NONLINEAR OPTICS – LECTURE 3

Nonlinear system/material which is supposed to be TIME INVARIANT, CAUSAL and with a <u>local</u> response.

$\vec{\mathcal{E}}(t)$ EXCITATION → NL medium → $\vec{P}(t) = \vec{P}^{(1)}(t) + \vec{P}^{(2)}(t) + \vec{P}^{(3)}(t) \ldots$ RESPONSE

⇒ 1^{st} order Response Term (LINEAR Respons)

$$\boxed{\vec{P}^{(1)}(t) = \int \underline{\underline{R}}^{(1)}(t-\tau)\, \vec{\mathcal{E}}(\tau)\, d\tau}$$

linear impulse response (Matrix)

assumptions:
- Invariant in Time
- causal system ⇒ $\underline{\underline{R}}^{(1)}(t) = 0$ if $t<0$

FT

$$\boxed{\vec{P}^{(1)}(\omega) = \varepsilon_0\, \underline{\underline{\chi}}^{(1)}(\omega)\, \vec{E}(\omega)}$$

↳ LINEAR susceptibility

⇒ <u>2^{nd} order NL response</u>

$$\vec{P}^{(2)}(t) = \iint \underline{\underline{R}}^{(2)}(\tau_1, \tau_2)\, \vec{E}(t-\tau_1)\, \vec{E}(t-\tau_2)\, d\tau_1 d\tau_2$$

↳ Tensor of Rank 3

as $\vec{E}(t-\tau_1) = \int \vec{E}(\omega)\, e^{-i\omega(t-\tau_1)}\, d\omega$

after substitution:

$$\vec{P}^{(2)}(t) = \iiiint \underline{\underline{R}}^{(2)}(\tau_1, \tau_2)\, e^{i\omega_2\tau_2}\, e^{i\omega_1\tau_1}\, d\tau_1 d\tau_2\; \vec{E}(\omega_1)\, \vec{E}(\omega_2)\, e^{-i(\omega_1+\omega_2)t}\, d\omega_1 d\omega_2$$

$FT_{2D}\left[\underline{\underline{R}}^{(2)}(\tau_1, \tau_2)\right]_{\omega_1, \omega_2}$

$= \underline{\underline{\chi}}^{(2)}(\omega_1, \omega_2)$ – 2^{nd} order NL susceptibility

$$\Rightarrow \vec{P}^{(2)}(t) = \varepsilon_0 \iint \underline{\underline{\chi}}^{(2)}(\omega_1, \omega_2)\, \vec{E}(\omega_1)\, \vec{E}(\omega_2)\, e^{-i(\omega_1+\omega_2)t}\, d\omega_1 d\omega_2$$

↳ the 2^{nd} order response Term contains oscillating terms @ $\omega_1 + \omega_2$

as $\vec{P}^{(2)}(\omega) = \frac{1}{2\pi}\int \vec{P}^{(2)}(t)\, e^{i\omega t}\, dt$

$$\Rightarrow \vec{P}^{(2)}(\omega) = \frac{\varepsilon_0}{2\pi}\iint \underline{\underline{\chi}}^{(2)}(\omega_1,\omega_2)\,\vec{E}(\omega_1)\,\vec{E}(\omega_2)\,d\omega_1 d\omega_2 \underbrace{\int e^{-i(\omega_1+\omega_2)t}\, e^{i\omega t}\, dt}_{\to\ \delta(\omega-\omega_1-\omega_2)}$$

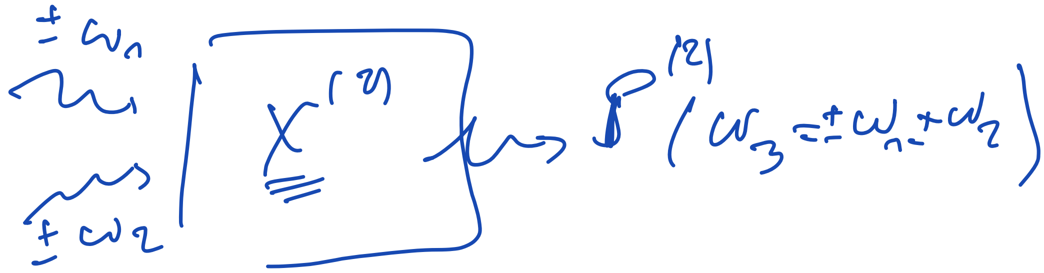
Finally

$$\boxed{P^{(2)}(\omega_3) = \varepsilon_0 \iint \underline{\underline{\chi}}^{(2)}(\omega_1,\omega_2)\,\vec{E}(\omega_1)\,\vec{E}(\omega_2)\,\delta(\omega_3-\omega_1-\omega_2)\,d\omega_1 d\omega_2}$$

CONCLUSION: The interaction between two waves @ ω_1 and ω_2 in a $\chi^{(2)}$ material generates a polarization term vibrating at $\omega_3 = \omega_1 + \omega_2$

IMPORTANT: Note that ω_1 and ω_2 can take negative values.

$\omega > 0$ or < 0


$\pm\omega_1$
$\pm\omega_2$
$\underline{\underline{\chi}}^{(2)}$
$P^{(2)}(\omega_3 = \pm\omega_1 \pm \omega_2)$