_{eff}} \chi^{(3)}_{eff}$

=> Solutions

$$\boxed{\begin{aligned} A_p(z) &= A_p(0) e^{i\gamma P_p z} \\ &= |A_p(0)| e^{i\theta} e^{i\gamma P_p z} \end{aligned}}$$

With θ = phase term for the pump

II - Four-wave Mixing

- **Degenerate FWM : parametric amplification**

$$\begin{cases} \dfrac{dA_s}{dz} = \dfrac{i\omega_s}{2n_s c} 3\chi^{(3)}_{eff} \left[2|A_p|^2 A_s + A_p^2 A_i^* e^{i\Delta\beta z}\right] \\ \dfrac{dA_i}{dz} = \dfrac{i\omega_i}{2n_i c} 3\chi^{(3)}_{eff} \left[2|A_p|^2 A_i + A_p^2 A_s^* e^{i\Delta\beta z}\right] \end{cases}$$

$$\boxed{\begin{aligned} A_p(z) &= A_p(0) e^{i\gamma P_p z} \\ &= |A_p(0)| e^{i\theta} e^{i\gamma P_p z} \end{aligned}}$$

$$\begin{cases} \dfrac{dA_s}{dz} = i\gamma P_p \dfrac{\omega_s}{\omega_p} \left[2A_s + A_i^* e^{2i\theta} e^{i(\Delta\beta + 2\gamma P_p)z}\right] \\ \dfrac{dA_i^*}{dz} = -i\gamma P_p \dfrac{\omega_i}{\omega_p} \left[2A_i^* + A_s e^{-2i\theta} e^{-i(\Delta\beta + 2\gamma P_p)z}\right] \end{cases}$$

New variables : $B_s(z) = A_s(z) e^{-2i\gamma P_p \frac{\omega_s}{\omega_p} z}$

$B_i^*(z) = A_i^*(z) e^{+2i\gamma P_p \frac{\omega_i}{\omega_p} z}$

II - Four-wave Mixing

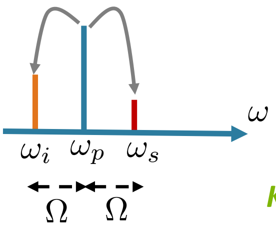
- **Degenerate FWM : parametric amplification**

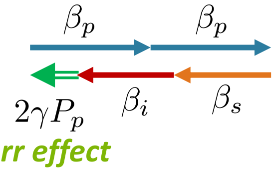
$$\begin{cases} \dfrac{dB_s}{dz} = i\gamma P_p \dfrac{\omega_s}{\omega_p} e^{2i\theta} B_i^*(z) e^{+iK'z} \\ \dfrac{dB_i^*}{dz} = -i\gamma P_p \dfrac{\omega_i}{\omega_p} e^{-2i\theta} B_s(z) e^{-iK'z} \end{cases}$$

$$K' = \Delta\beta + 2\gamma P_p \left[1 - \frac{\omega_s + \omega_i}{\omega_p}\right]$$

About the phase-matching condition :

$$K' \simeq \Delta\beta - 2\gamma P_p$$





$$\Delta\beta = 2\beta_p - \beta_s - \beta_i$$

$$\beta(\omega) = \beta_0 + \beta_1 \Delta\omega + \frac{\beta_2}{2}\Delta\omega^2 + \cdots$$

$$\Delta\beta \simeq -\beta_2 \Omega^2$$

$\Rightarrow$ **The phase matching condition is modified by the optical Kerr effect (nonlinear phase shift)**

$\Rightarrow$ **$K' = 0$ implies :** $\gamma > 0$ and $\beta_2 < 0$ **or** $\gamma < 0$ and $\beta_2 > 0$

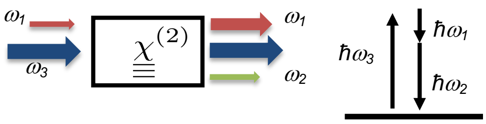
II - Four-wave Mixing

- **Degenerate FWM : parametric amplification**

$$\begin{cases} \dfrac{dB_s}{dz} = i\gamma P_p \dfrac{\omega_s}{\omega_p} e^{2i\theta} B_i^*(z) e^{+iK'z} \\ \dfrac{dB_i^*}{dz} = -i\gamma P_p \dfrac{\omega_i}{\omega_p} e^{-2i\theta} B_s(z) e^{-iK'z} \end{cases}$$

$$K' = \Delta\beta + 2\gamma P_p \left[1 - \frac{\omega_s + \omega_i}{\omega_p}\right]$$

Similitude with the difference frequency generation in $\chi^{(2)}$



$$\frac{dA_1}{dz} = \frac{\imath\omega_1}{2n_1c}\chi_{\text{eff}}^{(2)} A_3 A_2^\star e^{\imath\Delta kz}$$
$$\frac{dA_2}{dz} = \frac{\imath\omega_2}{2n_2c}\chi_{\text{eff}}^{(2)} A_3 A_1^\star e^{\imath\Delta kz}$$

II - Four-wave Mixing

- **Degenerate FWM : parametric amplification**

$$\begin{cases} \dfrac{dB_s}{dz} = i\gamma P_p \dfrac{\omega_s}{\omega_p} e^{2i\theta} B_i^*(z) e^{+iK'z} \\ \dfrac{dB_i^*}{dz} = -i\gamma P_p \dfrac{\omega_i}{\omega_p} e^{-2i\theta} B_s(z) e^{-iK'z} \end{cases}$$

$$K' = \Delta\beta + 2\gamma P_p \left[1 - \frac{\omega_s + \omega_i}{\omega_p}\right]$$

Boundary conditions :

$$A_s(z=0) = A_{s0}$$
$$A_i(z=0) = 0$$

Solutions based on the the difference frequency generation in $\chi^{(2)}$

$$A_s(z) = A_s(0)e^{-2i\gamma P_p z}\left[\cosh(g'z) - \frac{iK'}{2g'}\sinh(g'z)\right] e^{-i\frac{K'}{2}z}$$

Nonlinear phase shift through XPM

Parametric Amplification

$$\Gamma' = \gamma P_p \frac{n_p}{\omega_p}\sqrt{\frac{\omega_i\omega_s}{n_i n_s}} \simeq \gamma P_p \qquad g'^2 = \Gamma'^2 - \frac{K'^2}{4}$$

II - Four-wave Mixing

- **Degenerate FWM : parametric amplification**

Solutions

$$A_s(z) = \underbrace{A_s(0)e^{-2i\gamma P_p z}}_{\text{Nonlinear phase shift through XPM}} \underbrace{\left[\cosh(g'z) - \frac{iK'}{2g'}\sinh(g'z)\right] e^{-i\frac{K'}{2}z}}_{\text{Parametric Amplification}}$$

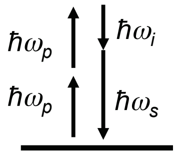
$$\begin{cases} P_s(z) = P_s(0)\left[1 + (1 + \dfrac{K'^2}{4g'^2})\sinh^2(g'z)\right] & \text{SIGNAL AMPLIFICATION} \\ P_i(z) = P_s(0)\left(1 + \dfrac{K'^2}{4g'^2}\right)\sinh^2(g'z) & \text{IDLER GENERATION} \end{cases}$$

II - Four-wave Mixing

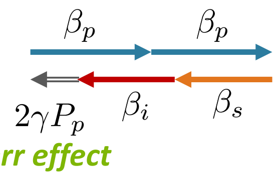
- **Degenerate FWM**

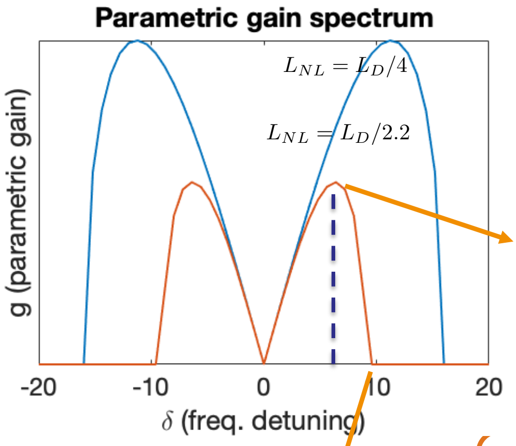
Parametric Gain

$$G(z) = 1 + \left(\frac{\gamma P_p}{g'}\right)^2 \sinh^2(g'z)$$



Phase matching condition





➜ **For n_2>0, amplification in the anomalous dispersion regime** $\beta_2 < 0$ $\gamma > 0$

$\Omega_{max}/\sqrt{2}$

Phase matching condition $\Delta\beta = |\beta_2|\Omega^2 = 2\gamma P_0$

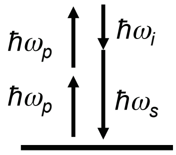
Amplification factor

$$G_{max}(z) \simeq 1 + \sinh^2(\gamma P_p z) = 1 + \sinh^2(\Phi_{NL}(z))$$

$$\Omega_{max} = \sqrt{\frac{4\gamma P_0}{|\beta_2|}}$$

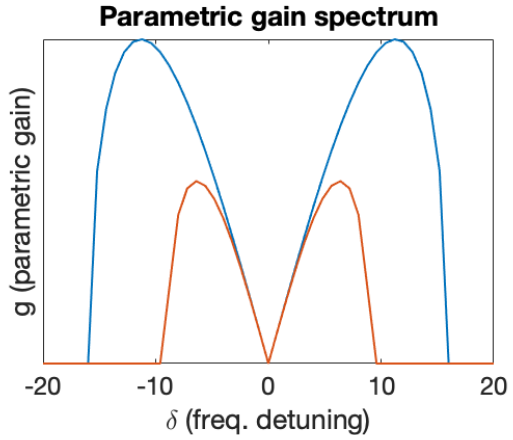
II - Four-wave Mixing

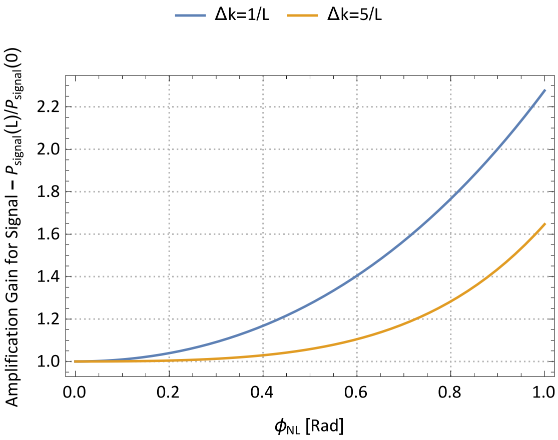
- **Degenerate FWM**



Parametric Gain

$$G(z) = 1 + \left(\frac{\gamma P_p}{g'}\right)^2 \sinh^2(g'z)$$

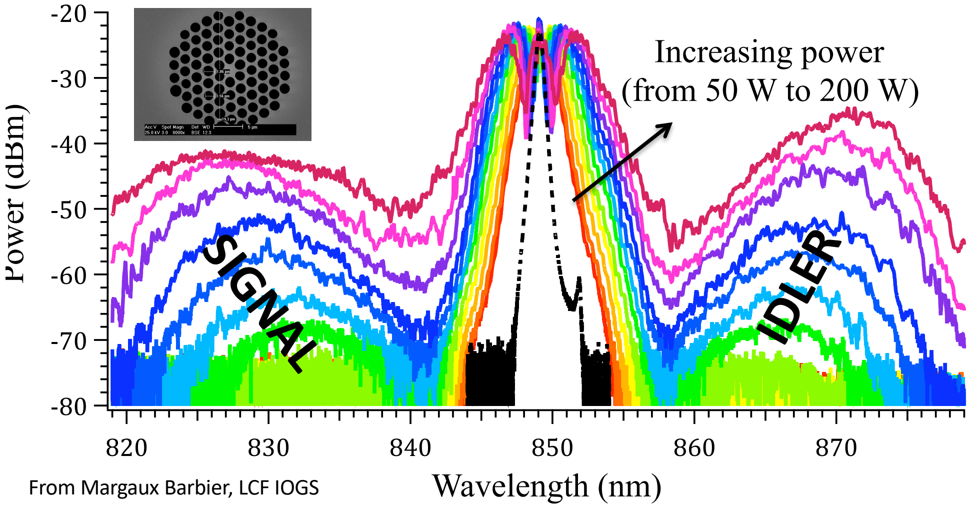




II - Four-wave Mixing

- **Degenerate FWM**

➜ ***Optical Parametric Fluorescence Effect***



From Margaux Barbier, LCF IOGS
PhD manuscript (2014)

➜ spontaneous generation of signal and idler photons : application in quantum optics (generation of photon pairs)

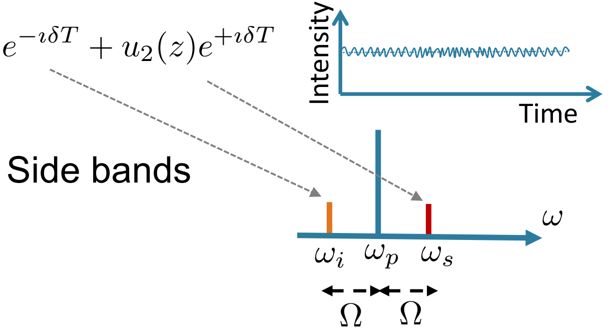
II - Four-wave Mixing

• Degenerate FWM ➜ Modulation instability

Standard model to describe the energy transfer between a CW pump and fluctuations (treated as perturbations)

Nonlinear propagation of the following wave (CW + noise) :

$$u(z,T) = u_0(z) + u_1(z)e^{-\imath\delta T} + u_2(z)e^{+\imath\delta T}$$



➜ For n_2>0 and in the anomalous dispersion regime, an amplification of the intensity fluctuations is expected (as illustrated by the following simulations)

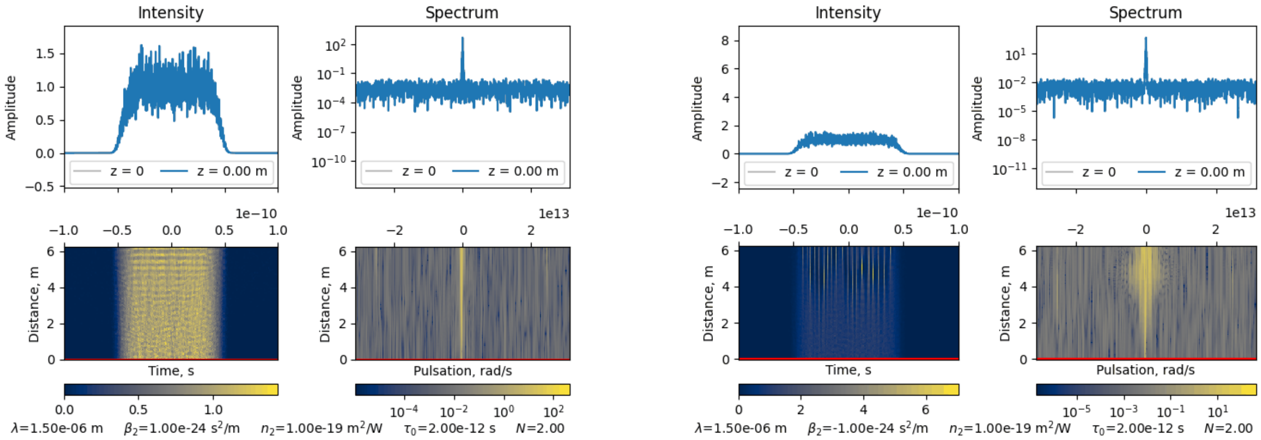
II - Four-wave Mixing

Modulation Instabilities

Evolution of a quasi-CW pulse through a 3rd order nonlinear waveguide

$\beta_2 > 0$ et $\gamma > 0$.

$\beta_2 < 0$ et $\gamma > 0$

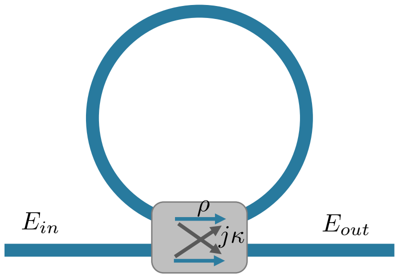


From, M. Dyatlov, LP2N - IOGS

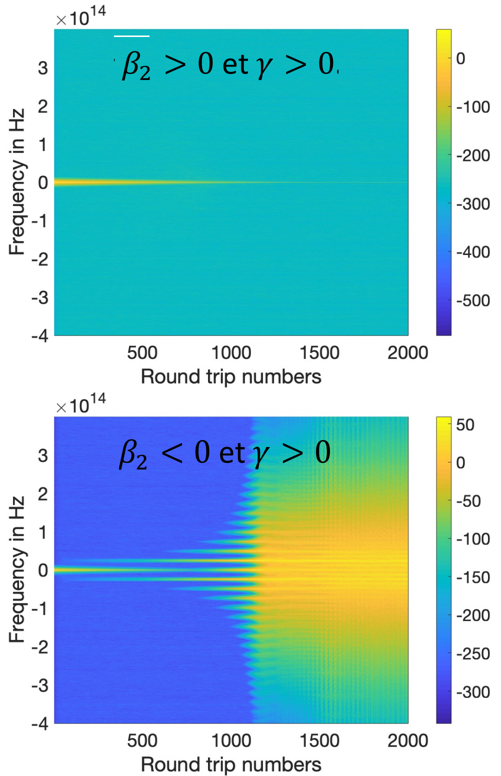
➜ Depending on the dispersion regime, nonlinear propagation can lead to modulation instability in intensities

II - Four-wave Mixing

Modulation Instabilities in microcavities

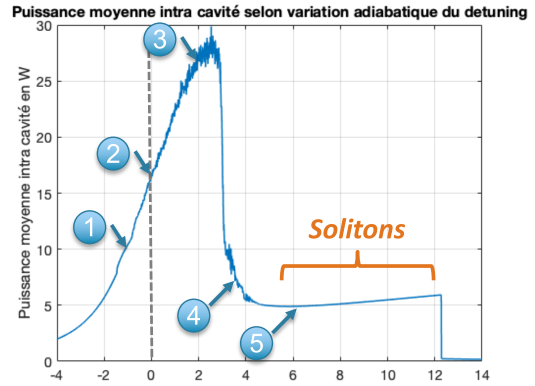


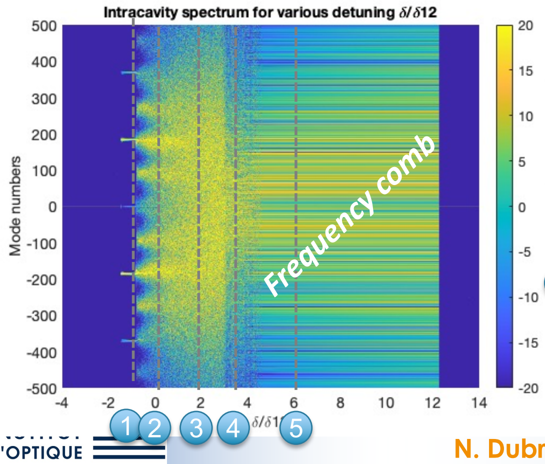
Intracavity spectrum evolution with the round trip numbers

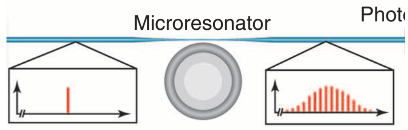


➜ **The modulation instability can lead to the generation of a periodic frequency comb**

Frequency Comb Generation in nonlinear µresonators – Numerical simulations

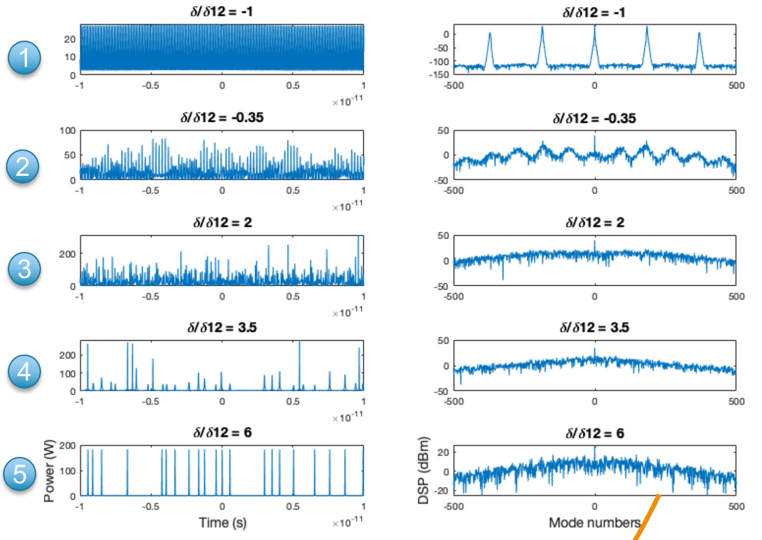






Intracavity Power dynamics

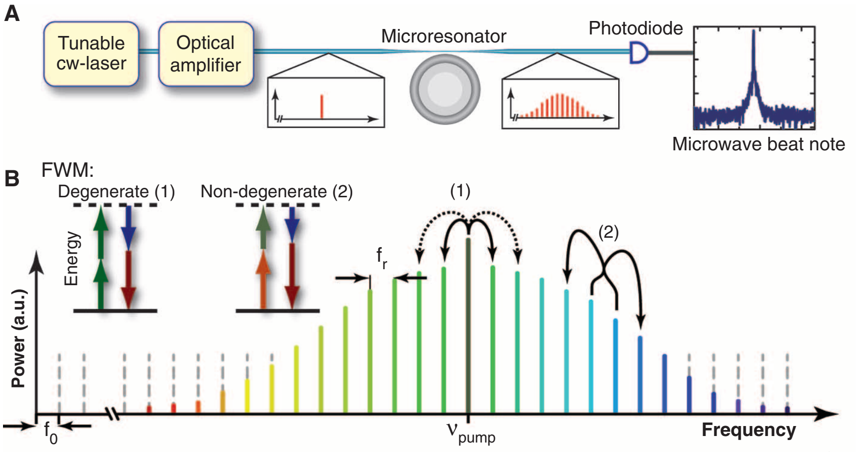
Intracavity Optical Spectra



Generation of a Frequency comb in a microresonator pumped by a CW laser

II - Four-wave Mixing

- **Generation of optical Frequency Combs in nonlinear micro-cavities**



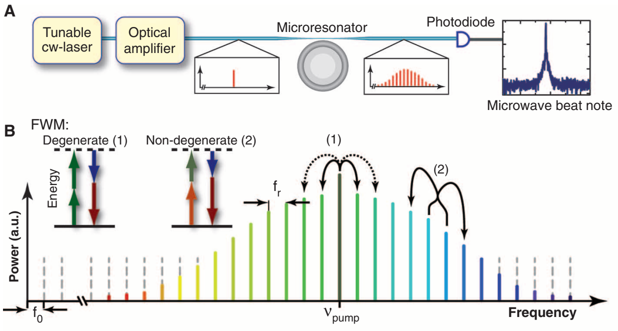
From Kippenberg, Science (2011)

Frequency comb generation in a CW pumped nonlinear µcavity

➜ Parametric Frequency conversion
  - Degenerate FWM
  - Followed by non-degenerate FWM

II - Four-wave Mixing

- **Generation of optical Frequency Combs in nonlinear micro-cavities**



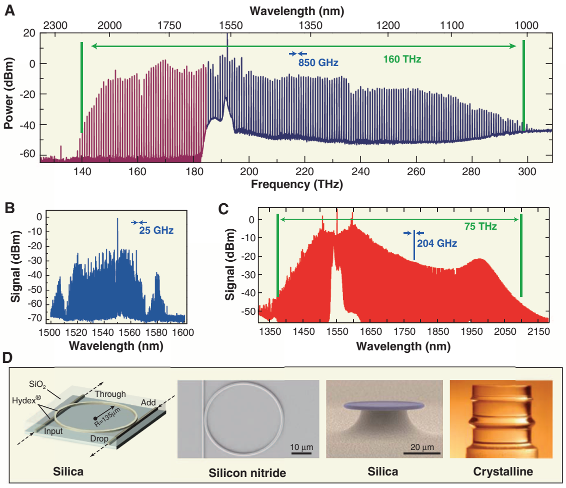
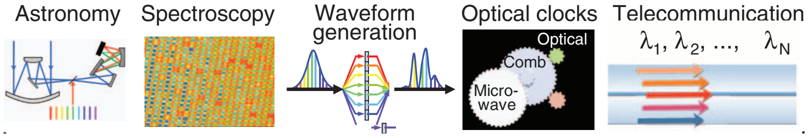


Fig. 3. Microresonator-based frequency combs. (**A**) Spectrum of an octave-spanning frequency comb generated using a silica microtoroidal resonator (*24*). (**B**) An optical frequency comb generated using a crystalline CaF_2 resonator with a mode spacing of 25 GHz (*27*). (**C**) Optical spectrum covering two-thirds of an octave (with a mode spacing of 204 GHz) generated using an integrated SiN resonator (*31*). (**D**) Experimental systems in which frequency combs have been generated (from left to right): Silica waveguides on a chip (Hydex glass) (*32*), chip-based silicon nitride (SiN) ring resonators (*30*) and waveguides, ultrahigh Q toroidal microresonators (*24*) on a silicon chip, and ultrahigh *Q* millimeter-scale crystalline resonators (*27*).

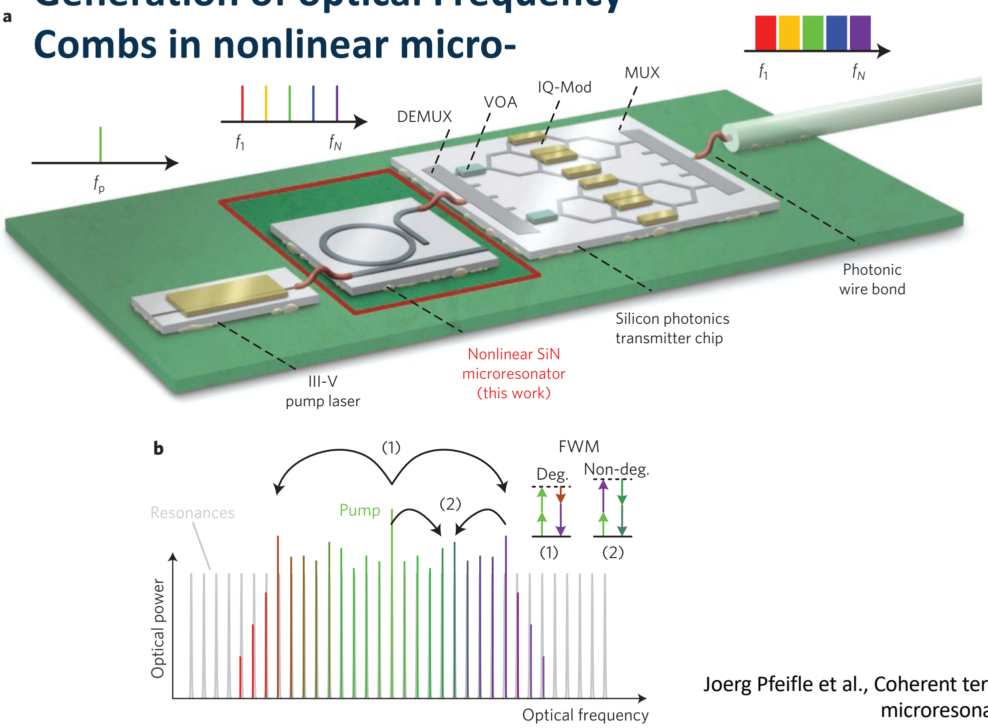
Applications :



From Kippenberg, Science (2011)

II - Four-wave Mixing

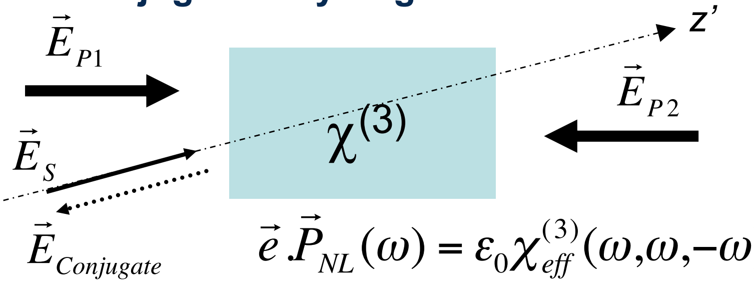
- **Generation of optical Frequency Combs in nonlinear micro-**



Joerg Pfeifle et al., Coherent terabit communications with microresonator Kerr frequency combs
NATURE PHOTONICS | VOL 8 | MAY 2014

II - Four-Wave Mixing

Phase Conjugation by Degenerate Four-Wave Mixing



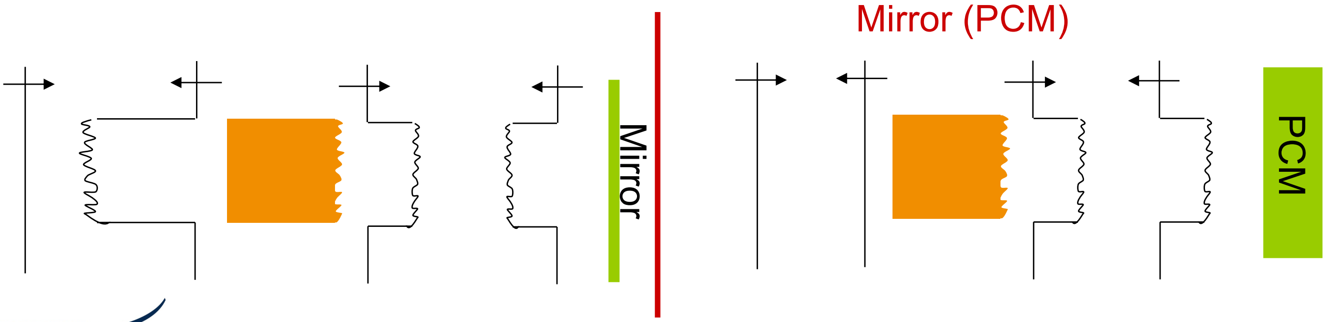
$$\vec{e}.\vec{P}_{NL}(\omega) = \varepsilon_0\chi^{(3)}_{eff}(\omega,\omega,-\omega)A_{P1}A_{P2}\boxed{A_S^* e^{-ikz'}}$$

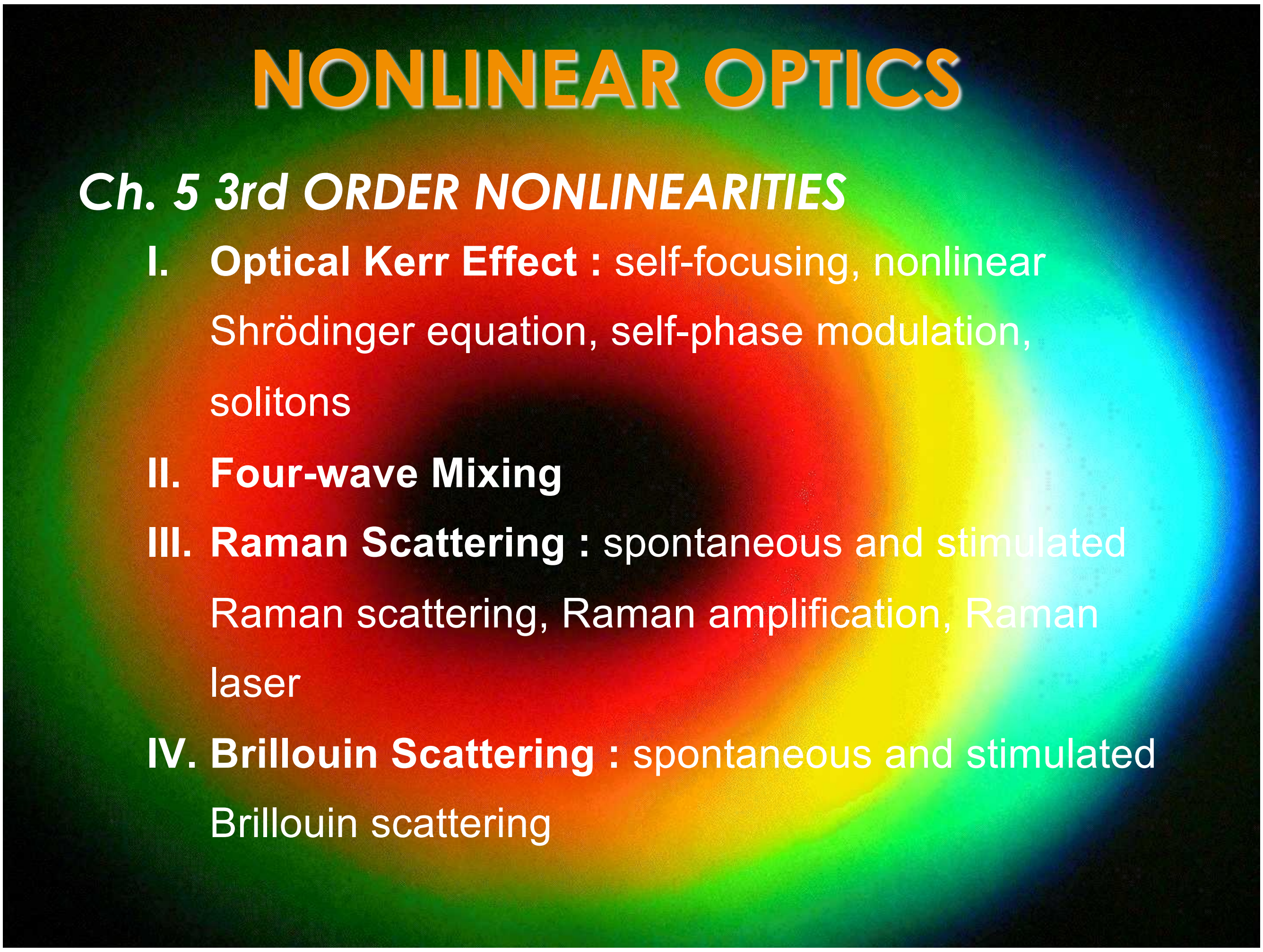
Generation of a conjugate wavefront

Automatic phase-matching with the conjugate wave

Reflection on a Mirror

Reflection on a Phase Conjugate Mirror (PCM)





NONLINEAR OPTICS

Ch. 5 3rd ORDER NONLINEARITIES

I. **Optical Kerr Effect :** self-focusing, nonlinear Shrödinger equation, self-phase modulation, solitons

II. **Four-wave Mixing**

III. **Raman Scattering :** spontaneous and stimulated Raman scattering, Raman amplification, Raman laser

IV. **Brillouin Scattering :** spontaneous and stimulated Brillouin scattering

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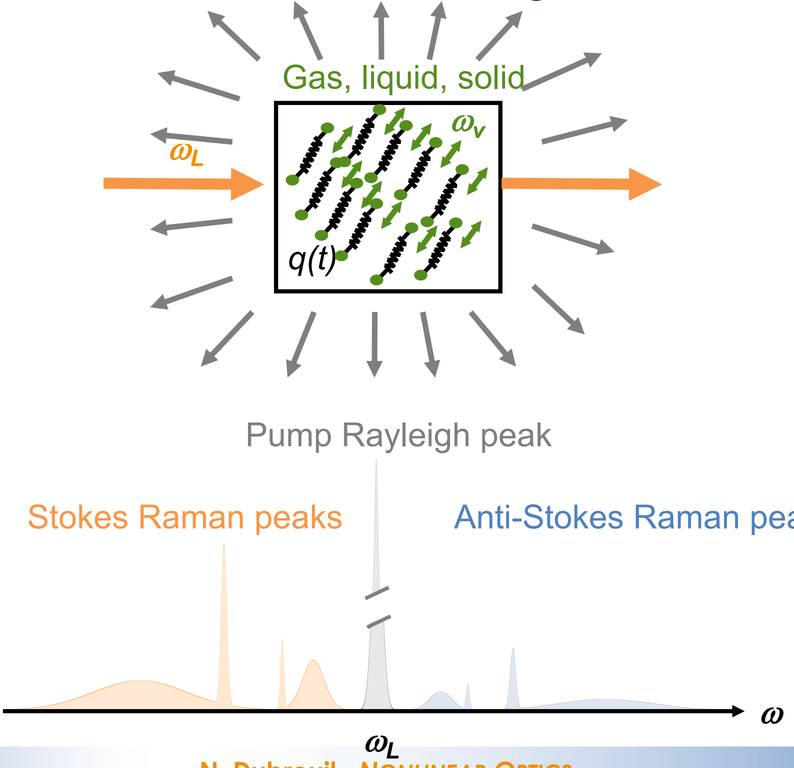
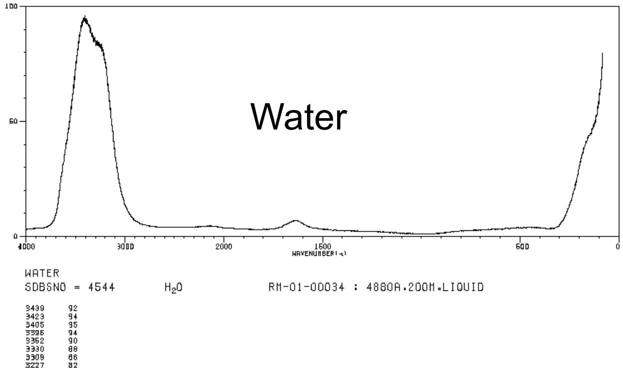
Raman Scattering

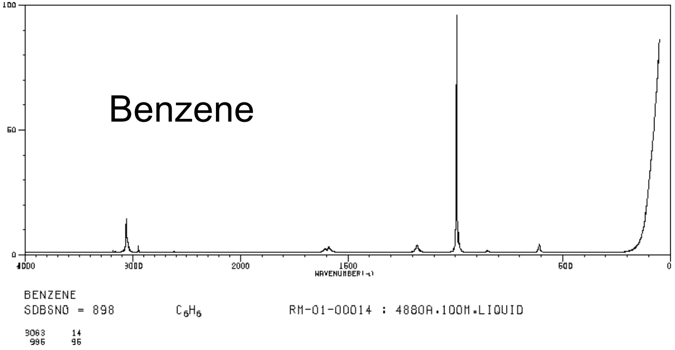
- ***Spontaneous Raman Scattering***

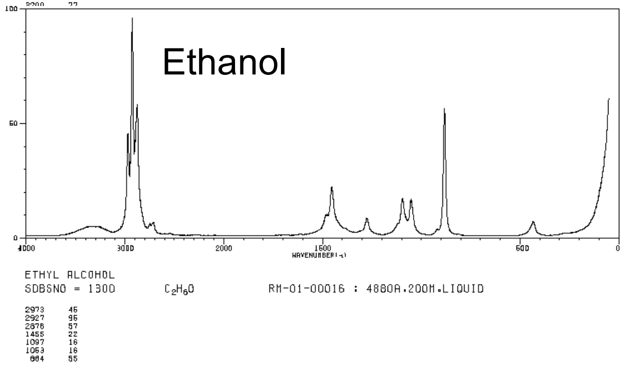


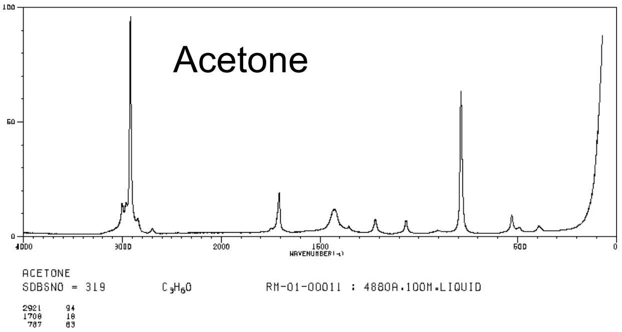
III - Raman Scattering

- *Raman spectra of various liquids*







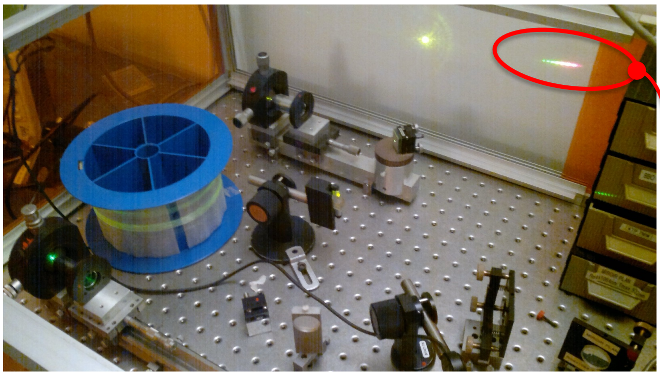


III - Raman Scattering


Laboratoire Charles Fabry de l'Institut d'Optique Equipe ELSA

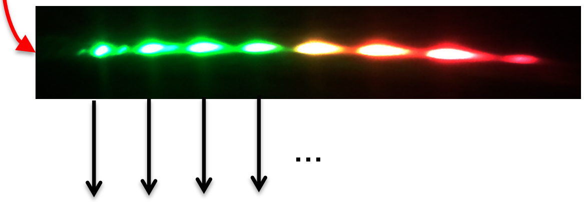
III - Raman Scattering

- *Raman scattering in a silica based optical fibre*



Output Optical Spectrum

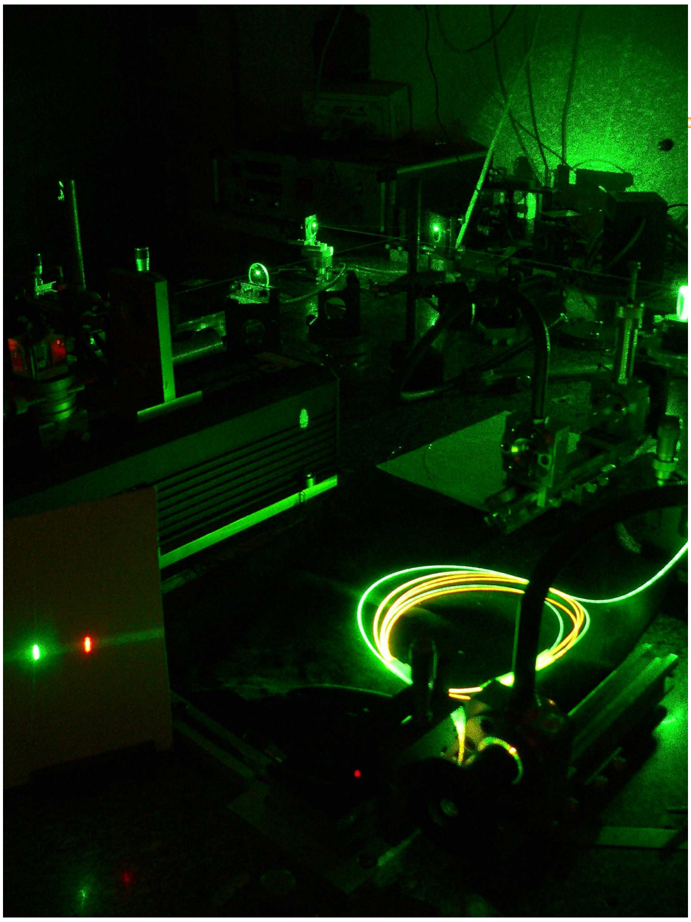
Crédit Photo : T. Claude, A. Volte (2015)



λ_{pump} λ_{s1} λ_{s2} λ_{s3} …

532 nm

Cascaded Raman Effect



Scattering

- ***Raman scattering in a hollow-core photonic crystal fibre filled with ETHANOL***
  (Sylvie LEBRUN, LCF)

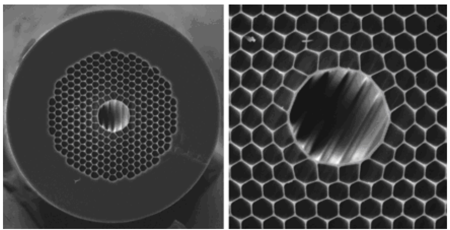
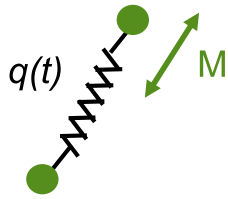


Photo : Univ. Bath

III - Raman Scattering

- ***Spontaneous Raman Scattering - Microscopic origin***



Molecule vibration

$$q - q_0 = q_1 \cos(\omega_v t)$$

Polarizability of the molecule $\alpha = \alpha_0 + \left(\frac{\partial \alpha}{\partial q}\right)_0 (q - q_0)$

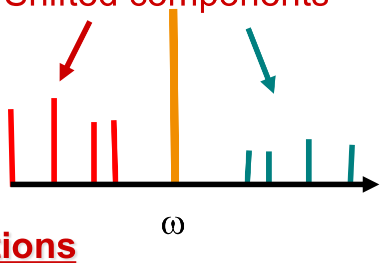
Incident monochromatic wave : $E = E_0 \cos(\omega t)$

Induced dipole : $p = \alpha\ E$ (α : polarizability of the molecule)

$p(\omega)$ $p(\omega + \omega_v)$ $p(\omega - \omega_v)$

AntiStokes **Stokes**

Shifted components

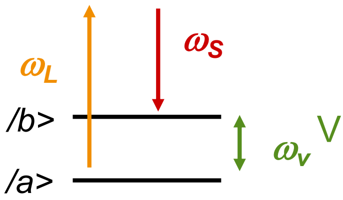


Origin of the process : <u>polarizability fluctuations</u>

III - Raman Scattering

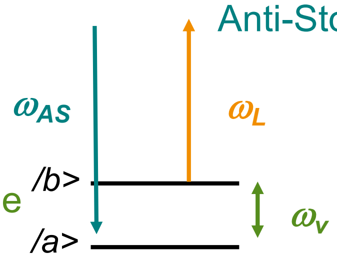
- ***Raman Scattering - Energy level diagrams***

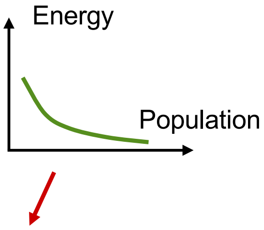
Stokes component



"optical" phonon
Vibrational mode of the molecules

Anti-Stokes component





Raman Stokes scattering :
Initial state for the medium = \a>
Creation of one Stokes photon ω_S is accompanied by the annihilation of one photon ω_L
Final state for the medium = \b>

Raman anti-Stokes scattering :
Initial state for the medium = \b>
Creation of one anti-Stokes photon ω_{AS} is accompanied by the annihilation of one photon ω_L
Final state for the medium = \a>

Boltzmann distribution

$$exp\left(\frac{-\hbar\omega_{ab}}{k_B T}\right)$$

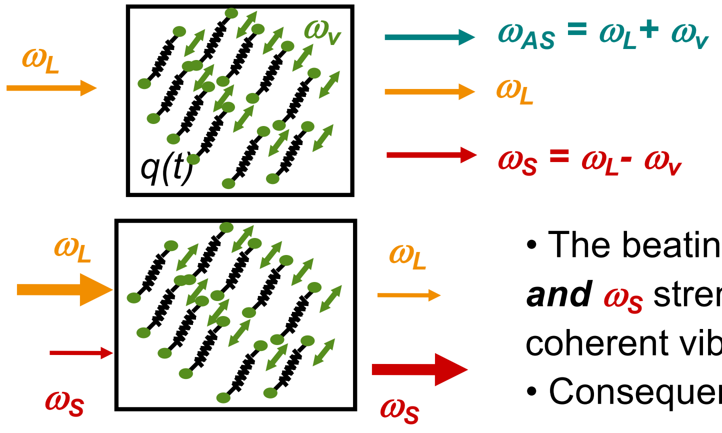
Since $\hbar\omega_{ab} \gg k_B T$

Stokes scattering is predominant

III - Raman Scattering

- ***Stimulated Raman scattering - Classical behavior***

Molecules in vibration



- The beating between the wave components ω_L ***and*** ω_S strengthens (drives) a collective and coherent vibration of the molecules @ ω_L
- Consequence : amplification of the signal @ ω_S

Energy required to drive the dipole oscillation : $W = \frac{1}{2}\langle \mathbf{p}(z,t).\mathbf{E}(z,t)\rangle$

Time average

$$\mathbf{p} = \alpha\mathbf{E} \quad \Rightarrow \quad W = \frac{1}{2}\alpha\langle \mathbf{E}^2(z,t)\rangle$$

III - Raman Scattering

- ***Stimulated Raman scattering - Classical behavior***

Driven force applied onto the oscillator :

$$F = \frac{dW}{dq} = \frac{1}{2}\frac{d\alpha}{dq}\langle \mathbf{E}^2(z,t)\rangle$$

Applied fields : $E = E_L \cos\omega_L t + E_S \cos\omega_S t$

Equation of motion of a classical harmonic oscillator :

$$\boxed{\frac{d^2q}{dt^2} + 2\gamma\frac{dq}{dt} + \omega_v^2 q = \frac{F(t)}{m}}$$

$$E(z,t) = A_L e^{i(k_L z - \omega_L t} + A_S e^{i(k_S z - \omega_S t} + CC$$

$$\frac{d^2q}{dt^2} + 2\gamma\frac{dq}{dt} + \omega_v^2 q = \frac{1}{m}\frac{d\alpha}{dq}[A_L A_S^* e^{i(Kz - \Omega t)} + CC]$$

$$K = k_L - k_S$$
$$\Omega = \omega_L - \omega_S$$

III - Raman Scattering

- ***Stimulated Raman scattering - Classical behavior***

$$\frac{d^2q}{dt^2} + 2\gamma\frac{dq}{dt} + \omega_v^2 q = \frac{1}{m}\frac{d\alpha}{dq}[A_L A_S^* e^{i(Kz-\Omega t)} + CC]$$

Driven solution of the form $q = q(\Omega)e^{i(Kz-\Omega t)} + CC$

solution $$q(\Omega) = \frac{\frac{1}{m}\left(\frac{d\alpha}{dq}\right)A_L A_S^*}{\omega_v^2 - \Omega^2 - 2i\Omega\gamma}$$

Expression of the macroscopic polarization

$$P(z,t) = N\alpha(z,t)E(z,t) = N[\alpha_0 + \left(\frac{d\alpha}{dq}\right)q(z,t)]E(z,t)$$

$$P_{NL}(z,t) = N\left(\frac{d\alpha}{dq}\right)[q(\Omega)e^{i(Kz-\Omega t)} + CC].[A_L e^{i(k_L z-\omega_L t} + A_S e^{i(k_S z-\omega_S t} + CC]$$

$$\boxed{P_{NL}(\omega_S) = \epsilon_0\chi_R^{(3)}(\omega_S;\omega_L,-\omega_L,\omega_S)|A_L|^2 A_S e^{ik_S z}}$$

III - Raman Scattering

- ***Stimulated Raman scattering - Classical behavior***

$$P_{NL}(\omega_S) = \epsilon_0\chi_R^{(3)}(\omega_S;\omega_L,-\omega_L,\omega_S)|A_L|^2 A_S e^{ik_S z}$$

$$\boxed{\chi_R^{(3)}(\omega_S) = \frac{\frac{N}{\epsilon_0 m}\left(\frac{d\alpha}{dq}\right)^2}{\omega_v^2 - (\omega_L - \omega_S)^2 + 2i(\omega_L - \omega_S)\gamma}}$$

Susceptibility expression shows that in a resonance case i.e. $\omega_v = \omega_L - \omega_S$

$\chi_R^{(3)}(\omega_S) =$ negative imaginary

III - Raman Scattering

- ***Stimulated Raman scattering***

Nonlinear polarization calculation **@ ω_S (ω_{AS}) et ω_L** solving the equation of motion of a classical harmonic oscillator **:**

$$P_{NL}(\omega_S) = \varepsilon_0\, \chi_R^{(3)}(\omega_S;\omega_L,-\omega_L,\omega_S)|A_L|^2 A_S\, e^{ik_S z}$$

$$P_{NL}(\omega_L) = \varepsilon_0\, \chi_R^{(3)}(\omega_L;\omega_S,-\omega_S,\omega_L)|A_S|^2 A_L\, e^{ik_L z}$$

Susceptibility expression shows that in a resonance case i.e. $\omega_V = \omega_L - \omega_S$

$$\chi_R^{(3)}(\omega_S) = \chi_R^{(3)}(\omega_L)^* = \text{ negative imaginary}$$

Coupled equations :

$$\begin{cases} \dfrac{\partial A_S}{\partial z} = g_R |A_L|^2 A_S & \text{Stokes wave amplification} \\[2ex] \dfrac{\partial A_L}{\partial z} = -\dfrac{\omega_L}{\omega_S} g_R |A_S|^2 A_L & \text{Pump wave depletion} \end{cases}$$

with $g_R = \dfrac{3\omega_S}{2nc}\chi_R^{(3)}(\omega_S)$

III - Raman Scattering

- ***Raman amplification :***

$$\begin{cases} \dfrac{dP_S}{dz} = -\alpha_S P_S + \dfrac{\gamma_R}{A_{eff}} P_L P_S \\[2ex] \dfrac{dP_L}{dz} = -\alpha_L P_L - \dfrac{\gamma_R}{A_{eff}} P_S P_L \end{cases}$$

with $\gamma_R = \dfrac{2\, g_R}{2nc\varepsilon_0}$ Raman Gain of the medium in m.W^{-1}

Aeff : effective mode area of the optical fiber

Solution in the undepleted pump approximation

$$P_L(z) = P_L(0)\, e^{-\alpha_L z}$$

$$\underbrace{\boxed{P_S(z) = P_S(0)\, e^{-\alpha_S z}\, \overbrace{e^{\frac{\gamma_R}{A_{eff}} P_L(0) L_{eff}}}^{\text{“ON-OFF” Gain}}}}_{\text{Net Gain}}$$

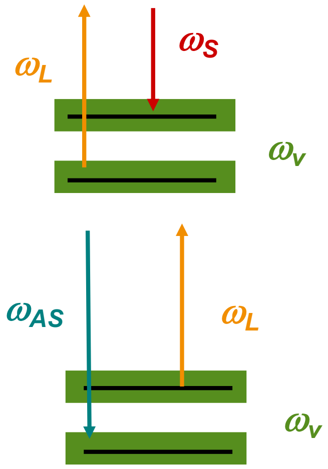
$$\text{Avec } L_{eff} = \frac{1}{\alpha_L}\left(1 - e^{-\alpha_L z}\right)$$

Effective length

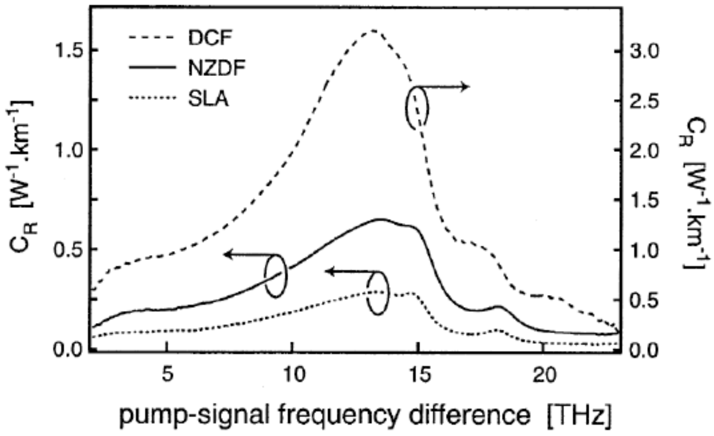
III - Raman Scattering

- *Example : Raman scattering in silica fiber*

ω_v = 13 THz



Spontaneous Raman Scattering spectra
for various doped silica



Bromage, JLT 22, 79-93 (2004)

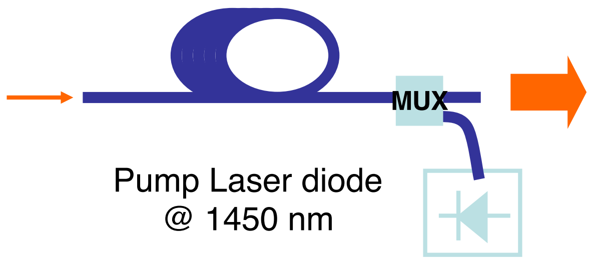
III - Raman Scattering

- ***Raman amplification :***

Structure

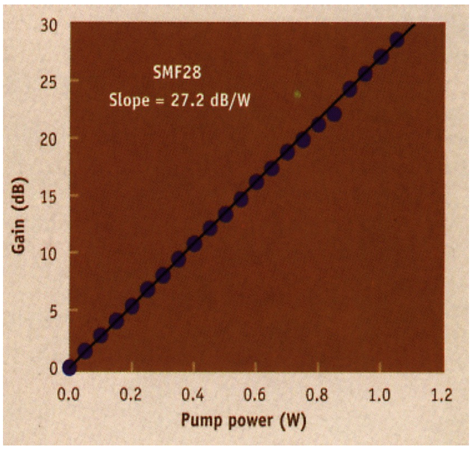
Stimulated Raman
scattering

Standard optical fiber



Pump Laser diode
@ 1450 nm

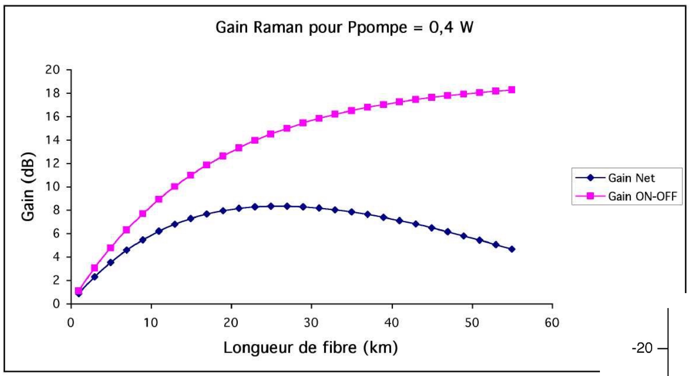
Raman gain amplification



III - Raman Scattering

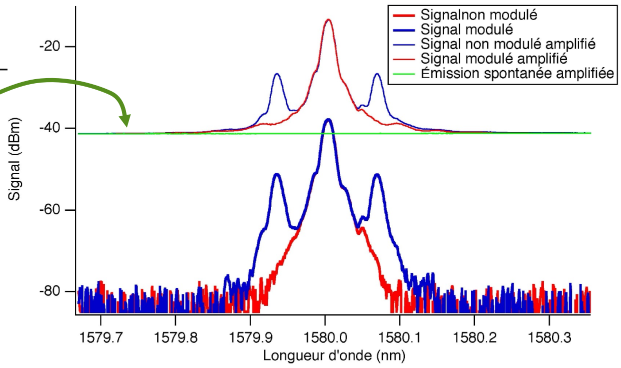
• *Raman amplification :*

Raman fiber amplification



Input and output optical spectra

Amplified spontaneous emission

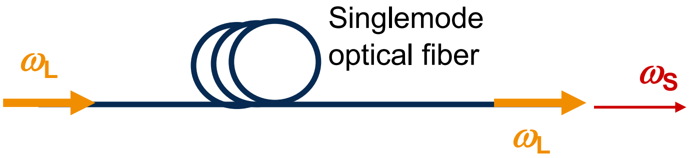


III - Raman Scattering

• *Amplified spontaneous Raman scattering*

For $P_s(0)=0$, spontaneous Raman scattering

Example :



Calculation of the number of Stokes photons created through the amplification of the photons initially created through spontaneous Raman scattering

$N_s(0)$ = 1 photon per mode

See exercise + [Agrawal, *Nonlinear fiber optics*,Ch8] + [Smith, Appl. Opt. **11**, 2489 (1972)]

Threshold condition : optical power for which $P_s(z) = P_L(z)$

For a lengthy fiber $L_{\text{eff}} \approx 1/\alpha_L \approx 20\ \text{km} \quad @1{,}55\ \mu\text{m}$

$A_{eff} = 50\ \mu m^2$

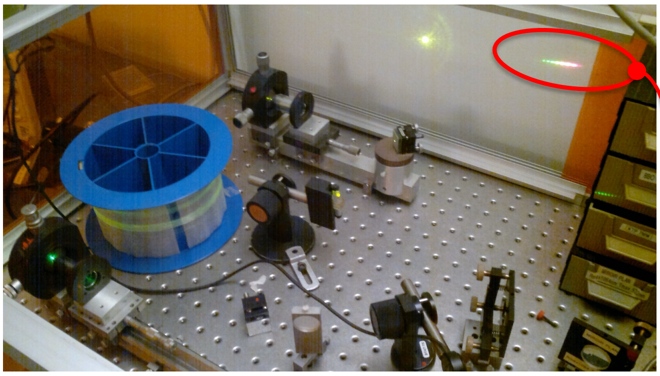
$P_{seuil} \approx 600$ mW (relatively high value)

III - Raman Scattering

Laboratoire Charles Fabry de l'Institut d'Optique Equipe ELSA

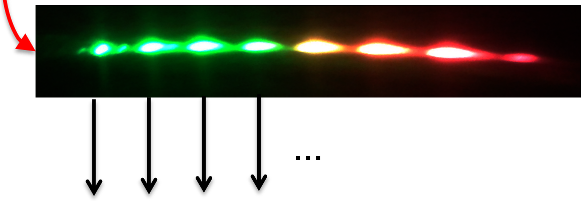
III - Raman Scattering

- ***Raman scattering in a silica based optical fibre***



Output Optical Spectrum

Crédit Photo : T. Claude, A. Volte (2015)



...

λ_{pump} λ_{s1} λ_{s2} λ_{s3} ...

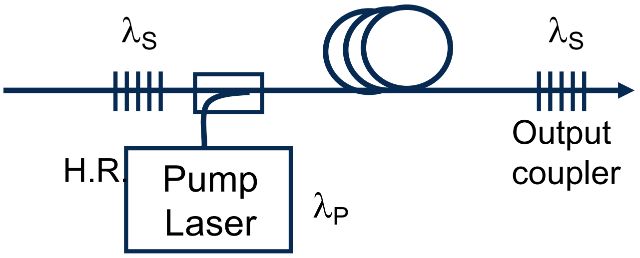
532 nm

Cascaded Raman Effect

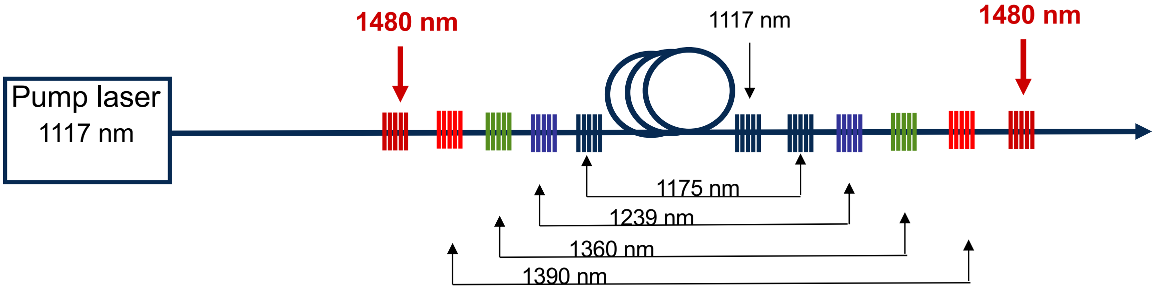
III - Raman Scattering

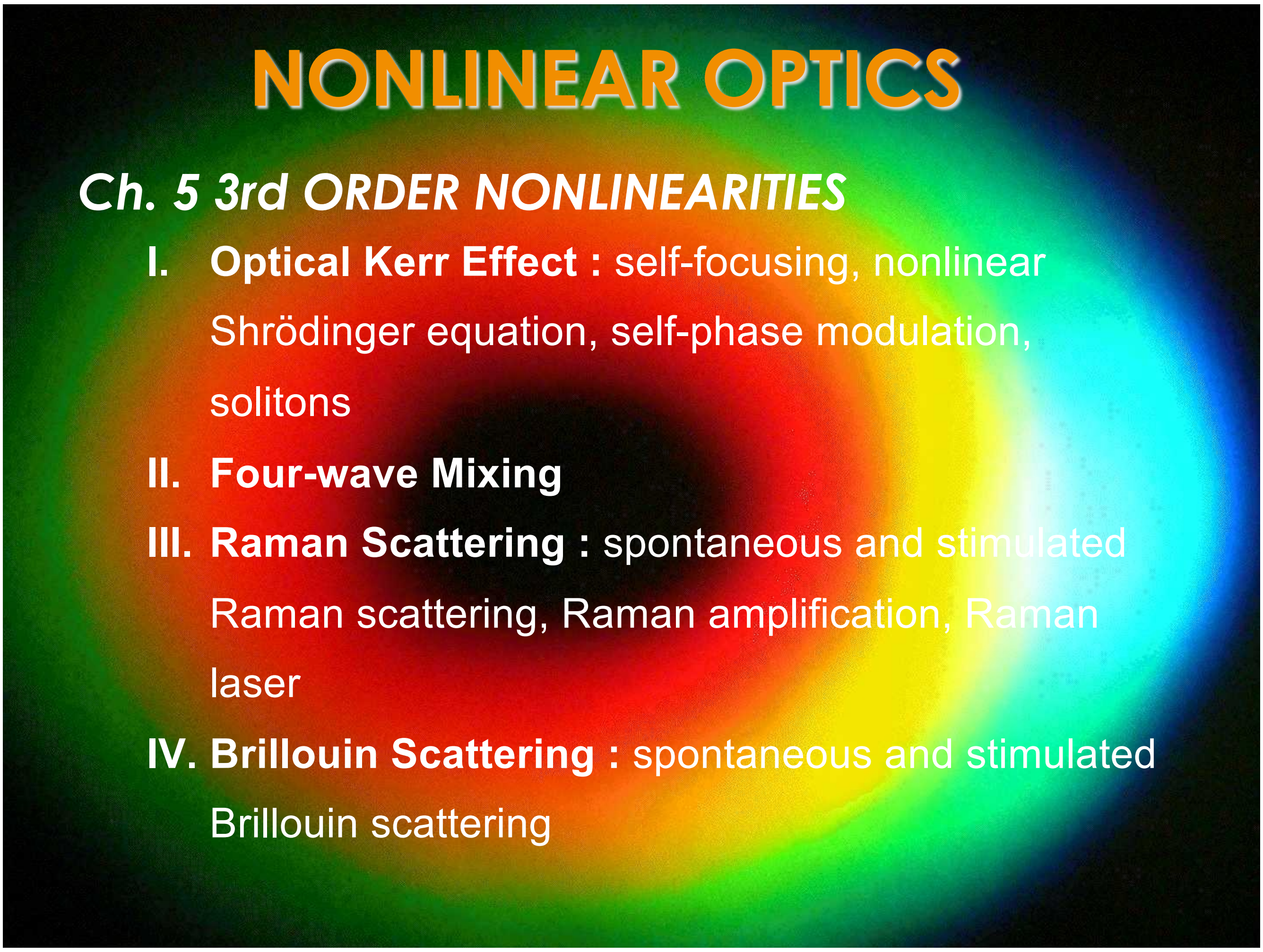
- ***Raman fiber laser***

$$\omega_v = \omega_L - \omega_S$$



Cascaded Raman fiber lasers





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II. **Four-wave Mixing**

III. **Raman Scattering :** spontaneous and stimulated Raman scattering, Raman amplification, Raman laser

IV. **Brillouin Scattering :** spontaneous and stimulated Brillouin scattering

IV - Brillouin Scattering

• *Origin of the Brillouin scattering*

Inelastic scattering due to the fluctuation of the density of the material

- presence of thermal fluctuations : spontaneous Brillouin scattering
-> scattering of light from acoustic phonons
- density fluctuations reinforced by the beating between two optical waves through electrostriction : stimulated Brillouin scattering

• *Spontaneous Brillouin scattering*

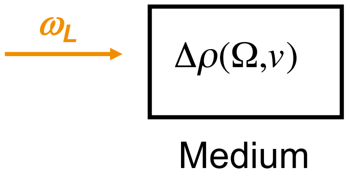
Density fluctuation

$$N = \rho_0 + \Delta\rho\, e^{i(\vec{q}.\vec{r}-\Omega t)} + c.c$$

Average density

Density fluctuation driven by a propagative sound wave

Inelastic scattering onto a sound wave



IV - Brillouin Scattering

Dispersion relation for sound waves

$$\Omega = v\,|q|$$

with *v* : sound velocity within the medium

Induced macroscopic polarization due to density fluctuations of the medium

$$\vec{P}_L = N(0,\Omega)\,\alpha\,\vec{E}_L(\omega_L)$$

$P(\omega_L)$

Phase matching condition

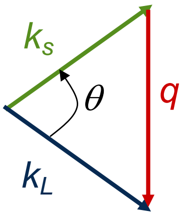
$P(\omega_S = \omega_L - \Omega)$ Stokes wave $\vec{k}_S = \vec{k}_L - \vec{q}$

$P(\omega_{AS} = \omega_L + \Omega)$ Anti-Stokes wave $\vec{k}_{AS} = \vec{k}_L + \vec{q}$

IV - Brillouin Scattering

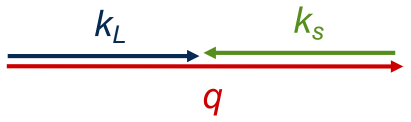
• *Typical orders of magnitude*

- amplitude of the acoustic wavevector $|q| > |k|$
- phase matching condition



$$\Omega_B = 2\frac{v}{c}\omega\sin(\theta/2)$$

The maximum frequency shift is achieved in a backscattering geometry (θ=180°)



- Brillouin frequency shift = acoustic wave frequency

$$\Omega_B \approx \frac{2\,\omega\,n\,v}{c}$$

Silica case : Ω_B = 12 GHz

- Linewidth of the Brillouin shift

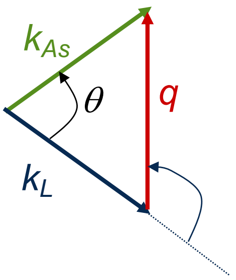
Silica : 10 MHz

IV - Brillouin Scattering

• *Phase-matching considerations*

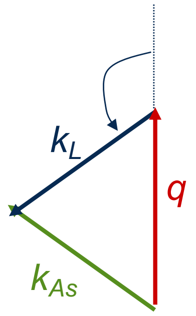
$|q| > |k|$

Anti-Stokes Scattering

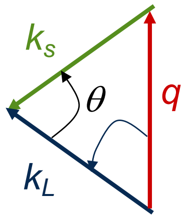


$$\vec{k}_{AS} = \vec{k}_L + \vec{q}$$

$$(\vec{k}_L, \vec{q}) > \pi/2$$

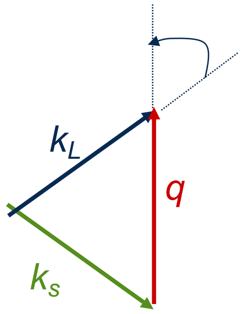


Stokes Scattering



$$\vec{k}_S = \vec{k}_L - \vec{q}$$

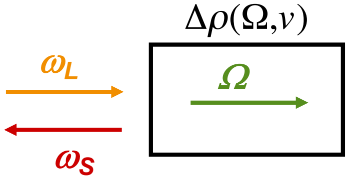
$$(\vec{k}_L, \vec{q}) < \pi/2$$



IV - Brillouin Scattering

- ***Stimulated Brillouin scattering***

Counterpropgating geometry



CW or quasi-CWregime : Resolution of the coupled optical and sound wave equations shows the following expression of the spatial intensity evolutions for pump and Stokes

For details See [Boyd, Ch.9]

$$\begin{cases} \dfrac{dI_S}{dz} = -g_B I_L I_S + \alpha I_S \\ \dfrac{dI_L}{dz} = -g_B I_L I_S - \alpha I_L \end{cases}$$

Equations similar to SRS, with a difference in the sign (contra-propa. géometry)

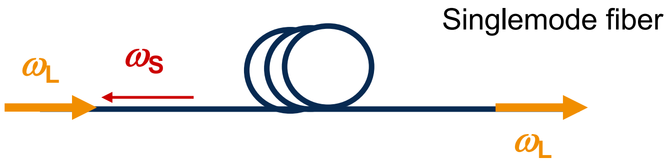
Ex. : silica $g_B = 5\ 10^{-11}$ m/W

IV - Brillouin Scattering

- ***Amplified spontaneous Brillouin scattering***

For $P_s(0)=0$, spontaneous Brillouin scattering

Example :



Calculation of the number of Stokes photons created through the amplification of the photons initially created through spontaneous Brillouin scattering

$N_s(0)$ = 1 photon per mode

Threshold condition : optical power for which $P_s(z) = P_L(z)$

For a lengthy fiber $L_{\text{eff}} \approx 1/\alpha_L \approx 20\ \text{km}$ @1,55 µm

$A_{\text{eff}} = 50\ \mu\text{m}^2$

$P_{\text{seuil}} \approx 1$ mW !!

Lower threshold than Raman process

Very easy to observe in CW regime