

# MATHS FOR DATA SCIENCE

## CLASS 5 - EXERCISES

### EXERCISE 1

a) SHOW THAT  $\|Ax\| \leq \|A\| \|x\|$  FOR ALL MATRIX  $A$ , VECTOR  $x$

HINT: USE THE ORIGINAL DEFINITION OF  $\|A\|$

b) SHOW THAT  $\|AB\| \leq \|A\| \|B\|$  FOR ALL MATRICES  $A, B$

HINT: USE THE ALTERNATIVE DEFINITION OF MATRIX NORM (THE ONE WITH  $\|x\| = 1$ ) AND PROPERTY a) TWICE

### EXERCISE 2

COMPUTE THE DETERMINANT OF THE MATRIX

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & -2 & 3 \\ 3 & 1 & 2 & 1 \\ 1 & -1 & 0 & 2 \end{pmatrix}$$

(ANSWER:  $\det(A) = -39$ )

### EXERCISE 3

SHOW THAT THE DETERMINANT OF AN UPPER TRIANGULAR MATRIX, THAT IS A MATRIX OF THE TYPE

$$A = \begin{pmatrix} a_{11} & * & \dots & * \\ 0 & a_{22} & \dots & * \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & 0 & a_{mm} \end{pmatrix}, \text{ IS } \det(A) = a_{11} \cdot a_{22} \cdot \dots \cdot a_{mm}$$