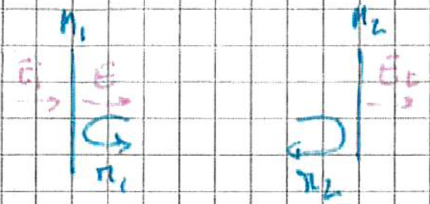
TD n°2 : FABRY-PEROT et MIROIR

EXERCICE 1 : CAVITE FABRY-PEROT



① $\mathcal{E} = t_1 \mathcal{E}_i + r_1 r_2 e^{2ikd} \mathcal{E}$

② $$\mathcal{E} = \frac{t_1 \mathcal{E}_i}{1 - r_1 r_2 e^{2ikd}}$$

③ $\mathcal{E} = t_1 \mathcal{E}_i + t_1 r_1 r_2 e^{2ikd} \mathcal{E}_i + \dots + t_1 (r_1 r_2 e^{2ikd})^n \mathcal{E}_i + \dots$

④ $$\mathcal{E}_t = \frac{t_1 t_2 e^{ikd}}{1 - r_1 r_2 e^{2ikd}} \mathcal{E}_i \quad \text{car} \quad \mathcal{E}_t = t_2 e^{ikd} \mathcal{E}$$

$$\mathcal{E}_r = -r_1 \mathcal{E}_i + t_1 r_2 \mathcal{E} e^{2ikd} \Rightarrow \mathcal{E}_r = \left(-r_1 + \frac{r_2 t_1^2 e^{2ikd}}{1 - r_1 r_2 e^{2ikd}} \right) \mathcal{E}_i$$

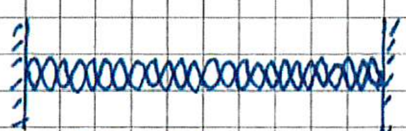
⑤ $$\mathcal{T} = |\mathcal{E}_t / \mathcal{E}_i|^2 = \frac{T_1 T_2}{1 + R_1 R_2 - 2 r_1 r_2 \cos 2kd} \quad \text{or} \quad \cos 2kd = 1 - 2\sin^2 kd$$

$$\mathcal{T} = \frac{T_1 T_2}{(1 - r_1 r_2)^2 + 4 r_1 r_2 \sin^2 kd} = \underbrace{\frac{T_1 T_2}{(1 - r_1 r_2)^2}}_{\mathcal{T}_{max}} \frac{1}{1 + \underbrace{\frac{4 r_1 r_2}{(1 - r_1 r_2)^2}}_{m} \sin^2 kd}$$

⑥ Résonance : $\mathcal{T} = \mathcal{T}_{max}$ quand $\sin kd = 0 \Rightarrow 2\pi \frac{\nu}{c} d = p\pi \quad p \in \mathbb{N}^*$

$$\boxed{\nu = p \frac{c}{2d}} \qquad \boxed{ISL = \frac{c}{2d}}$$

⑦



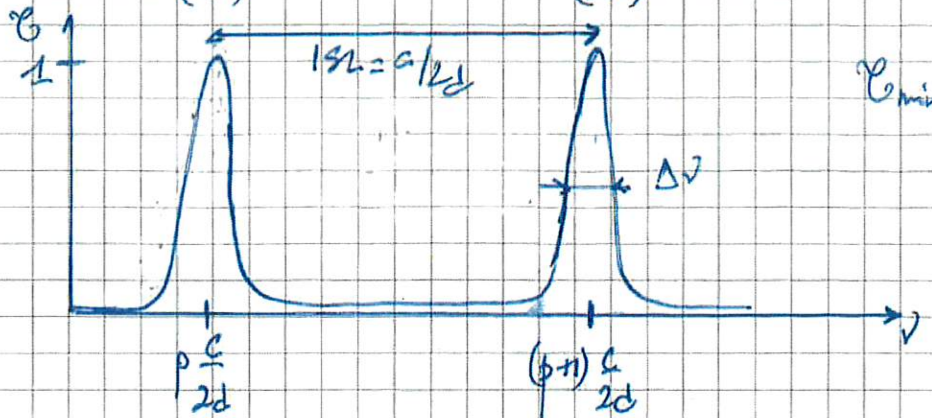
Onde stationnaire

Nombre de noeuds : $\frac{d}{\lambda/2} = \frac{2d\nu}{c} = \frac{2d}{c} p \frac{c}{2d} = p$

Ordre de grandeur : $\left. \begin{array}{l} d = 1\,m \\ \lambda/2 = 0{,}5\,\mu m \end{array} \right| \Rightarrow p = 2 \cdot 10^6$

$ISL = 150$ MHz

(8) $\mathcal{T}_{max} = \frac{T^2}{(1-R)^2} = 1$ $\quad m = \frac{4R}{(1-R)^2} = 4R/T^2$



$\mathcal{T}_{min} = \frac{1}{1+m}$

(9) Si $T \ll 1$ alors $m \gg 1$

A mi-hauteur : $\sin^2 \frac{2\pi}{c}\frac{\Delta\nu}{2} d = 1/m$

On linéarise le sin car $m \gg 1$: $\Delta\nu = \frac{c}{\pi d}\frac{1}{\sqrt{m}} = \frac{c}{\pi d}\frac{T}{2\sqrt{R}}$

$\mathcal{F} = \frac{c/2d}{\Delta\nu} = \frac{c}{2d}\frac{\pi d\, 2\sqrt{R}}{c\, T}$ $\quad \mathcal{F} = \frac{\pi\sqrt{R}}{T} \simeq \frac{\pi}{T} = \frac{2\pi}{\text{Pertes par aller-retour}}$

(10) A résonance : $\mathcal{T} = \mathcal{T}_{max} = 1$

$I = I_i/T = \frac{\mathcal{F} I_{in}}{\pi}$ $\quad$ Il y a une surtension de $\frac{\mathcal{F}}{\pi}$

(11) $T = 10^{-5}$, $d = 1\,m$

$c/2d = 150$ MHz

$\mathcal{F} = 3.1 \times 10^5$

$\mathcal{F}/\pi = 10^5$ surtension

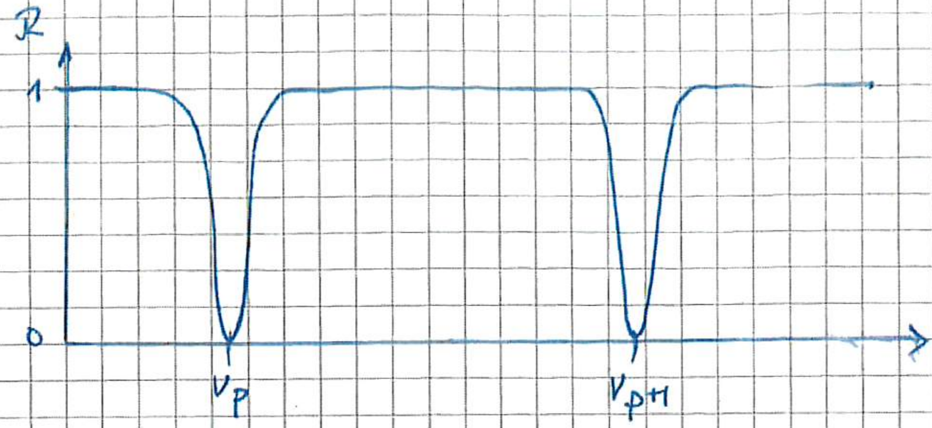
$\Delta\nu = \frac{150 \text{ MHz}}{3.10^5} = 480$ Hz

$\frac{\nu}{\Delta\nu} = \frac{1}{480}\frac{3.10^8}{0{,}5\times10^{-6}} = 1{,}2\times10^{12}$

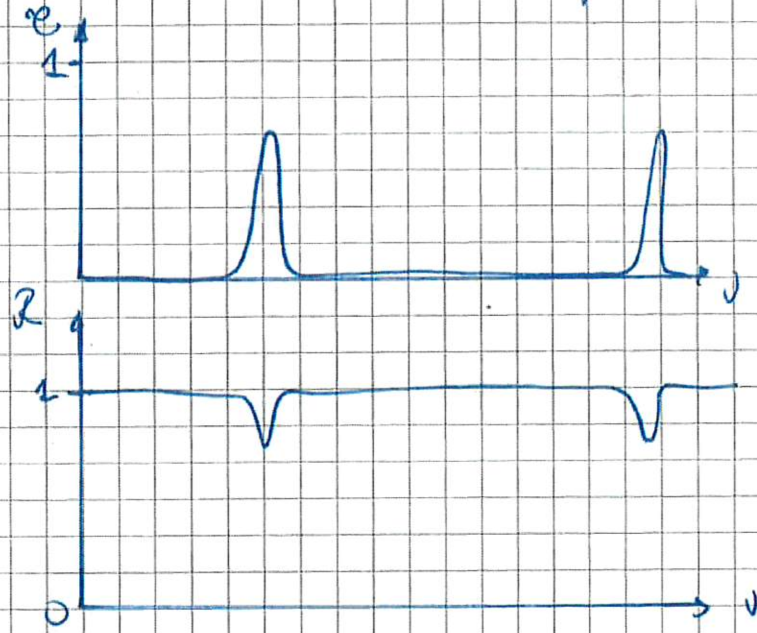
(12) A résonance : $kd = p\pi \Rightarrow e^{2ikd} = 1$

$\mathcal{E}_r = \left(-r + \frac{rTe^{2ikd}}{1-Re^{2ikd}}\right)\mathcal{E}_i = \left(\frac{-r + r\overbrace{(R+T)}^{1}e^{2ikd}}{1-Re^{2ikd}}\right)\mathcal{E}_i = \frac{r(e^{2ikd}-1)}{1-Re^{2ikd}}\mathcal{E}_i$

A résonance : $\mathcal{E}_r = 0$



(13) Cas général : $\mathcal{R} + \mathcal{T} + \text{dissipation} = 1$



EXERCICE 3 : MIROIR MULTI-COUCHE DIÉLECTRIQUE

(1) Relations de continuité : $(\vec{D_1}-\vec{D_2}).\hat{n}=0$, $(\vec{B_1}-\vec{B_2}).\hat{n}=0$ — Continuité des composantes $\perp$ à l'interface

$\hat{\vec{n}}$ = normale à l'interface

$(\vec{H_1}-\vec{H_2})\times\hat{n}=0$, $(\vec{E_1}-\vec{E_2})\times\hat{n}=0$ — Continuité des composantes // à l'interface

(2) Les champs $\vec{E}$ et $\vec{H}$ sont tangentiels aux surfaces, donc continus. Donc :

$$\mathcal{E}_{j-1} = \mathcal{E}_{j-1}^+ + \mathcal{E}_{j-1}^- = \mathcal{E}_j^+ e^{-i\phi_j} + \mathcal{E}_j^- e^{i\phi_j} \quad (1)$$

$$\mathcal{H}_{j-1} = \mathcal{H}_{j-1}^+ + \mathcal{H}_{j-1}^- = \mathcal{H}_j^+ e^{-i\phi_j} + \mathcal{H}_j^- e^{i\phi_j} \quad (2)$$

(3) Impédance du milieu j : $Z_j \mathcal{H}_j^+ = \mathcal{E}_j^+ \quad (3)$

$$-Z_j \mathcal{H}_j^- = \mathcal{E}_j^- \quad (4)$$

D'où vient le signe moins ?

On a (eq. 2.34 du poly) pour une onde plane : $\vec{k}\times\vec{\mathcal{H}} = -\omega\varepsilon\vec{\mathcal{E}}$

Sens + : si $\vec{\mathcal{E}} = \mathcal{E}\hat{x}$, $\vec{k} = k\hat{z}$, $\vec{\mathcal{H}} = \mathcal{H}\hat{y}$ $\Rightarrow k\mathcal{H}\underbrace{\hat{z}\times\hat{y}}_{-\hat{x}} = -\omega\varepsilon\mathcal{E}\hat{x}$ $\Rightarrow -k\mathcal{H} = -\omega\varepsilon\mathcal{E}$

$$\mathcal{E}/\mathcal{H} = \frac{k}{\omega\varepsilon} = \frac{n}{c\varepsilon} = \frac{\sqrt{\varepsilon_0\mu_0}}{\varepsilon_0\sqrt{\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon}} = Z$$

Sens − : si $\vec{\mathcal{E}} = \mathcal{E}\hat{x}$, $\vec{k} = -k\hat{z}$, $\vec{\mathcal{H}} = \mathcal{H}\hat{y}$ $\Rightarrow -k\mathcal{H}\underbrace{\hat{z}\times\hat{y}}_{-\hat{x}} = -\omega\varepsilon\mathcal{E}\hat{x}$ $\Rightarrow k\mathcal{H} = -\omega\varepsilon\mathcal{E}$

$$\Rightarrow \mathcal{E}/\mathcal{H} = -Z$$

(3),(4) $\Rightarrow \mathcal{H}_j = \mathcal{H}_j^+ + \mathcal{H}_j^- = \dfrac{\mathcal{E}_j^+ - \mathcal{E}_j^-}{Z_j}$

De plus, $\mathcal{E}_j = \mathcal{E}_j^+ + \mathcal{E}_j^-$

$$\Rightarrow \begin{cases} \mathcal{E}_j^+ = \dfrac{\mathcal{E}_j + Z_j\mathcal{H}_j}{2} \\ \mathcal{E}_j^- = \dfrac{\mathcal{E}_j - Z_j\mathcal{H}_j}{2} \end{cases}$$

On injecte dans (1), (2) :

$$\mathcal{E}_{j-1} = e^{-i\phi_j}\,\frac{\mathcal{E}_j + Z_j \mathcal{H}_j}{2} + e^{i\phi_j}\,\frac{\mathcal{E}_j - Z_j \mathcal{H}_j}{2}$$

$$= \cos\phi_j\,\mathcal{E}_j - i Z_j \sin\phi_j\,\mathcal{H}_j$$

$$\mathcal{H}_{j-1} = \mathcal{H}_j^+ e^{-i\phi_j} + \mathcal{H}_j^- e^{i\phi_j}$$

$$= \frac{1}{Z_j}\mathcal{E}_j^+ e^{-i\phi_j} - \frac{1}{Z_j}\mathcal{E}_j^- e^{i\phi_j}$$

$$= \frac{1}{Z_j}\left[\frac{\mathcal{E}_j + Z_j\mathcal{H}_j}{2} e^{-i\phi_j} - \frac{\mathcal{E}_j - Z_j\mathcal{H}_j}{2} e^{i\phi_j}\right]$$

$$= \frac{1}{Z_j}\left(Z_j \mathcal{H}_j \cos\phi_j - i\,\mathcal{E}_j \sin\phi_j\right)$$

Donc
$$\begin{pmatrix}\mathcal{E}_{j-1}\\ \mathcal{H}_{j-1}\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\phi_j & -iZ_j\sin\phi_j\\ -\frac{i}{Z_j}\sin\phi_j & \cos\phi_j\end{pmatrix}}_{M_j}\begin{pmatrix}\mathcal{E}_j\\ \mathcal{H}_j\end{pmatrix}$$

Il y a une coquille dans le poly !

(4) $$M = M_1 M_2 \cdots M_N = \prod_{j=1}^{N} M_j$$

(5) $$\begin{pmatrix}1+R\\ \frac{1-R}{Z_0}\end{pmatrix} = M\begin{pmatrix}\mathcal{T}\\ \mathcal{T}/Z_N\end{pmatrix} \qquad M = \begin{pmatrix}A & B\\ C & D\end{pmatrix}$$

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$$M = (M_A M_B)^M = \left[\begin{pmatrix} \cos\phi & -iZ_A\sin\phi \\ -\frac{i}{Z_A}\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \cos\phi & -iZ_B\sin\phi \\ -\frac{i}{Z_B}\sin\phi & \cos\phi \end{pmatrix}\right]^M$$

$$= \begin{pmatrix} \cos^2\phi - \frac{Z_A}{Z_B}\sin^2\phi & -i(Z_A+Z_B)\sin\phi\cos\phi \\ -i\left(\frac{1}{Z_A}+\frac{1}{Z_B}\right)\sin\phi\cos\phi & \cos^2\phi - \frac{Z_B}{Z_A}\sin^2\phi \end{pmatrix}^M$$

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si $\phi = \pi/2$:

$$M = \begin{pmatrix} -Z_A/Z_B & 0 \\ 0 & -\frac{Z_B}{Z_A} \end{pmatrix}^M \qquad \text{or } Z = \sqrt{\mu/\varepsilon} = \frac{\sqrt{\mu_0/\varepsilon_0}}{n}$$

$$M = \begin{pmatrix} -n_B/n_A & 0 \\ 0 & -n_A/n_B \end{pmatrix}^M$$

$$\left.\begin{aligned} 1+R &= \left(-\frac{n_B}{n_A}\right)^M \mathcal{C} \\ 1-R &= \left(-\frac{n_A}{n_B}\right)^M \mathcal{C} \end{aligned}\right| \Rightarrow \left(-\frac{n_A}{n_B}\right)^M (1+R) = \left(-\frac{n_B}{n_A}\right)^M (1-R)$$

$$\Downarrow$$

$$R = \frac{\left(-\frac{n_B}{n_A}\right)^M - \left(-\frac{n_A}{n_B}\right)^M}{\left(-\frac{n_B}{n_A}\right)^M + \left(-\frac{n_A}{n_B}\right)^M}$$

$$R = \frac{\left(\frac{n_B}{n_A}\right)^{2M} - 1}{\left(\frac{n_B}{n_A}\right)^{2M} + 1} \rightarrow 1 \text{ quand } M \rightarrow \infty$$

In[9]:= refl[nA_, nB_, m_] := $\dfrac{\left(\frac{nB}{nA}\right)^{2 m} - 1}{\left(\frac{nB}{nA}\right)^{2 m} + 1}$

In[18]:= nA := 1.5

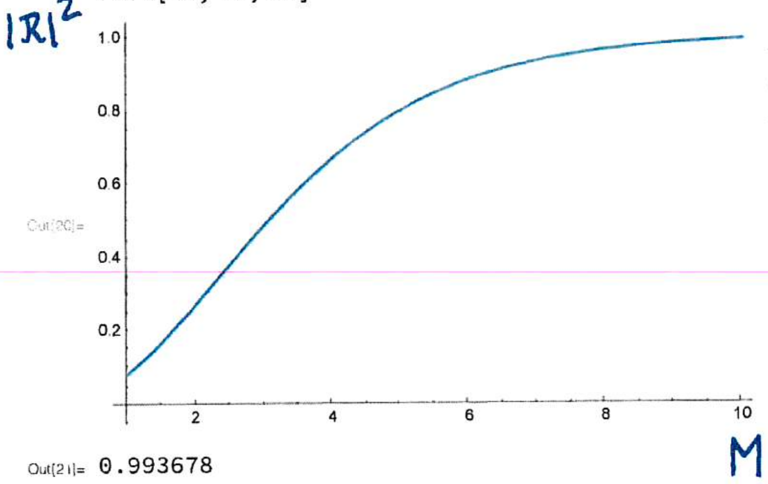
nB := 2.0

Plot[refl[nA, nB, m]^2, {m, 1, 10}]

refl[nA, nB, 10]

refl[nA, nB, 20]

Out[20]=



Out[21]= 0.993678

Out[22]= 0.99998