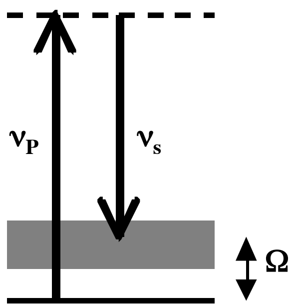
Raman Scattering Effect in an Optical Fiber

The stimulated Raman scattering (SRS) effect corresponds to the conversion of a laser beam at ν_P to a beam whose frequency is shifted to $\nu_S = \nu_P - \Omega$ (Stokes wave), where Ω is associated to a vibrational transition in the medium. In particular in optical fibers, due to their amorphous structure, the vibrational transitions of the silica SiO_2 are spectrally large, leading to an amplification range around $\Omega_0 = 13{,}2$ THz with a total width of $\Delta\Omega \cong 20$ THz.



I. Coupled equations of the stimulated Raman scattering

The stimulated Raman scattering corresponds to the coupling between the pump and the signal waves, at the frequencies ν_P and ν_S, such that $\nu_S = \nu_P - \Omega$ and Ω coincides with the Raman shift for silica. We have shown during the course that this interaction is governed by a 3rd-order nonlinear susceptibility that takes the form[1] :

$$\chi_R^{(3)}(\omega_s) = \frac{2\Omega\gamma\,\chi_{R0}^{(3)}}{\Omega^2 - \left(\omega_p - \omega_s\right)^2 + 2i\left(\omega_p - \omega_s\right)\gamma}$$

where γ is the related to Raman transition linewidth.

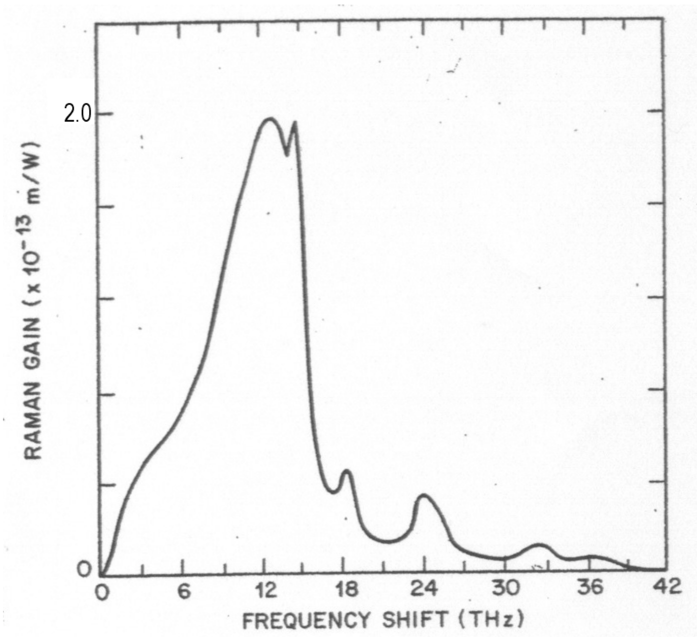
1. Write down the equation of evolution of the signal wave during its propagation in the fiber. Show in particular that the phase matching condition is automatically fulfilled, whatever is the material.
2. Considering a resonant situation, i.e. $\nu_S = \nu_P - \Omega$, what is the expected evolution for the signal ? Derive a relation for the light intensity variation I_S at ν_S along the direction of propagation. This relation will be given in terms of a Raman gain coefficient g_R to be defined.
3. Using the energy conservation, determine the equation of evolution of the pump intensity I_P.

II. Raman amplifier

We consider a silica fiber with a length of L = 50 m, it has a core diameter of ϕ =4 μm, and it is pumped at λ_P = 532 nm by an intense laser beam of 2 mW average power, a repetition rate of 10 kHz and a pulse duration of 50 ns. We neglect the attenuation of the pump wave during the propagation in the fiber. The maximum Raman gain in the silica is $g_R = 2 \times 10^{-13}$ m/W. The spectral dependence of the Raman gain of a silica fiber is depicted in the figure below. [Agrawal, Non-linear Fiber Optics].

[1] Refer to the demonstration based on a classical harmonic oscillator (see lecture slides on 3rd order nonlinearities)

4. For which wavelength λ_S of the signal wave is the Raman gain maximal?



5. Calculate numerically the peak intensity of the pump beam, which is injected into the fiber.

6. We consider a continuous (cw) signal beam being injected into the fiber, having a relatively weak power of $P_S(0)$ = 0,1 mW at λ_S. By neglecting the attenuation of the pump due to the Raman scattering, find an expression for the power of the generated wave at λ_S at the end of the fiber L. What temporal dependence shows the beam at λ_S at the exit of the fiber.

III. Threshold of the spontaneous Raman scattering

Hereafter, we do not inject a beam at λ_S into the fiber. We want to show that, above a certain pump power at the entrance of the fiber, a spontaneous conversion of the pump photons to the Stokes photons takes place. The so-called *spontaneous Raman scattering* can be described as the amplification of a fictious photon in the frequency range [ν;ν+dν], i.e. we can describe the spontaneous Raman scattering by the amplification of a beam with the intensity:

$$\mathrm{dI_s}(0) = \frac{h\nu}{\pi\phi^2/4}d\nu$$

in the spectral range [ν;ν+dν].

7. For simplification, we approximate the spectral dependance of the Raman gain $g_R(\Omega)$ by a square function with a constant gain $g_R(\Omega_0)$ over the width $\Delta\Omega$ = 10 THz. Find the expression of the total power of the Stokes wave (around $\nu_S = \nu_P - \Omega$) which is generated by spontaneous Raman scattering at the output fiber of length L.

8. The Raman threshold refers to the input pump intensity $I_P(0)$ that is necessary to generate at the fiber output a Stokes intenseity $I_S(L)$ equal to 10% of the intensity of the pump $I_P(L)$. Show that $I_P(0)$ is the implicit solution of an analytical expression that can be solved graphically.

9. Explain why, for the pump intensities above the previous threshold value, the spectrum of the light at the exit of the fiber exhibits, besides a peak at 532 nm, a large peak around 545 nm, a second one around 558 nm, etc …