TD | Stimulated Raman Scattering (SRS) in optical Fiber

1) $\vec{e}_s \cdot \vec{P}_{NL}(\omega_s) = \varepsilon_0 \left[\vec{e}_s \cdot \underline{\underline{\chi}}^{(3)}(\omega_s; \omega_s, -\omega_s, \omega_s) \vec{e}_s \vec{e}_s \vec{e}_s \, |A_s|^2 A_s e^{ik_s z} \right.$

$\left. + 6\, \vec{e}_s \cdot \underline{\underline{\chi}}^{(3)}(\omega_s; \omega_p, -\omega_p, \omega_s) \vec{e}_p \vec{e}_p \vec{e}_s \, |A_p|^2 A_s e^{ik_s z} \right]$

$= \varepsilon_0 \left[\chi^{(3)}_{eff\,K}(\omega_s) |A_s|^2 A_s e^{ik_s z} + \chi^{(3)}_{eff\,R}(\omega_s) |A_p|^2 A_s e^{ik_s z} \right]$

Accounting that $|A_p|^2 \gg |A_s|^2$

and $\chi^{(3)}_{eff\,R} \gg \chi^{(3)}_{eff\,K}$

$\hookrightarrow$ Kerr (non resonant)

Resonant interaction (Raman contribution + Kerr

$$\Rightarrow \boxed{\vec{e}_s \cdot \vec{P}_{NL}(\omega_s) \simeq \varepsilon_0 \chi^{(3)}_{eff\,R}(\omega_s) |A_p|^2 A_s e^{ik_s z}}$$

Similarly

$$\boxed{\vec{e}_p \cdot \vec{P}_{NL}(\omega_p) \simeq \varepsilon_0 \chi^{(3)}_{eff\,R}(\omega_p) |A_s|^2 A_p e^{ik_p z}}$$

$\rightsquigarrow$ Non linear wave Eq @ ω_s

$$\boxed{\frac{\partial A_s}{\partial z} = \frac{i\omega_s}{2nc} \chi^{(3)}_{eff\,R}(\omega_s) |A_p|^2 A_s}$$

- phase matching: automatically satisfied $\Delta k = 0$
- For a resonant interaction, i.e. $\omega_p = \omega_s + \Omega$ one can show that $\chi^{(3)}_{eff\,R}(\omega_s) = i \chi^{(3)''}_{eff\,R}$

Negative imaginary — Negative

2) $I_s = 2nc\varepsilon_0 |A_s|^2 \Rightarrow \frac{\partial I_s}{\partial z} \propto A_s \frac{\partial A_s^*}{\partial z} + A_s^* \frac{\partial A_s}{\partial z}$

$$\Rightarrow \boxed{\frac{\partial I_s}{\partial z} = -\frac{\omega_s \chi^{(3)''}_{eff\,R}}{2 n_p n_s c^2 \varepsilon_0} I_p I_s = g_R I_p I_s}$$

Amplification of the STOKES signal @ ω_s.

3) ω_p, ω_s, Ω — For each generated STOKES photon $\Rightarrow$ one annihilated PUMP

$\Rightarrow \frac{dN_s}{dz} = -\frac{dN_p}{dz} \Rightarrow \frac{d(I_p/\hbar\omega_p)}{dz} = -\frac{d\frac{I_s}{\hbar\omega_s}}{dz}$

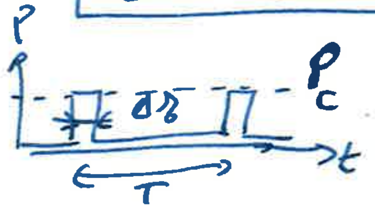
$$\Rightarrow \boxed{\frac{dI_p}{dz} = -\frac{\omega_p}{\omega_s} g_R I_s I_p}$$

Raman Amplification

4) $\nu_s = \nu_p - \Omega \quad (\hookrightarrow \text{Hz}) \quad \Rightarrow \quad \frac{1}{\lambda_s} = \frac{1}{\lambda_p} - \frac{\Omega}{c}$

$\Rightarrow \boxed{\lambda_s = 544{,}75\ \text{nm}}$

5)



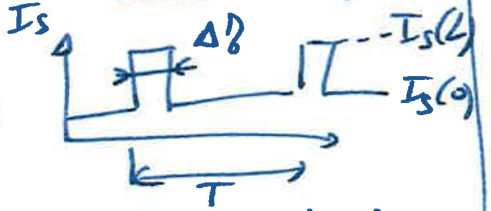
$\Rightarrow \boxed{I_c = \frac{P_c}{A_{eff}} = \frac{T\langle P\rangle}{\Delta z\, \frac{\pi\phi^2}{4}}}$

$I_c \simeq 32\ \text{MW/cm}^2$

ϕ: Mode Field diameter

6) $I_s(L) = I_s(0)\, \underbrace{e^{g_R I_p L}}_{G_R} \quad \Rightarrow \quad \boxed{G_R = 24}$

At the output of the Fiber $\rightarrow$

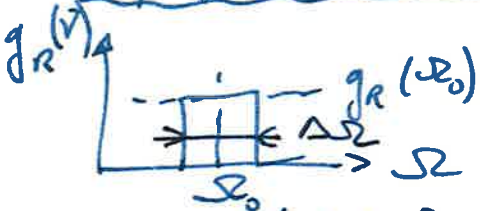


Average Output Power

$\Rightarrow \langle P_s\rangle = \langle P_s(0)\rangle + I_s(L)\frac{\Delta z}{T}A_{eff}$

$= \langle P_s(0)\rangle\left[1 + \underbrace{G_R \frac{\Delta z}{T}}_{1{,}2\%!!}\right]$

Amplified Raman Scattering: Threshold



$dI_s(0) = \frac{h\nu}{\pi\phi^2/4}\, d\nu$

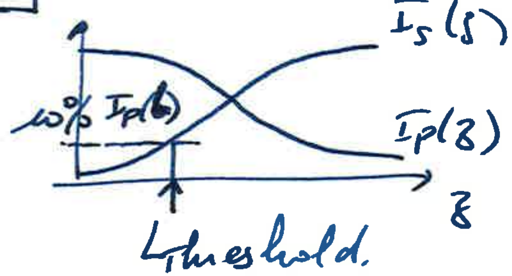
7) $I_s = \int I(\nu)\, d\nu = \int I_s(0, d\nu)\, G_R(\nu)\, d\nu$

$I_{s\,TOT}(L) = \frac{G_R}{\pi\phi^2/4}\int_{\nu_p - \Omega_0 - \frac{\Delta\Omega}{2}}^{\nu_p - \Omega_0 + \Delta\Omega/2} h\nu\, d\nu$

$\Rightarrow \boxed{P_{s\,TOT}(L) = G_R h\nu_s \Delta\Omega}$

8) $\frac{dI_p}{dz} = -\frac{\omega_p}{\omega_s}\frac{dI_s}{dz}$

$\frac{dI_s}{dz} = g_R I_p I_s$



$I_s = \frac{h\nu}{\pi\phi^2/4}\, G_R \Delta\Omega = 0{,}1\ I_p(L)$

Now $I_p(L) - I_p(0) = -\frac{\omega_p}{\omega_s}\left(I_s(L) - I_s(0)\right)$

$\hookrightarrow 10\, I_s(L)$

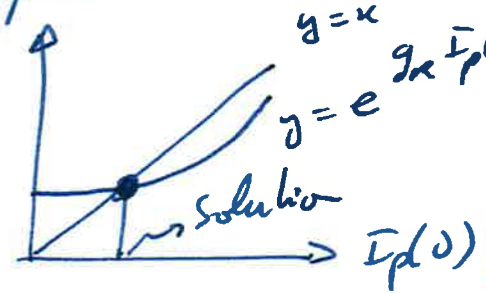
and considering that $N_p(0) \gg N_s(0)$

$\Rightarrow I_p(0) \gg \frac{\omega_p}{\omega_s} I_s(0)$

$\Rightarrow \left[10 + \frac{\omega_p}{\omega_s}\right] I_s(L) \simeq I_p(0)$

$\Rightarrow \boxed{\left[10 + \frac{\omega_p}{\omega_s}\right]\frac{h\nu_s \Delta\Omega}{\pi\phi^2/4}\, e^{g_R I_p(0) L} \simeq I_p(0)}$

Graphical Solution



$\boxed{\begin{array}{l} I_p(0) \simeq 130\ \text{MW/cm}^2 \\ \hookrightarrow I_s(L) \simeq 12\ \text{MW/cm}^2 \end{array}}$