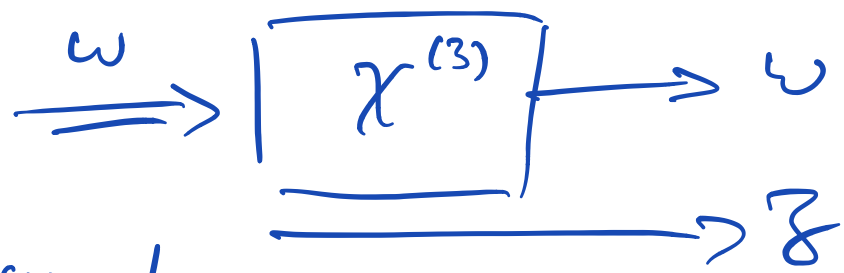
OPTICAL KERR EFFECT



Assumptions:

↳ loss less material $\chi_{eff}^{(3)} \in \mathrm{Re}$

↳ consequence: No absorption.

↳ non resonant interaction

$\Rightarrow$ $\chi^{(3)}$ = constant with ω

$\chi^{(3)}$ does not vary with ω

$\Rightarrow$ Instantaneous Nonlinear Response

Degeneracy Factor

$$\vec{P}_{NL}(\omega) = 3\,\varepsilon_0\,\chi^{(3)}(\omega,-\omega,\omega)\,\vec{e}\,\vec{e}\,\vec{e}\;E(\omega)E(-\omega)E(\omega)$$

with $\vec{E}(\omega) = \vec{e}\,A(\omega)\,e^{ikz}$

Total Polarization in the Material @ ω

$$\vec{P}(\omega) = \vec{P}^{(1)}(\omega) + \vec{P}^{(3)}(\omega)$$

$$= \varepsilon_0 \left[\underline{\underline{\chi}}^{(1)} \vec{e} + 3 \underline{\underline{\underline{\underline{\chi}}}}^{(3)} \vec{e}\,\vec{e}\,\vec{e}\, |E(\omega)|^2 \right] E(\omega)$$

$$\boxed{\underset{\Sigma}{\vec{e}} \cdot \vec{P}(\omega) = \varepsilon_0 \left[\underbrace{\chi_{eu}^{(1)}} + \underbrace{3 \chi_{eu}^{(3)} |A|^2} \right] A e^{ikz}}$$

linear suscept.

$\Delta \chi^{(1)}$ Modif. of the linear suscept. $\propto$ Intensity

Because we have assumed a loss less material $\Rightarrow$ $\begin{cases} \chi_{eu}^{(1)} \in \mathbb{R} \\ \chi_{eu}^{(3)} \in \mathbb{R} \end{cases}$

$\Rightarrow$ Modification of the refractive index of the material proportionally to the wave intensity

This effect refers to **OPTICAL KERR EFFECT**

$$n_0(\omega) = \sqrt{1 + \chi_{eff}^{(1)}(\omega)}$$

$$\Rightarrow n^2(\omega) = 1 + \chi_{eff}^{(1)}(\omega) + 3\chi_{eff}^{(3)}|A|^2$$

now the wave intensity

$$I = 2nc\varepsilon_0|A|^2$$

$$\Rightarrow n^2(\omega) = n_0^2(\omega) + \frac{3\chi_{eff}^{(3)}}{2n_0\varepsilon_0 c} I$$

$$= n_0^2\left[1 + \underbrace{\frac{3\chi_{eff}^{(3)}}{2n_0^3\varepsilon_0 c} I}_{\varepsilon \ll 1}\right]$$

Small perturbation

$$\Rightarrow \boxed{n_{(\vec{r})} \simeq n_0 + n_2 \, I(\vec{r})}$$

Nonlinear refractive Index $\quad n_2 \left[\frac{m^2}{W} \right]$

with
$$\boxed{n_2 = \frac{3 \chi^{(3)}_{ee}}{4 n_0^2 \varepsilon_0 c}}$$

Ex: Silica $\begin{cases} n_0 \simeq 1{,}45 \\ n_2 = 3 \; 10^{-20} \, m^2/W \, !! \end{cases}$

CONSEQUENCES ?

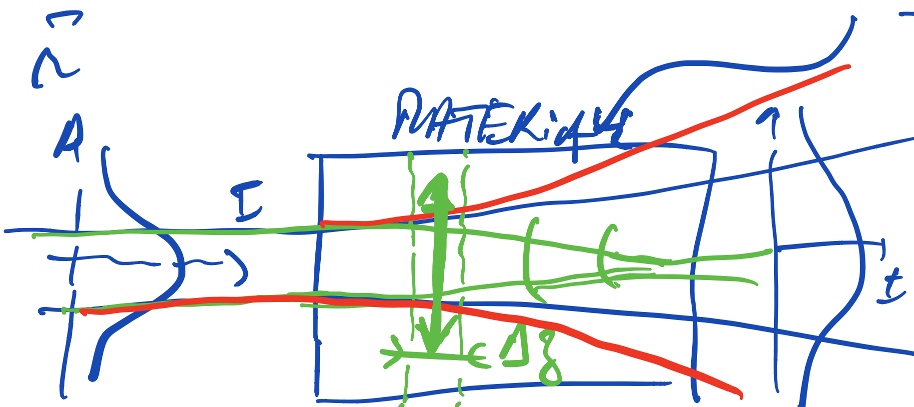
↳ Along the propagation, an optical beam is able to modify the refractive index of the material proportionally to its Intensity

$\Longrightarrow$ Phase shift $\propto I$

↳ Propagation with "wave packet"

in TIME Domain = **PULSE**

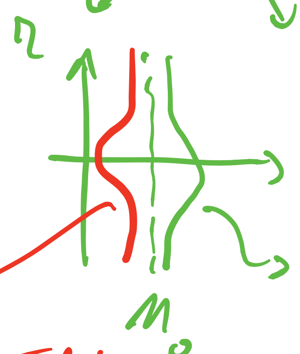
SPATIAL domain = **BEAM**



LINEAR Regime

optical Diffraction

NL Regime



$n_2 > 0$

$\Delta n(r) = n_2 I(r)$

$n_2 < 0$

$\Delta n(r) = n_2 I(r)$

SELF-DEFOCUSING EFFECT

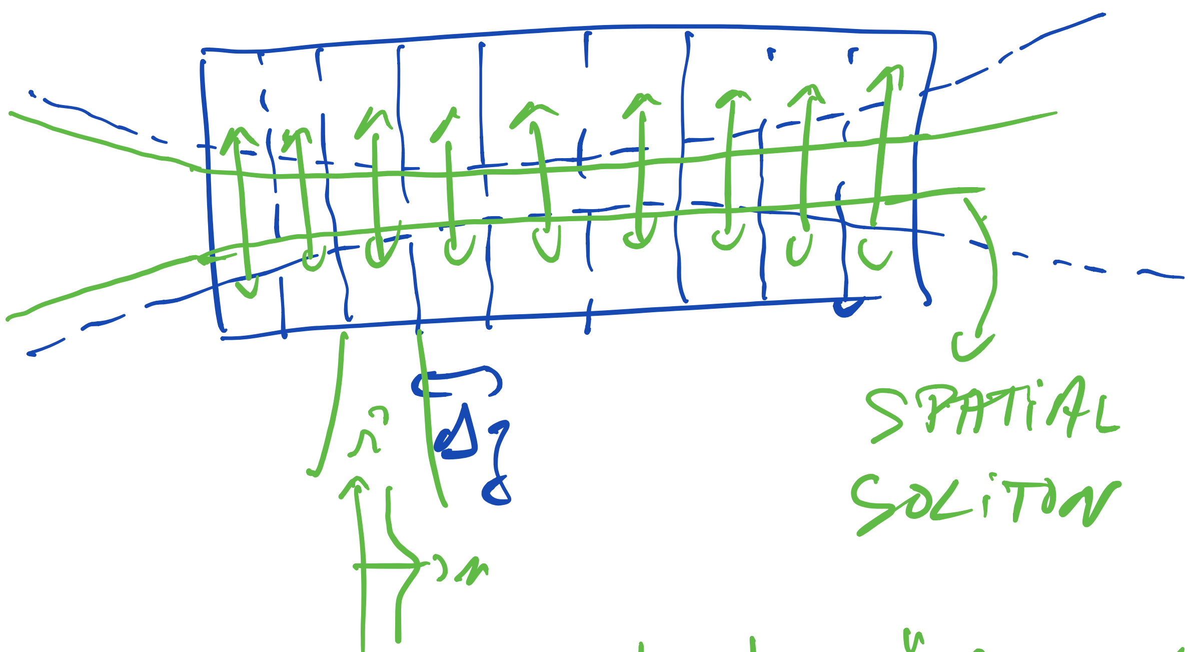
⇒ **SELF-FOCUSING EFFECT**

KERR-LENS EFFECT

↳ STRICT Compensation between DIFFRACTION and SELF-FOCUSING EFFECTS

⇒ Beam propagates <u>without</u> any deformation.

⇒ $\boxed{\text{SOLITON EFFECT}}$



⇒ equivalent to a "waveguide" (a gradient index waveguide)

⊗ ‖ ⇒ the gradient index is recorded by the Beam itself.

SELF-Phase Modulation Effect

Hereafter, we consider a wave packet = | spatial beam
or Temporal pulse.

described by:

$$\vec{\psi}_0(\xi,z) = \vec{e}\,A(\xi,z)e^{-i(\omega_0 t - kz)} + cc$$

where $\xi = \vec{r}$ or t
space ↳ time

$$\vec{S}_{NL}(\xi,z) = 3\varepsilon_0\,\underline{\underline{\underline{\chi}}}^{(3)}\vec{e}\vec{e}\vec{e}\,|A(\xi)|^2 A(\xi)e^{-i(\omega_0 t - kz)} + cc$$

Assuming an instantaneous Nonlinear Response.

Nonlinear Wave Equation:

$$\boxed{\frac{\partial A}{\partial z} = \frac{i\omega}{2nc}\, 3\chi^{(3)}_{yyyy}\, |A(\xi)|^2 A(\xi)}$$

COMMENT: the phase-matching is automatically fulfilled with optical Kerr Effect.

Now $I = 2nc\varepsilon_0 |A|^2$

$$\boxed{\frac{\partial A}{\partial z} = i k_0 n_2 I(\xi, \cancel{z}) A} \quad (1)$$

with $n_2 \in \mathbb{R}$

$$\frac{\partial |A|^2}{\partial z} = A^* \frac{\partial A}{\partial z} + A \frac{\partial A^*}{\partial z} = 0$$

$\Rightarrow$ $I(\xi, z)$ does not vary with z

Integration of (1) is straightforward

à re $\Rightarrow$
$$\boxed{A(\xi, z) = A(\xi, 0)\, e^{\overbrace{i k_0 n_2 I(\xi) z}^{}}} \quad (2)$$

Nonlinear Phase shift $\propto I(\xi)$

$$\phi_{NL}(\xi) = k_0 n_2 I(\xi) L$$

$\Rightarrow$ SELF-PHASE Modulation effect

COMMENTS

① $|A(\xi, z)| = |A(\xi, 0)|$

$\Rightarrow$ Intensity distribution is $c^{\underline{st}}$

$\Rightarrow$ No modification of the intensity distribution of the wave packet

② The fact that $\phi_{NL}(I)$ implies a spectral Broadening effect

$\Rightarrow$ See TUTORIAL

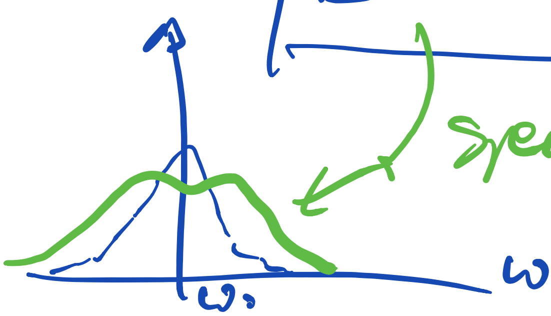
Ex: pulse

$$e^{i(\omega_0 t + \varphi_{NL}(t))} \simeq e^{i(\omega_0 t + \frac{\partial \varphi_{NL}}{\partial t} t + \dots)}$$

$$\Delta\omega = -\frac{\partial \phi_{NL}}{\partial t}$$

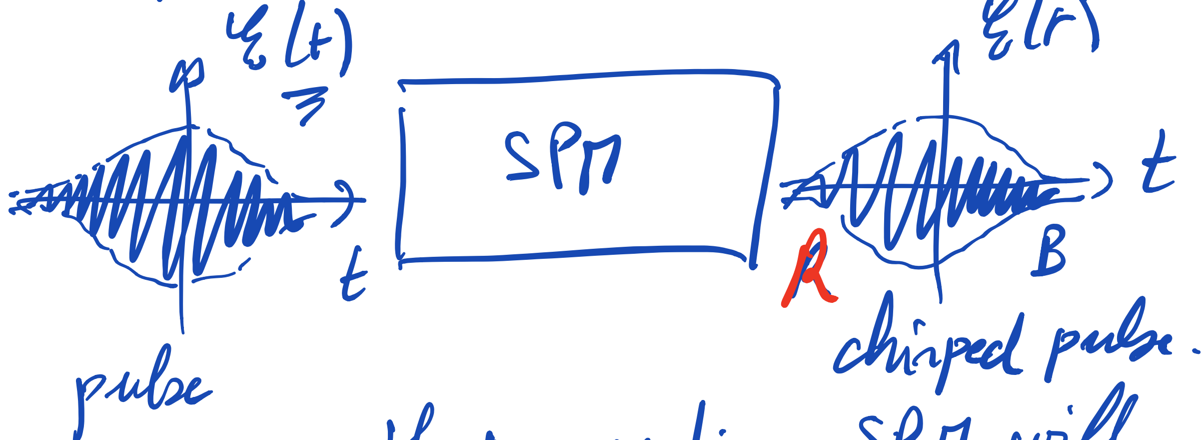
spectral broadening is proportional to $\frac{\partial I(t)}{\partial t}$

$$\boxed{\Delta\omega = -k_0 n_2 L \frac{dI}{\partial t}}$$



spectral broadening under Self phase Modulation (SPM)

3) Why the spectral broadening does not lead to a shorter pulse?



During the propagation, SPM will generate a spectral broadening of the pulse. $\Delta\omega = -k_0 n_2 \frac{dI}{dt}$

$\Rightarrow$ at the front edge of the pulse:

$$\frac{dI}{dt} > 0, \; n_2 > 0 \Rightarrow \Delta\omega < 0$$

$\Rightarrow$ generation of Red components

$\rightarrow$ at the tailing edge of the pulse

$$\frac{dI}{dt} < 0, \; n_2 > 0 \Rightarrow \Delta\omega > 0$$

$\Rightarrow$ Instantaneous Freq. of the pulse varies with <u>TIME</u>

$\Rightarrow$ <u>NONLINEAR SCHRÖDINGER EQUATION</u>

<u>Objective</u>: derivation of a wave equation that describes the propagation of a pulse through a Kerr + Dispersive Material.

In the following, we use a very basic description
For a complete description $\Rightarrow$ LECTURE Notes.

1st case : Non linear effect — $\beta_2 = 0$

we showed that

$$\boxed{\frac{\partial A}{\partial z} = i k_0 n_2 I(t) A(t, z)}$$

2nd case : $n_2 = 0$, $\beta_2 \neq 0$ DISPERSION

Linear Regime.

↳ Origin = Dispersion of the refractive index $\Rightarrow n(\omega)$

$$\underset{\approx}{\beta(\omega)} = \beta_0 + \Delta\omega\beta_1 + \frac{\Delta\omega^2}{2}\beta_2 + \cdots$$

group velocity (delay)

$$\Rightarrow \boxed{\frac{\partial \hat{A}(\omega)}{\partial z} = i \frac{\beta_2}{2} \Delta\omega^2 \hat{A}(\omega)}$$

Spectral domain

$$\left(\frac{\partial \hat{A}}{\partial z} = i\beta(\omega) \hat{A} \right)$$

$\Big\downarrow$ FT

Time domain

LINEAR Regime.

$$\boxed{\frac{\partial A}{\partial z} + \underbrace{i\,\frac{\beta_2}{2}\,\frac{\partial^2 A}{\partial \tau^2}}_{\text{2nd order dispersion effect}}(z,\tau) = 0}$$

3rd case : Combining Kerr effect
+ 2nd order dispersion effect

$$\Rightarrow \boxed{\frac{\partial A}{\partial z} + \underbrace{i\,\frac{\beta_2}{2}\,\frac{\partial^2 A}{\partial \tau^2}}_{\text{DISPERSION effect}} = \underbrace{i\,k_0 n_2 I(z) A}_{\text{KERR effect}}}$$

Non Linear Schrödinger Equation

$\Rightarrow$ usually, the resolution is achieved by Numerical SIMULATION

See TUTORIAL