MATHS FOR DATA SCIENCE

CLASS 3 - EXERCISES

EXERCISE 1

SOLVE THE FOLLOWING LINEAR SYSTEMS USING GAUSSIAN ELIMINATION:

$$\begin{cases} x_1 + 3x_2 + x_3 - x_4 = 1 \\ 3x_1 + 9x_2 + 4x_3 + x_4 = 1 \\ 2x_1 + x_2 + 5x_3 + 2x_4 = 0 \\ x_2 - x_3 - x_4 = 2 \end{cases}$$

(EXAMPLE WITH ROWS EXCHANGE, TEST THE CHOICE NOT DONE IN CLASS)

$$\begin{cases} 2x_1 + 6x_2 + 3x_3 + 2x_4 = 4 \\ x_1 - 2x_2 + \frac{1}{2}x_3 + \frac{9}{4}x_4 = 1 \\ -x_1 + x_2 - \frac{1}{2}x_3 - x_4 = \frac{2}{5} \end{cases}$$

$$\begin{cases} 2x_1 + 6x_2 + 3x_3 = 4 \\ x_1 - 2x_2 + \frac{1}{2}x_3 = 1 \\ -x_1 + x_2 - \frac{7}{10}x_3 = \frac{2}{5} \end{cases}$$

EXERCISE 2

SHOW THAT FOR ALL $x, y \in \mathbb{R}^m$

$$\|x+y\|_2^2 = \|x\|_2^2 + 2\,x\cdot y + \|y\|_2^2 .$$

EXERCISE 3

LET $v_1, \dots, v_m$ BE A LIST OF ORTHOGONAL NON ZERO VECTORS, THAT IS
$v_i \cdot v_j = 0$ FOR ALL $i \neq j$, $i, j = 1, \dots, m$.

PROVE THAT THEY ARE LINEARLY INDEPENDENT.