Tutorial n°6
Performance of communication channels

Some notions to know:

We have seen in the lectures that for the case of binary transmission (two possible symbols: a_1 and a_2), the probability of binary error is expressed by:

$$p_B = \int_{\frac{a_1-a_2}{2\sigma_0}}^{+\infty} \frac{1}{\sqrt{2\pi}}.e^{-\frac{u^2}{2}} du = Q(\frac{a_1 - a_2}{2\sigma_0})$$

$$Q(x) \approx \frac{1}{\sqrt{2\pi}} \int_{x}^{+\infty} e^{-\frac{u^2}{2}} du$$

Remember that a_1-a_2 is related to signal power and σ_0 is related to noise power.

More generally, $p_b = \mathrm{Q}(\sqrt{\frac{d_{12}^2}{2\,N_0}})$

Where d_{12} is the distance between the two points a_1 and a_2.

For M-ary signals, there is also another way of representing performance: the symbol error rate (SER) as a function of the SNR or as a function of the E_s/N_0 ratio. P_m: average error probability, it is bounded by:

$$P_m \leq (M-1)Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \leq \frac{M-1}{2} e^{-\frac{d_{min}^2}{4N_0}}$$

Performance of M-ASK modulations:

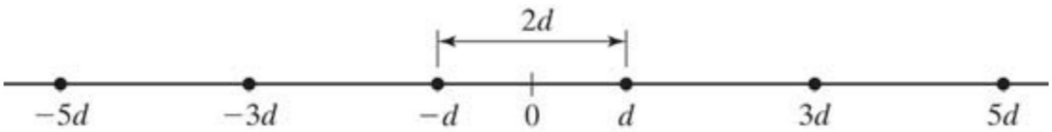

Figure 1: Example of M-ASK modulation (case M=6)

$$\overline{P}_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2d^2}{N_0}}\right)$$

$$= \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6(\log_2 M)}{(M^2-1)}} SNR\right)$$

Performance of M-PSK modulations:

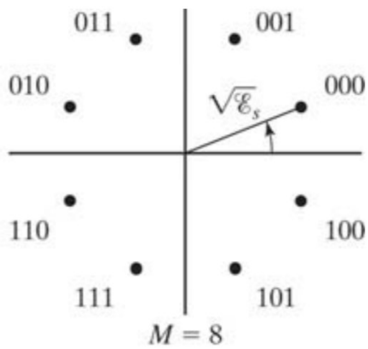

Figure 2: Example of M-PSK modulation (case M=8)

$$d_{min} = 2\sqrt{\mathcal{E}_s} \sin\frac{\pi}{M}$$

$$P_M = 1 - \int_{-\pi/M}^{\pi/M} f_\Theta(\theta) d\theta$$

Expression difficult to evaluate analytically, estimated numerically.

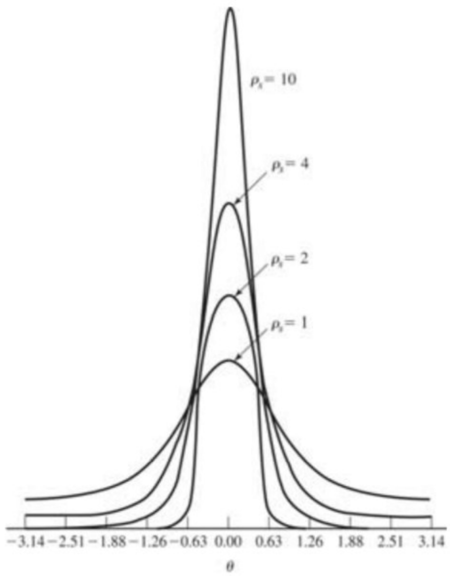

Figure 3: Numerical estimates of f_Θ for M-PSK modulation (for different values of the SNR)

Exercise 1: Binary case

We consider a digital communication system that transmits signals $s_1(t)$ and $s_2(t)$.

At the predetection point we have $z(T) = a_i + n_0$, where the « signal » part is equal to $a_1 = +1$ or $a_2 = -1$, and the « noise » part n_0 is uniformly distributed.

Conditional PDFs (likelihoods) are given by:

$$p(z|s_1) = \begin{cases} \frac{1}{2} & \text{for } -0.2 \leq z \leq 1.8 \\ 0 & \text{otherwise} \end{cases}$$

$$p(z|s_2) = \begin{cases} \frac{1}{2} & \text{for } -1.8 \leq z \leq 0.2 \\ 0 & \text{otherwise} \end{cases}$$

We assume that signals $s_1(t)$ and $s_2(t)$ are equiprobable, and that the optimal decision threshold has been used.

Under these conditions, calculate the probability of binary error p_B.

Exercise 2: M-ary case

The figure below shows the bit error rate as a function of the ratio E_s/N_0. The term E_s is the energy used to transmit a symbol (measured at the receiver input). It is desired to use modulation on a radio channel. The available frequency bandwidth is 150 KHz.

1- What is the theoretical maximum modulation speed?

We use a modulation speed of 1×10^5 bauds. The ratio E_s/N_0 can be written as (with PM the signal power at the receiver):

$$\frac{E_s}{N_0} = \frac{P_M}{R \cdot N_0}$$

2- Demonstrate this formula.

In the channel, we measure $N_0 = 2 \times 10^{-21}$ W/Hz. The transmission power is such that $P_M = 5 \times 10^{-15}$ W. The application using the channel tolerates a maximum bit error probability of 10^{-5}. We want, of course, to maximize throughput.

3- What is the value of E_s/N_0?

4- Which modulation will be chosen? The transmission rate? (Attention: the x-axis of the figure is in dB).

We want to limit ourselves to the use of PSK (thus excluding QAM).

5- What is the reason for this? What will be the valence then? The rate of transmission?

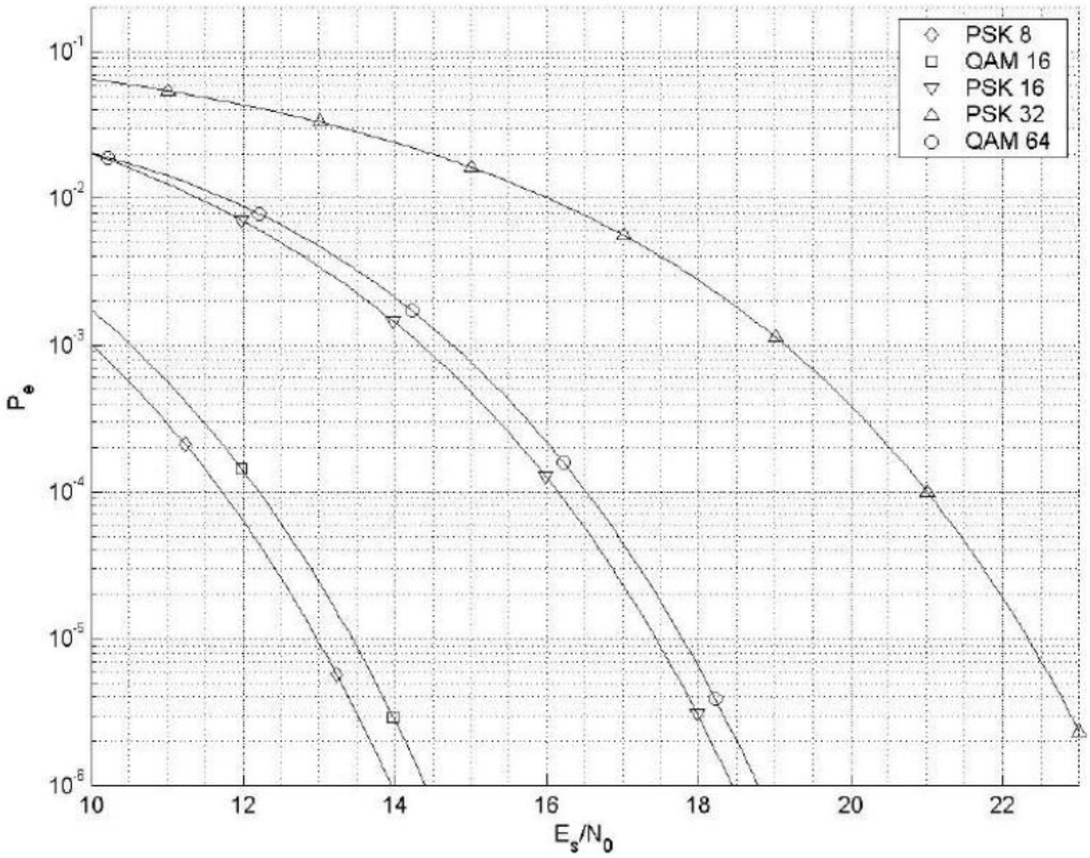

Figure 4: Probability of binary error as a function of the E_s/N_0 ratio on the channel (case M>2)