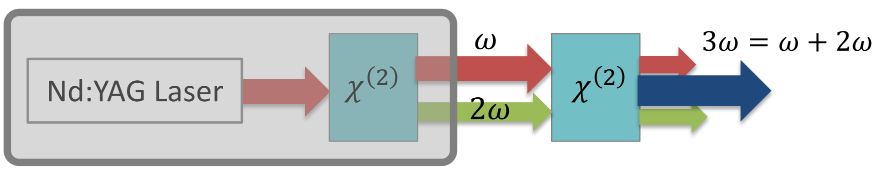
NONLINEAR OPTICS - Tutorial
Three-wave mixing interactions

16 novembre 2022

1 Sum-Frequency Generation in a BBO crystal

Hereafter, we study the generation of a sum-frequency beam after the propagation of two linearly polarized beams through a 2nd order nonlinear material. As shown in the figure below, the two-initial beams provide from a Nd:YAG laser at 1064 nm followed by a second harmonic generation stage in a $\chi^{(2)}$ material. Then, they are sent through a 10 mm thick BBO (β-barium borate, β-BaB_2O_4) crystal to perform at the end the third harmonic generation of the Nd:YAG laser.

The objective is to calculate the pulse energy generated at 3ω. All beams are described with a circular cross section with a beam radius of 200 μm. The effective susceptibility is taken equal to $\chi^{(2)}_{eff} = 4$ pm/V. Both incident beams at 1064 nm and 532 nm deliver 10 ns pulse duration with energies respectively equals to 50 μJ and 10 μJ.



1. What is the wavelength of the sum-frequency beam?

Nonlinear wave equations

We remind the notations used during the course to describe the electric field amplitude of a wave at ω_j, which propagates along the direction (Oz): $\boldsymbol{\mathcal{E}}_j(z,t) = \mathbf{E}(\omega_j)e^{-\imath\omega_j t} + C.C.$, with $\mathbf{E}(\omega_j) = A_j(z)\, e^{\imath k(\omega_j)z}\mathbf{e}_j$. The related field intensity is given by: $I_j = 2n(\omega_j)c\,\epsilon_0|A_j(z)|^2$.

2. Write the 3 coupled nonlinear wave equations for the envelop amplitudes $A_1(z)$, $A_2(z)$ and $A_3(z)$, respectively at ω, 2ω and 3ω. Each equation will be provided in terms of an effective nonlinear susceptibility $\chi^{(2)}_{eff,i}$ to be explicited.
3. What is the phase matching condition?
4. Derive the equations for the intensity variations: $dI_1(z)/dz$, $dI_2(z)/dz$ and $dI_3(z)/dz$.
5. Assuming that the BBO crystal is a lossless material at those wavelengths, what is the necessarily relation between the 3 intensity variations?
6. What does it imply for the different effective susceptibilities $\chi^{(2)}_{eff,i}$ used in the nonlinear wave equations?

Subsequently, we consider the case of a perfect phase matching (this condition is studied in the supplementary section 3).

7. Give the solution for $A_3(z)$ assuming the undepleted pump approximation, which implies that the intensities at ω and 2ω do not significantly vary along the BBO crystal.
8. Calculate the beam intensities at ω, 2ω and 3ω at the crystal output. Calculate the pulse energy delivered at 3ω. Comment about the validity of the undepleted pump approximation.

Phase matching consideration

9. Remind the phase matching relation derived previously from the nonlinear wave equations. Why is it necessary to fulfill the phase matching condition?

2 Second-harmonic generation

You are in charge of a new project in your company: the development of a laser source at 532 nm based on the PicoYAG laser module, already available in your product catalogue. The PicoYAG delivers sub-ns pulse duration at 1064 nm with a 1 kW peak-power.

You seek to evaluate an order of magnitude for the second-harmonic generation (SHG) efficiency throughout a 2 mm long lithium niobate ($LiNbO_3$) crystal. The refractive indices at 1064 nm and 532 nm are respectively equal to $n_\omega = 2.155$ and $n_{2\omega} = 2.234$. In the following, the effective nonlinear susceptibility is taken equal to 18 pm/V. The beam diameter inside the crystal is kept equal to 170 μm for both beams at ω and 2ω. Subsequently, the undepleted pump approximation is assumed.

1. Derive the phase matching condition. Can it be fulfilled?
2. Give the solution for the evolution of the intensity at 2ω along the propagation distance z. Comment on the evolution of $I_{2\omega,z}$ for various phase matching Δk situations.
3. For $\Delta k \neq 0$, show that the maximum SHG efficiency is reached for $L_c = \pi/\Delta k$, the coherence length. Calculate L_c and the maximum expected SHG efficiency.
4. Derive a relation for the SHG efficiency $\eta_{\text{SHG}} = \frac{I_{2\omega}}{I_\omega}$. Assuming a perfect phase matching condition, calculate the expected SHG efficiency. Comment about the validity of the undepleted pump approximation.

$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

3 Sum-Frequency Generation in a BBO crystal : SUPPLEMENT

The most common method to achieve phase matching is based on the birefringence properties of crystals. More precisely, it takes into account the fact that the refractive index of birefringent crystals depends on wavelengths AND on the polarization state of the light. There are two different types of phase matching:

– **Type I phase matching:** the polarization states of the two incident waves (both linearly polarized) at ω and 2ω are aligned either both along the ordinary axis or both along the extraordinary axis. The resulting polarization direction at 3ω is perpendicular to the two others.

- **Type II phase matching:** the polarization state of one of the two incident waves is ordinary, while the second is extraordinary. The resulting polarization at 3ω is parallel to one of the two incident polarizations.

As depicted in the figure (1), we consider only a collinear configuration. The following table provides the ordinary and extraordinary refractive indices for BBO (a negative uniaxial crystal) at the different wavelengths.

λ	n_o	n_e
1064 nm	1.6545	1.5392
532 nm	1.6742	1.5547
?? nm	1.7054	1.5765

1. In case of a type I phase matching, and assuming a normal dispersion, which are the three wave polarization states? Give the relation to be satisfied by the refactive indices of the three waves at ω, 2ω and 3ω.
2. Same question for a type II phase matching.
3. For a type I phase matching in BBO, and in the specific case of the sum frequency generation between two beams at 1064 nm and 532 nm, calculate the refractive index that should experience the wave at 3ω. Would it be possible to reach this condition? How?

Supplementary: consider the case of a positive ($n_e > n_o$) uniaxial crystal for questions 10 and 11.