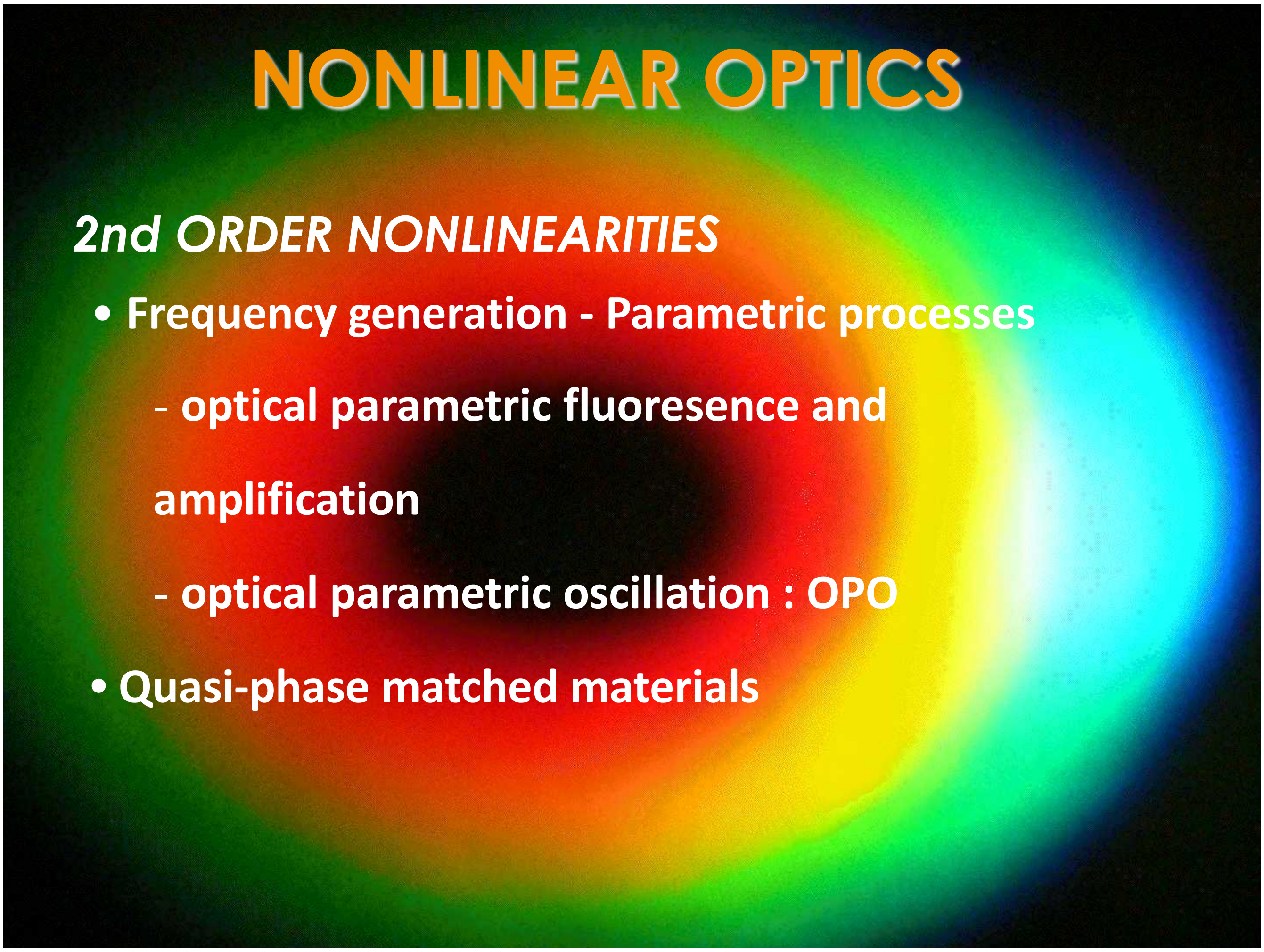


Lecture 5 /7 : learning outcomes

By the end of this lecture, students will be able to...

- Evaluate nonlinear interaction performances/efficiencies under approximations that should be specified, explained and justified (T2)

By the end of this lecture, students will be skilled at...

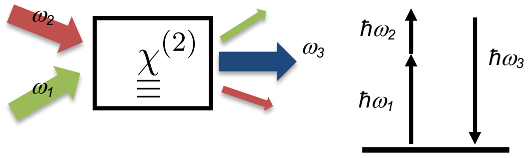
- deriving and solving the nonlinear wave equation in a parametric situation under the undepleted pump approximation (S3)
- Determining the phase matching conditions for a given nonlinear interaction and achieving/fulfilling this condition by exploiting birefringence properties of materials and/or QPM technique (S2)
- Calculating nonlinear interaction performances/efficiencies in situations governed by analytical solutions or expressions (S4)

By the end of this lecture, students will understand ...

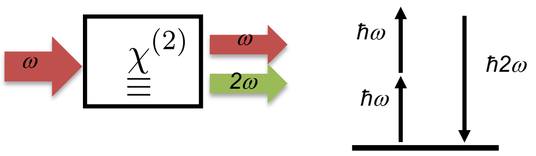
- Nonlinear effects are a key points in the development of many applications in photonics (especially in relation with laser physics) (U1)
- Nonlinear interactions lead to energy transfer between optical beams, and/or between matter and beams, enabling in some cases the realization of nonlinear optical amplification and/or oscillation (U3)

Three-wave Mixing Interactions

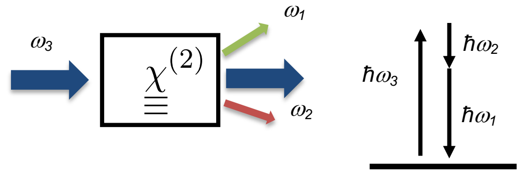
Sum-Frequency Generation (SFG)



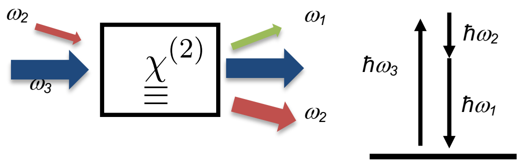
Second Harmonic Generation (SHG)



Optical Parametric Fluoresence

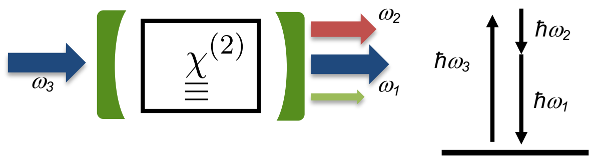


Optical Parametric Amplification (APO)



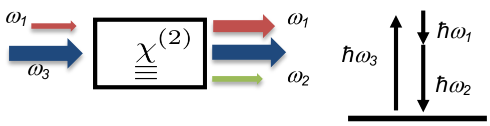
Today's Lecture

Optical Parametric Oscillation (OPO)



3 - Optical Parametric Amplification and Oscillation

Difference frequency generation



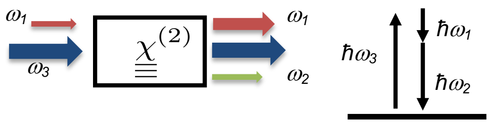
1. Parametric Amplification

Undepleted pump approximation: $A_3(z)$=Cste

$$\frac{dA_1}{dz} =$$

$$\frac{dA_2}{dz} =$$

Difference frequency generation



$$\omega_2 = \omega_3 - \omega_1$$

Idler Pump Signal

1. Parametric Amplification

Undepleted pump approximation: $A_3(z)$=Cste

$$\frac{dA_1}{dz} = \frac{\imath\omega_1}{2n_1c}\chi_{\text{eff}}^{(2)} A_3 A_2^\star e^{\imath\Delta kz}$$

$$\frac{dA_2}{dz} = \frac{\imath\omega_2}{2n_2c}\chi_{\text{eff}}^{(2)} A_3 A_1^\star e^{\imath\Delta kz}$$

$$\Delta k = k_3 - k_2 - k_1$$

$$\chi_{\text{eff}}^{(2)} = 2 \times e_1 \cdot \underline{\underline{\chi}}^{(2)}(\omega_1;\omega_3,-\omega_2)e_3e_2 = 2 \times e_2 \cdot \underline{\underline{\chi}}^{(2)}(\omega_2;\omega_3,-\omega_1)e_3e_1$$

Resolution

$$a_1(z) = \sqrt{\frac{n_1}{\omega_1}} A_1(z) e^{-\imath\Delta kz/2}$$

$$a_2^\star(z) = \sqrt{\frac{n_2}{\omega_2}} A_2^\star(z) e^{+\imath\Delta kz/2}$$

$$a_3(z) = \sqrt{\frac{n_3}{\omega_3}} A_3(z).$$

$$\frac{da_1}{dz} + \imath\frac{\Delta k}{2}a_1 = \imath\gamma_0 a_2^\star,$$

$$\frac{da_2^\star}{dz} - \imath\frac{\Delta k}{2}a_2^\star = \imath\gamma_0^\star a_1,$$

$$\gamma_0 = \frac{\chi_{\text{eff}}^{(2)}}{c}\sqrt{\frac{\omega_1\omega_2}{n_1n_2}}A_3$$

$$\frac{d^2a_{1,2}}{dz^2} - \gamma^2 a_{1,2} = 0$$

$$\gamma^2 = |\gamma_0|^2 - \left(\frac{\Delta k}{2}\right)^2$$

3 - ...Parametric Amplification ...

Resolution

$$\frac{d^2 a_{1,2}}{dz^2} - \gamma^2 a_{1,2} = 0$$

$$\gamma^2 = |\gamma_0|^2 - \left(\frac{\Delta k}{2}\right)^2$$

- $\gamma^2 < 0$: strong phase miss-matching - oscillating solution
  => no amplification

- $\gamma^2 > 0$: solution of the form

$$a_{1,2}(z) = C_{1,2}\exp(-\gamma z) + D_{1,2}\exp(+\gamma z)$$

Boundary conditions :

$$a_1(z=0) = \sqrt{\frac{n_1}{\omega_1}} A_1(0) \text{ and } a_2(z=0) = \sqrt{\frac{n_2}{\omega_2}} A_2(0)$$

3 - ...Parametric Amplification ...

Solutions : Signal & Idler

$$A_{1,2}(z) = A_{1,2}(0)\left[\cosh(\gamma z) - \frac{\imath \Delta k}{2\gamma}\sinh(\gamma z)\right] e^{\imath \Delta k z/2} + \imath \frac{K_{1,2}}{\gamma} A^{\star}_{2,1}(0)\sinh(\gamma z) e^{\imath \Delta k z/2}$$

$$K_{1,2} = \frac{\omega_{1,2}}{n_{1,2}c}\chi^{(2)}_{\text{eff}} A_3$$

Comments :

- For a given wavevector missmatch, one can set the pump intensity to a threshold value in order to achieve $\gamma^2 = 0$:

$$I_{3th} = \frac{(\Delta k)^2 n_1 n_2 n_3 c^3 \varepsilon_0}{2(\chi_{eff})^2 \omega_1 \omega_2}$$

- I_{3th} is minimum for $\omega_1 = \omega_2 = \frac{\omega_3}{2}$

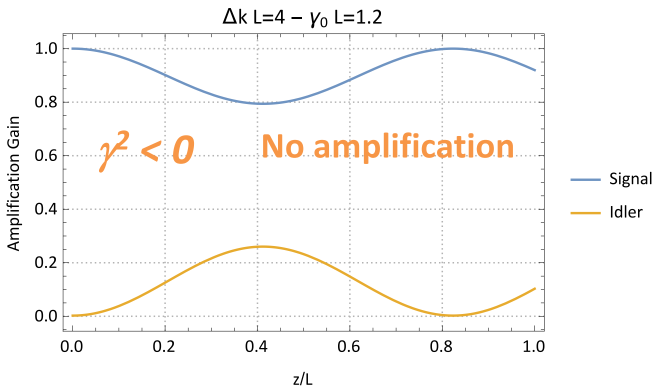
- and equal to 0 for $\Delta k = 0$

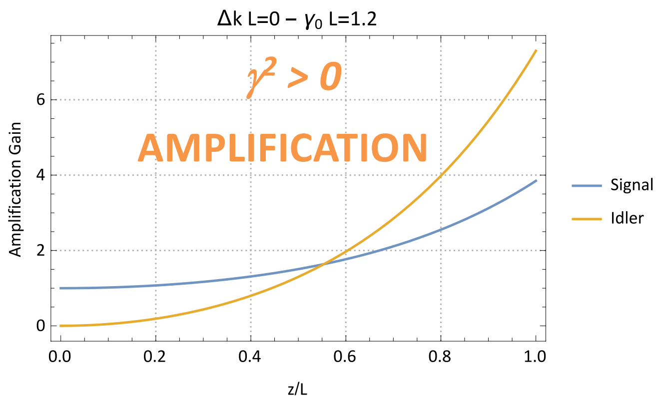
3 - ...Parametric Amplification ...

Solutions : Signal & Idler

$$A_{1,2}(z) = A_{1,2}(0)\left[\cosh(\gamma z) - \frac{\imath\Delta k}{2\gamma}\sinh(\gamma z)\right] e^{\imath\Delta kz/2} + \imath\frac{K_{1,2}}{\gamma}A^{\star}_{2,1}(0)\sinh(\gamma z)e^{\imath\Delta kz/2}$$

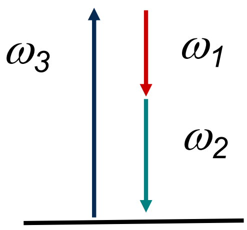
$$K_{1,2} = \frac{\omega_{1,2}}{n_{1,2}c}\chi^{(2)}_{\text{eff}}A_3$$





3 - ...Parametric Amplification ...

2. Parametric Fluorescence

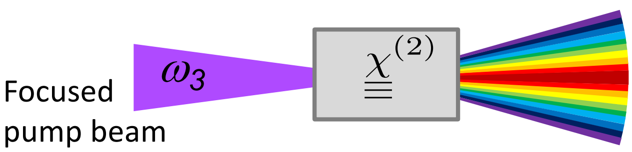


Boundary conditions : $A_1(0) = 0$, $A_2(0) = 0$

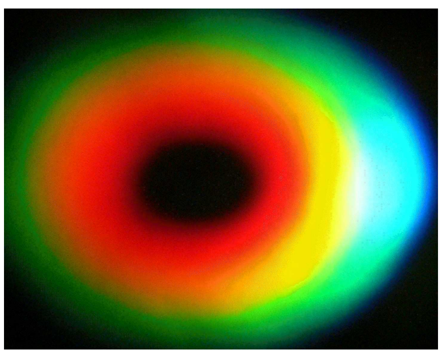
⇒ Solutions : $A_1(z) = 0$, $A_2(z) = 0$

⇒ *No generation of signal and idler is expected*

⇒ *in contradiction with the experimental observation*

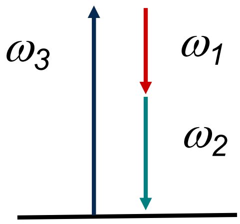


Spontaneous emission of signal and idler photons through a parametric fluorescence effect



2. Parametric Fluorescence

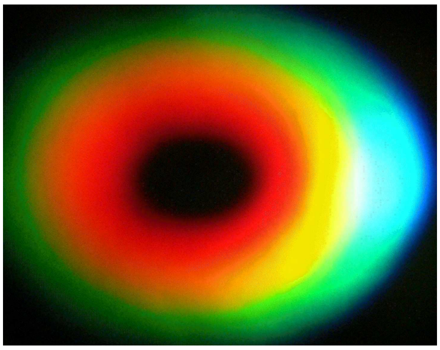
Boundary conditions : $A_1(0) = 0, A_2(0) = 0$



$\Rightarrow$ ***QUANTUM DESCRIPTION** is required (*)*

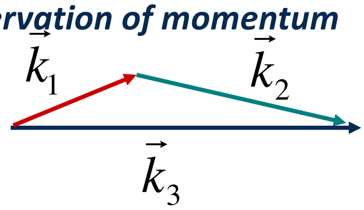
The annihilation of a photon @ ω_3 is accompanied by the creation ***simultaneously*** of one photon @ ω_1 and one photon @ ω_2

$\omega_3 = \omega_1 + \omega_2$: conservation of energy

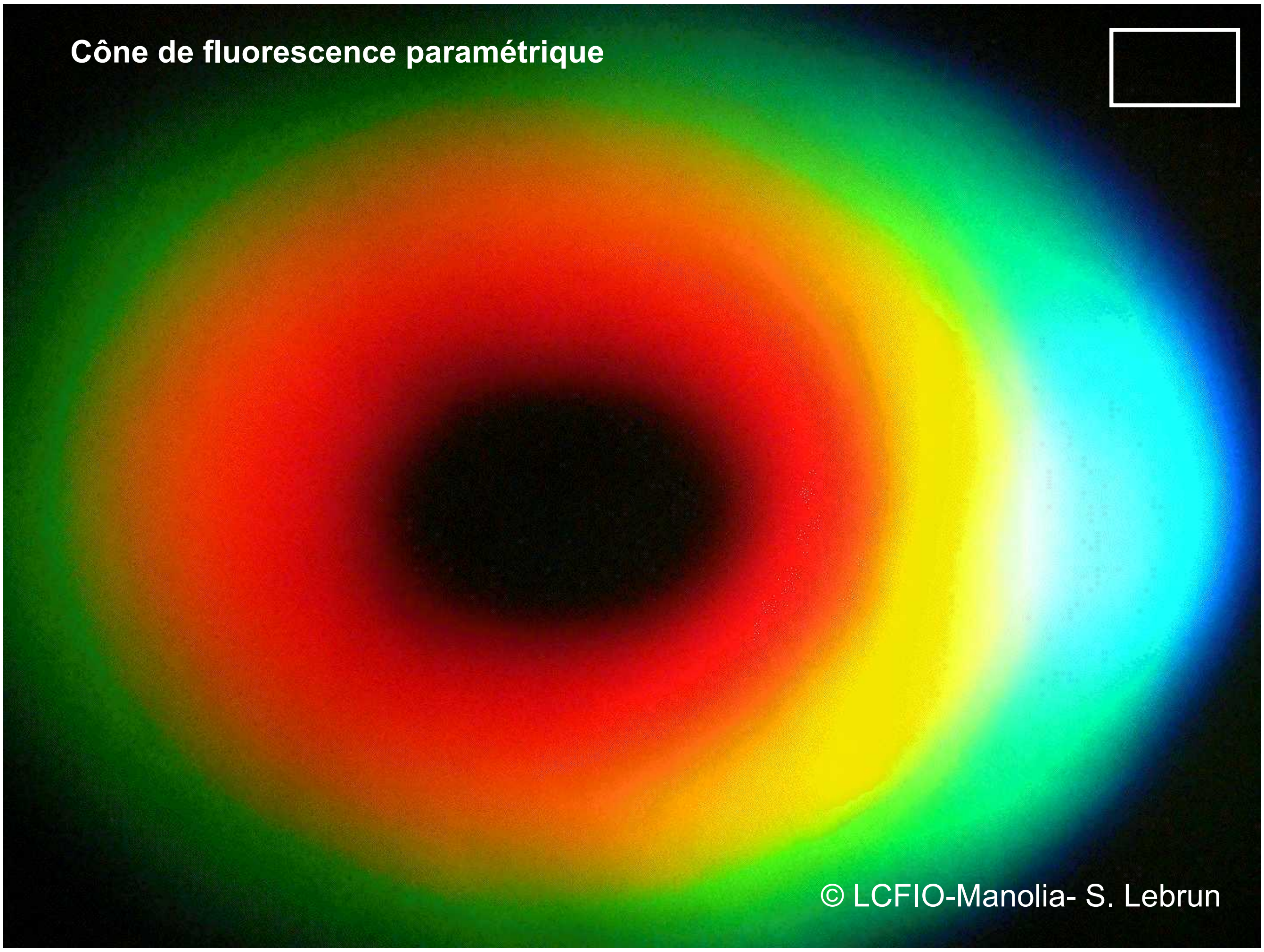


$\vec{k}_3 = \vec{k}_1 + \vec{k}_2$ ***: the phase-matching relation is a VECTORIAL relation and*** is interpreted as the ***law of conservation of momentum***

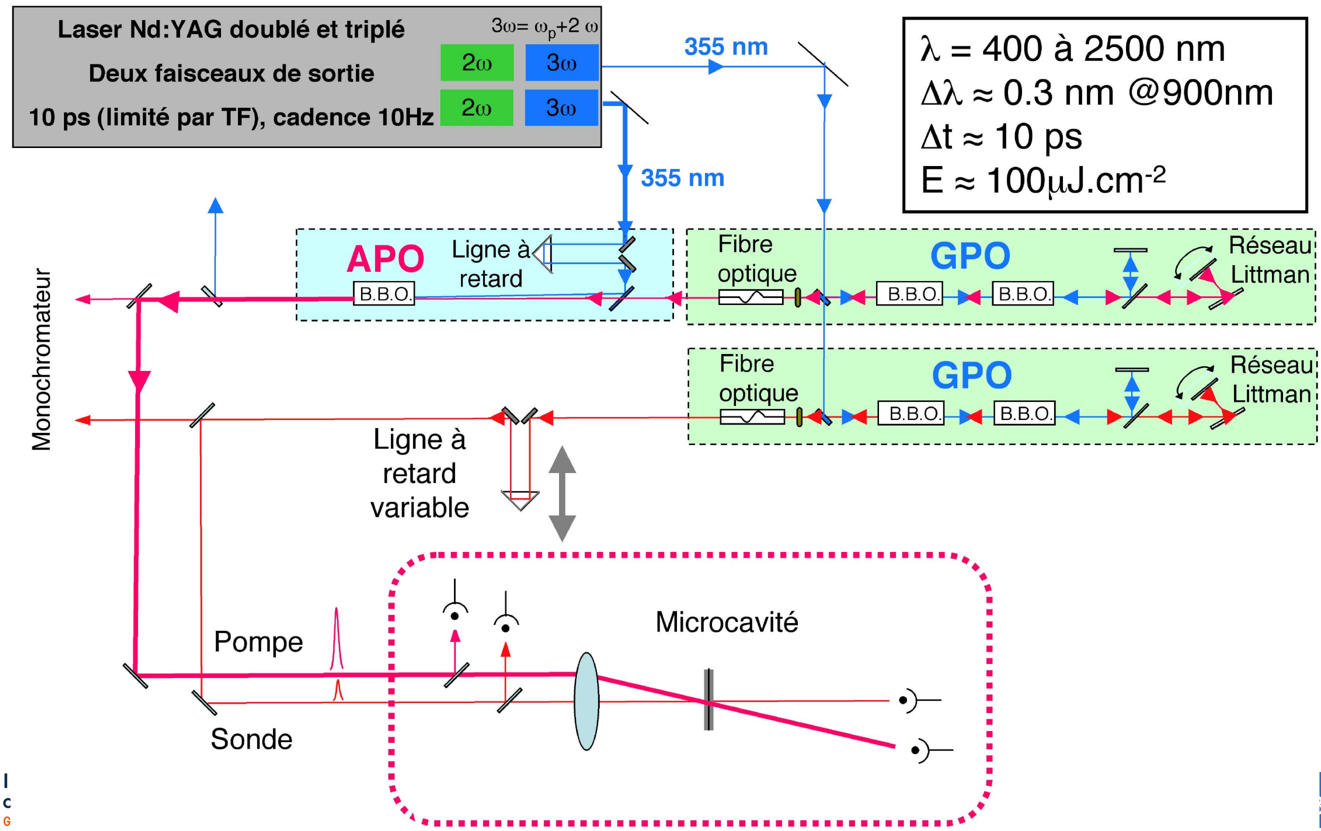
This last relation indicates the directions of emission the signal and idler waves

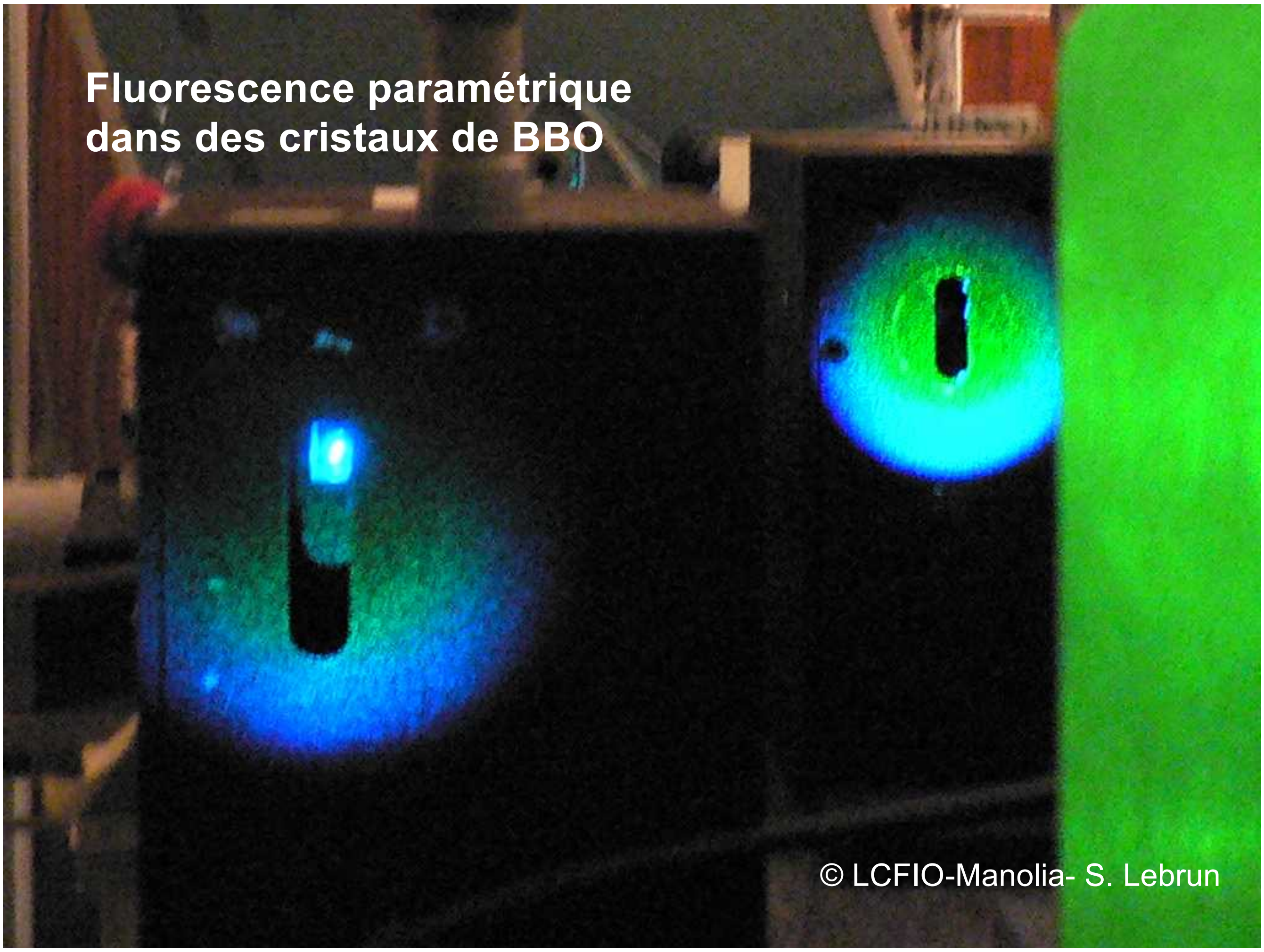


(*) : see Quantum Optics course of next year



Optical Parametric Generation

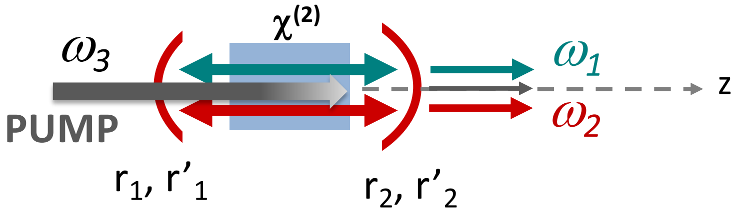




3 - Optical Parametric ... Oscillation

Considering a 2nd order NL medium placed inside an optical cavity - The amplification is realized through pumping @ $\boldsymbol{\omega}_3$

Schematic OPO cavity



- Mirrors with high reflectivity @ ω_1 and ω_2

& antireflection coating @ ω_3 (PUMP)

- Pump beam propagates only along the forward direction (+z)
- Amplification occurs only if the phase matching condition is satisfied

➜ Phase matching condition ?

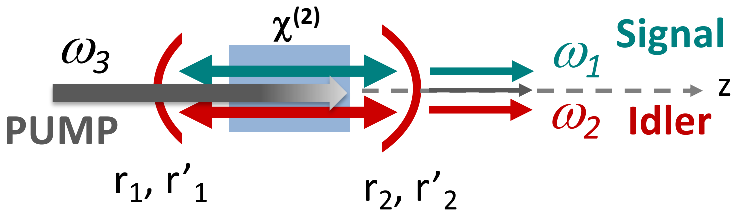
➜ Threshold condition ?

➜ Differences between LASERS & OPO ?

3 - Optical Parametric ... Oscillation

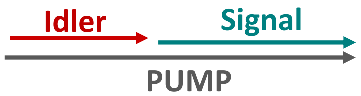
Considering a 2nd order NL medium placed inside an optical cavity - The amplification is realized through pumping @ $\boldsymbol{\omega}_3$

Schematic OPO cavity



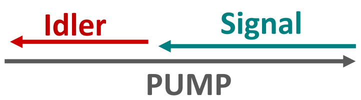
➜ Phase matching condition ?

- **Forward waves interaction**



$$\vec{k}_3 = \vec{k}_1 + \vec{k}_2 \Rightarrow \Delta\vec{k} = 0$$

- **Backward waves interaction**



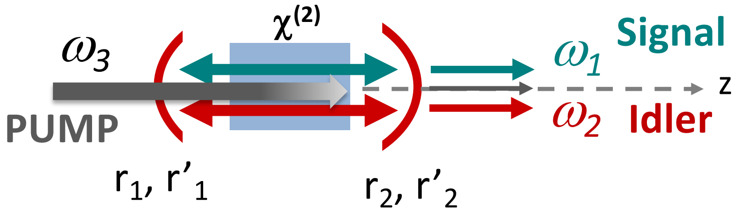
$$\vec{k}_3 = -\vec{k}_1 - \vec{k}_2 \Rightarrow \Delta\vec{k} \neq 0$$

Amplification only for <u>forward</u> signal and idler waves

3 - Optical Parametric ... Oscillation

Considering a 2nd order NL medium placed inside an optical cavity - The amplification is realized through pumping @ $\boldsymbol{\omega_3}$

Schematic OPO cavity



➜ Threshold condition ?
(assuming a perfect phase matching condition)

Singly resonant cavity (@ ω_1)

$$r_1 r_1' \cosh(\gamma_0 L) = 1$$

(see exercise on eCampus)

Doubly resonant cavity (@ ω_1 and ω_2)

$$\cosh(\gamma_0 L) = \frac{1 + r_1 r_1' r_2 r_2'}{r_1 r_1' + r_2 r_2'}$$

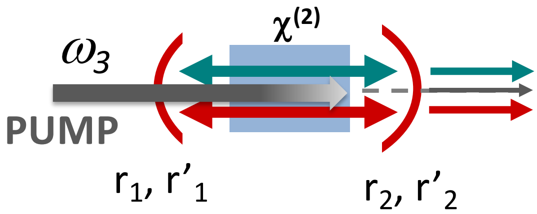
with $\gamma_0 = \frac{\chi_{\text{eff}}^{(2)}}{c}\sqrt{\frac{\omega_1\omega_2}{n_1 n_2}} A_3$ ← **Pump amplitude**

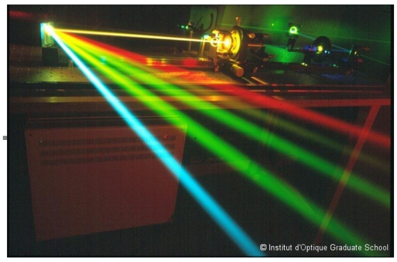
(parametric gain)

3 - Optical Parametric ... Oscillation

Considering a 2nd order NL medium placed inside an optical cavity - The amplification is realized through pumping @ $\boldsymbol{\omega_3}$

Schematic OPO cavity





➜ Differences between LASERS & OPO ?

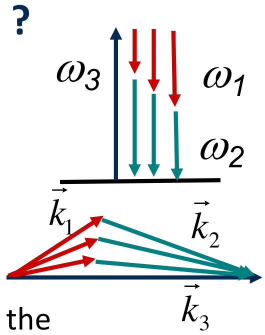
Generation of a coherent optical beam

LASERS

- Amplification = stimulated emission of radiation
- Resonant interaction with active medium = HIGH efficiency
- Limited tunability (limited by the gain linewidth)
- One output beams

OPO

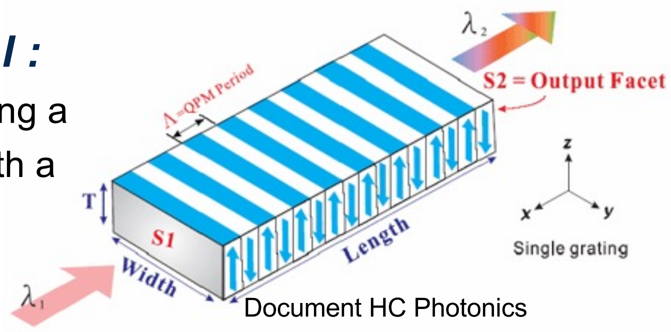
- Amplification = parametric down conversion
- NON-Resonant interaction with NL material = LOW efficiency
- **Wide tunability** assisted by tuning the phase matching condition
- Generation of two output beams : signal + idler with two distinct frequency values



4 - Quasi-Phase Matched Materials

Periodically poled material :

c-axis is alternatively oriented along a positive and negative direction with a period Λ =2 $\times$ Coherent Length



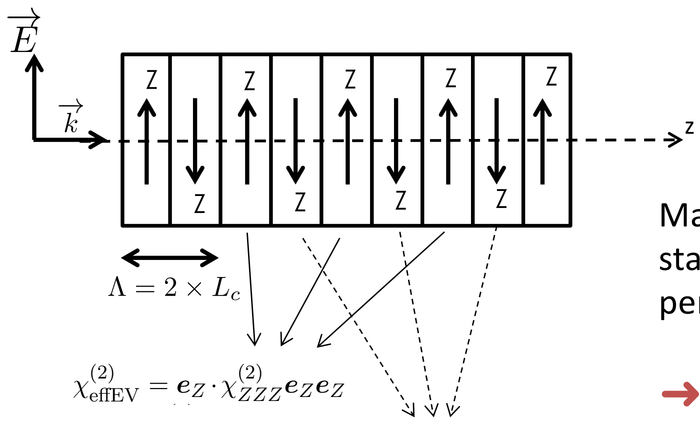
Achievement of a quasi-phase matching condition :

- in materials for which birefringent phase matching can not be achieved

- OR enables configuration along a large nonlinear coefficient, $\chi^{(2)}_{ZZZ}$ for which all the waves must be polarized in the same direction

4 - Quasi-Phase Matched Materials

Periodically poled material :



$$\chi^{(2)}_{\text{effEV}} = \boldsymbol{e}_Z \cdot \chi^{(2)}_{ZZZ} \boldsymbol{e}_Z \boldsymbol{e}_Z$$

$$\chi^{(2)}_{\text{effOD}} = (-\boldsymbol{e}_Z) \cdot \chi^{(2)}_{ZZZ} (-\boldsymbol{e}_Z)(-\boldsymbol{e}_Z) = -\chi^{(2)}_{\text{effEV}}$$

Material poling is achieved by applying a static field between electrodes, leading to a permanent domain reversal of the material.

➜ Periodic inversion of the effective NL susceptibility sign

4 - Quasi-Phase Matched Materials

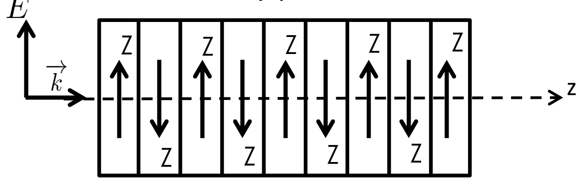
Illustration : SHG in a configuration that exploits the strong NL coef $\chi^{(2)}_{ZZZ}$

Nonlinear wave equation @ 2ω

$$\frac{dA_{2\omega}(z)}{dz} = \frac{\imath(2\omega)}{2n_{2\omega}c}\chi^{(2)}_{\text{eff}}\; A^2_{\omega}e^{\imath\Delta\boldsymbol{k}\cdot\boldsymbol{z}}$$

with $\chi^{(2)}_{\text{eff}} = \boldsymbol{e}_Z \cdot \chi^{(2)}_{ZZZ}\boldsymbol{e}_Z\boldsymbol{e}_Z$

Periodically poled material



➜ Introduction of a periodic spatial variation : $\chi^{(2)}_{\text{eff}}(z) = \chi^{(2)}_{\text{eff},0}\cos(Kz)$

$$\frac{dA_{2\omega}(z)}{dz} = \frac{\imath(2\omega)}{2n_{2\omega}c}\chi^{(2)}_{\text{eff},0}\; A^2_{\omega}\;\cos(Kz)e^{\imath\Delta\boldsymbol{k}\cdot\boldsymbol{z}},$$
$$= \frac{\imath(2\omega)}{2n_{2\omega}c}\chi^{(2)}_{\text{eff},0}\; A^2_{\omega}\;\frac{e^{\imath Kz}+e^{-\imath Kz}}{2}e^{\imath\Delta\boldsymbol{k}\cdot\boldsymbol{z}}$$

➜ A quasi-phase matched (QPM) situation is achieved since

$$K = \Delta\boldsymbol{k} = 2\boldsymbol{k}_{\omega} - \boldsymbol{k}_{2\omega}$$

➜ CONCLUSION : a periodic inversion of the sign the NL susceptibility, with a period

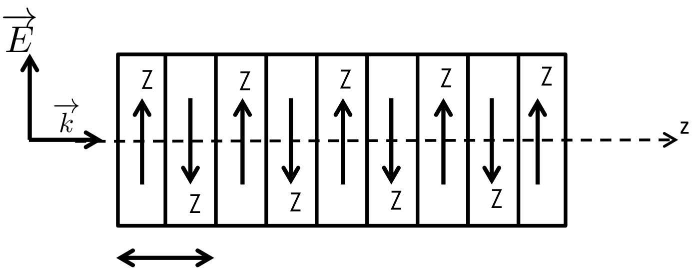
$$\Lambda = 2 \times L_c$$

leads to a growth in the intensity @ 2ω

Reminder : Lc refers to the coherence length $\pi/\Delta k = L_c$

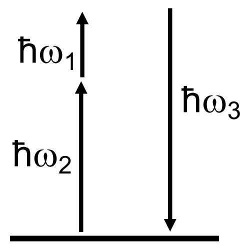
4 - Quasi-Phase Matched Materials

3 wave mixing configuration

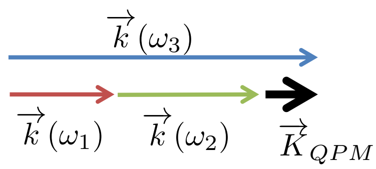


$\Lambda = 2 \times L_c$

Energy conservation



Phase-matching condition under QPM



Estimation of QPM efficiency consists in developing the effective nonlinear susceptibility with a square wave function S(z) of period $\Lambda = 2 \times L_c$

Using the Fourier expansion $S(z) = \frac{4}{\pi}\sum_p \frac{1}{2p+1}\sin\left((2p+1)2\pi z/\Lambda\right)$

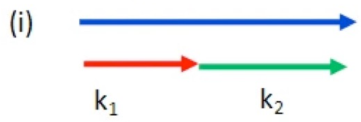
the effective nonlinear susceptibility is then equal to $i\frac{\chi^{(2)}_{\text{eff}}}{\pi/2}$

Conclusion :

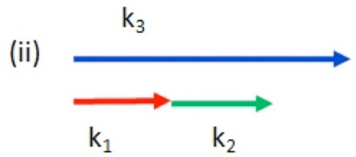
A QPM can be considered a a bulk material with an effective susceptibility set to $2\chi^{(2)}/\pi$

4 - Quasi-Phase Matched Materials

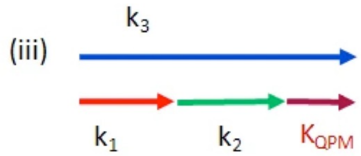
Comparison in the power evolution under perfect phase matching (i), non-phase matching (ii) and quasi-phase matching (iii) situations. The signs represent the sign of the effective nonlinear susceptibility



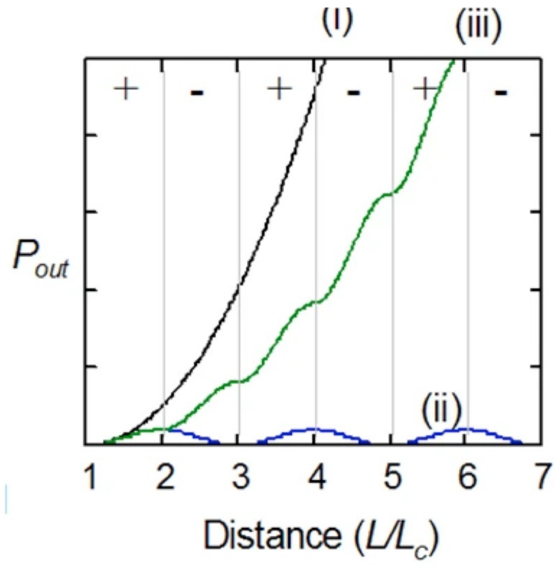
Phase-Matching
$\Delta\beta = k_3 - k_1 - k_2 = 0$



Non-Phase-Matching
$\Delta\beta = k_3 - k_1 - k_2 \neq 0$



Quasi-Phase-Matching
$\Delta\beta_{QPM} = k_3 - k_1 - k_2 - K_{QPM} = 0$
where $K_{QPM} = 2\pi/\Lambda$ & $\Lambda = 2L_c$

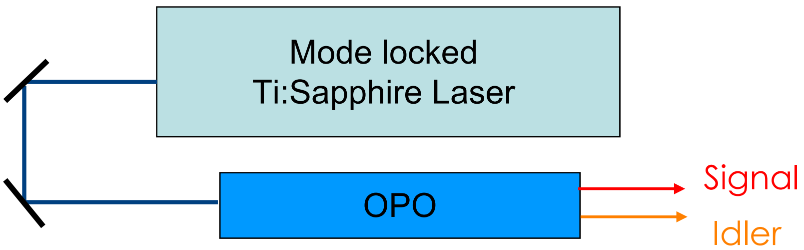


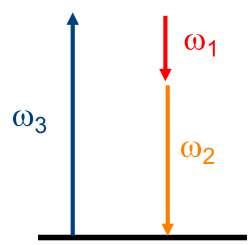
Document HC Photonics - https://www.hcphotonics.com/ppln-guide-overview

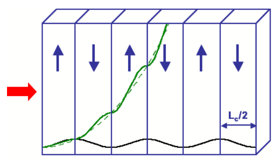
EXAMPLE : Singly resonant OPO cavity

Development of a tunable Optical Parametric Oscillator (OPO) @ 1550 nm

- short pulse (1-10ps) with narrow spectral linewidth
- high repetition rate
- Tunable : [1530-1600 nm]







QPM nonlinear crystal

A. Ryasnyanskiy, et al. JEOS. RP 3 :08037, 2008.

EXAMPLE : Singly resonant OPO cavity

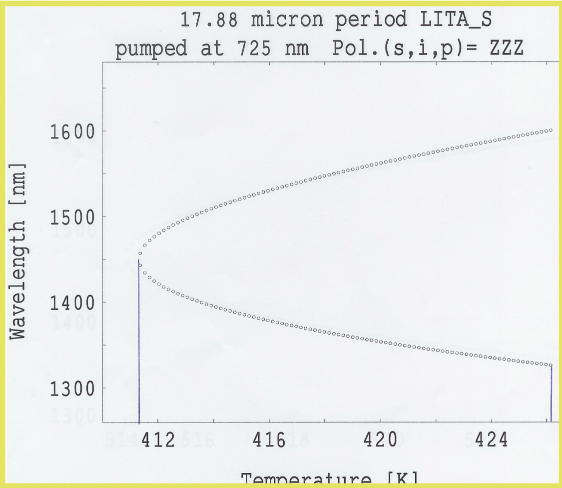
Quasi phase matching + Temperature tuning

QPM crystal : PP-SLT

Lower nonlinear coefficient d_{33} than PPLN
BUT higher refractive damage resistance

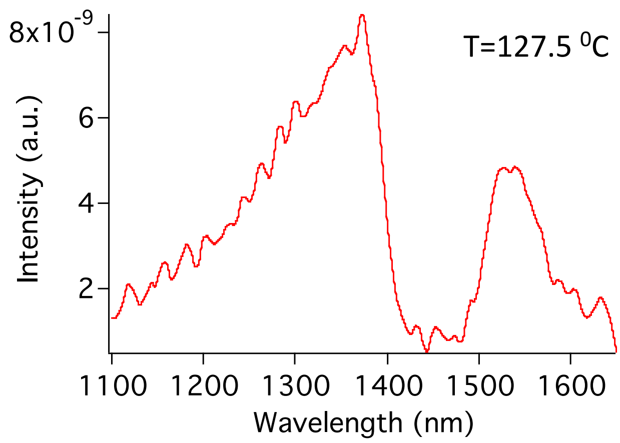
λ_{pump}=725 nm

- Crystal length 20 mm
- period : 18 µm



Data from SNLO software

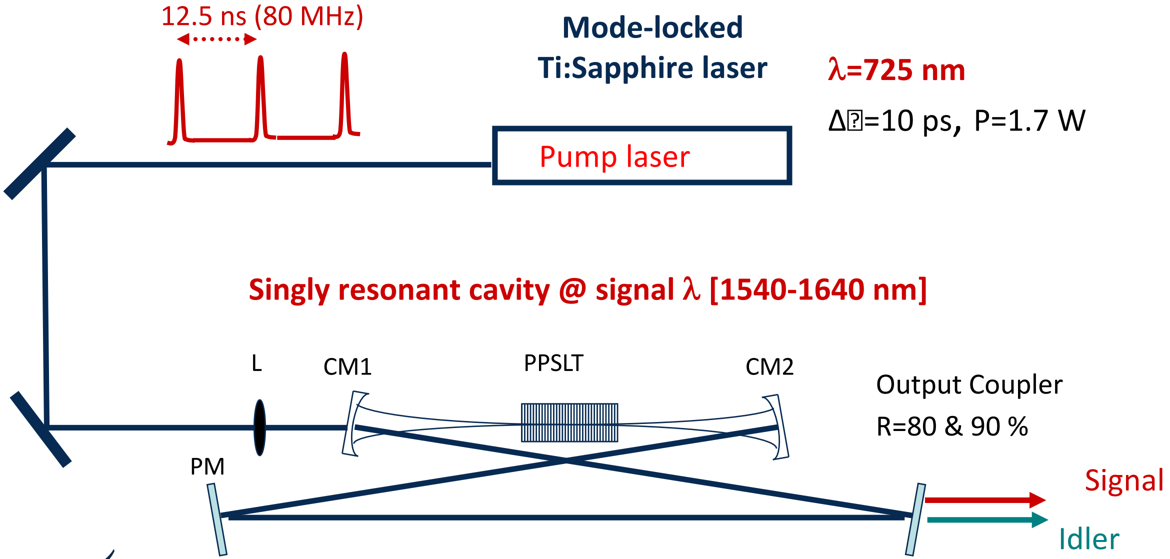
Fluorescence spectrum



EXAMPLE : Singly resonant OPO cavity

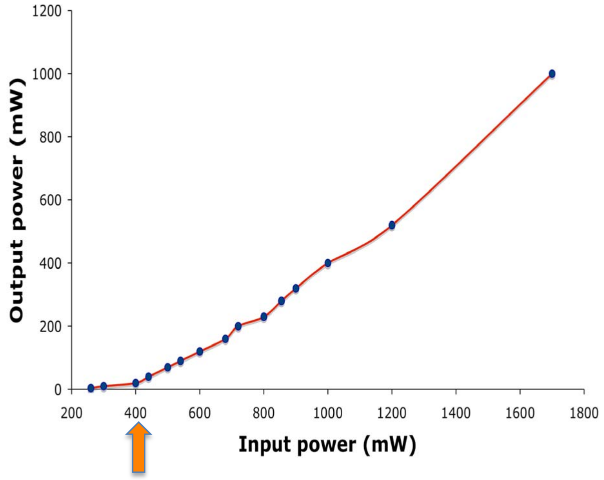
- Stable bow tie singly resonant cavity
- Synchronously pumped cavity : cavity round trip matches the repetition rate of the pump laser **L_{OPO} = 3.75 m**

(radius of curvature of the concave mirrors : 25.9 cm)



EXAMPLE : Singly resonant OPO cavity

- **OPO characteristics**



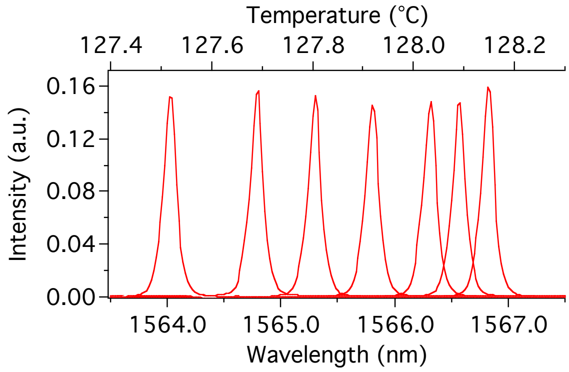
Threshold power :

P_{th}=400 mW (Averaged Power)

Peak-power = ×1000

- **Tuning range : Signal [1530-1640 nm] & Idler [1300-1375 nm]**
- **Keeping pulse duration around 10 ps**

''Fine'' T° tuning



A. Ryasnyanskiy, et al. JEOS. RP 3 :08037, 2008.