NONLINEAR OPTICS - Tutorial
2nd Harmonic Generation in KDP crystal

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This problem adresses the study and optimisation of the second harmonic generation in a KDP crystal for the conversion process 1064 nm $\rightarrow$ 532 nm, in a collinear configuration. The crystal is a negative uniaxial medium, with the principal refractive indices at room temperature given by :

$$\begin{array}{lll} \lambda = 1064\ nm & n_0 = 1.4942 & n_e = 1.4603 \\ \lambda = 532\ nm & n_0 = 1.5129 & n_e = 1.4709 \end{array}$$

The KDP belongs to the symmetry group $\overline{4}2m$, characterized by a $\underline{\underline{\chi}}^{(2)}$ tensor in which only six terms (with the intrinsic permutation) are non-zero: $\chi^{(2)}_{XYZ} = \chi^{(2)}_{YXZ} = \chi^{(2)}_{ZXY} = 1\,\mathrm{pm/V}$.
The tensor is defined in the cartesian frame of the principal axis directions, direction Z coinciding with the optical axis.

1 SHG study : perfect phase matching situation

In this section, we will consider that the angular difference between the vectors **D** and **E** of the electromagnetic fields along the extraordinary axis is negligible. This is equivalent to neglect the walk-off angle. *The consequence of this approximation will be discussed in section 2.*

1.1 Type I phase matching achievement in KDP.

1. Find the relation between the refractive indices at ω and 2ω for a type I phase matching in KDP.

2. Calculate the angle θ between the propagation direction of the beams and the optical axis of the crystal (see figure 1 p. 3).

1.2 Optimization of the nonlinear interaction

The phase matching condition enables to determine the angle θ and one has to determine the second angle ϕ to completely defined the direction of the beam propagation (see figure 1(a) p. 3). This angle is settled by optimizing the effective nonlinear susceptibility $\chi^{(2)}_{eff}$.

1. For a type I phase matching case, give the expression for the nonlinear polarization components at 2ω ($\mathbf{P}_{NL}(2\omega)$) long the crystallographic principal axes (X, Y, Z).

2. Write the nonlinear wave equation for the wave at 2ω in terms of an effective nonlinear coefficient $\chi^{(2)}_{eff}$ to be defined.

3. Find the value(s) of ϕ at which the nonlinear interaction is maximized.

1.3 Conversion efficiency

In the following, the KDP crystal has been cut such as the pump beam arrives perpendicularly to the incident facet, coinciding with a direction set by the angles θ and ϕ previously calculated. In this part a low conversion efficiency is assumed justifying the undepleted approximation.

1. By solving the nonlinear wave equation set in section (1.2), determine a relation for the conversion efficiency $\frac{I_{2\omega}(L)}{I_{\omega}(0)}$, where $I_{2\omega}(L)$ and $I_{\omega}(0)$ are the intensity of the doubled and fundamental beams respectively, and L the crystal length.

2. Evaluate the conversion efficiency of the second harmonic generation in a 10 mm long KDP crystal using a Nd:YAG laser at $\lambda = 1064$ nm, delivering pulses with a duration $\tau = 300$ ps and an energy of 1 mJ. The beam diameter along the crystal is supposed constant and set equal to 1 mm.

3. Comment about the validity of the undepleted pump approximation.

1.4 Limit of the low conversion efficiency approximation

In a strong conversion efficiency situation, the undepleted pump approximation is no longer valid and one needs to solve the two coupled nonlinear wave equations at ω and 2ω. It leads then to the following relation for the conversion efficiency :

$$\frac{I_{2\omega}(L)}{I_{\omega}(0)} = \tanh^2\left(\frac{\sqrt{2}\pi\chi^{(2)}_{eff}L}{\sqrt{\epsilon_0 c n_{2\omega} n_{\omega}^2}.\lambda}\sqrt{I_{\omega}(0)}\right) \tag{1}$$

1. How the conversion efficiency varies with the pump intensity at low and high pump power regimes ?

2. What would be the conversion efficiency in the case considered at question 2 in (1.3). Conclusion ?

2 Conversion efficiency limitation

2.1 Angular acceptance

The effect of a deviation $\delta\theta$ of the incident beam with respect to the phase matching angle θ is equivalent to a phase mismatch Δk.

1. Show that the conversion efficiency after a length L is then reduced by a factor :

$$\left[\frac{\sin(\Delta k L/2)}{\Delta k L/2}\right]^2 \tag{2}$$

2. Determine the angular acceptance $\delta\theta_{\frac{1}{2}}$, for which the conversion efficiency is divided by a factor 2. Note that $(\sin(\xi)/\xi)^2 = 1/2$ for $\xi = 1.39$.

3. In case of a phase matching angle achieved around $\theta = \pi/2$, why do we refer to a "non-critical" phase matching situation ?

2.2 Walk-off effect

1. Using the figure 1(b), show that the direction of propagation for the beam at 2ω is different from that of the beam at ω. Determine the walk-off angle between these two rays.

2. What is the consequence for the conversion efficiency ?

3. Compare with a non-critical phase matching situation.

RESOURCES

1. The electric field amplitude of a wave at ω_j, which propagates along the direction (Oz), is denoted : $\boldsymbol{\mathcal{E}}_j(z,t) = \mathbf{E}(\omega_j)e^{-\imath\omega_j t} + C.C.$, with $\mathbf{E}(\omega_j) = A_j(z)\, e^{\imath k(\omega_j)z}\mathbf{e}_j$. The related field intensity is given by: $I_j = 2n(\omega_j)c\,\epsilon_0|A_j(z)|^2$, with $\epsilon_0 = 8.85\ 10^{-12}$ F/m.

2. The amplitude variation along the propagation distance z of a wave at ω is governed by the following nonlinear wave equation :

$$\frac{\partial A(\omega)}{\partial z} = \frac{\imath\,\omega}{2nc\epsilon_o}\boldsymbol{e}.\mathbf{P}_{NL}(\omega)\exp(-ik(\omega)z)$$

3. The interacting waves have to be projected along the ordinary and extraordinary eigen modes for which the directions of polarization are denoted $\mathbf{e}_o$ and $\mathbf{e}_\theta$, respectively. In the crystallographic principal axes, they can be expressed as:

$$\mathbf{e}_o = \left| \begin{array}{c} \sin\phi \\ -\cos\phi \\ 0 \end{array} \right. , \quad \mathbf{e}_\theta = \left| \begin{array}{c} -\cos\theta\cos\phi \\ -\cos\theta\sin\phi \\ \sin\theta \end{array} \right. ;$$

The extraordinary refractive index n_θ is given by : $\left(\frac{1}{n_\theta}\right)^2 = \left(\frac{\cos\theta}{n_o}\right)^2 + \left(\frac{\sin\theta}{n_e}\right)^2$

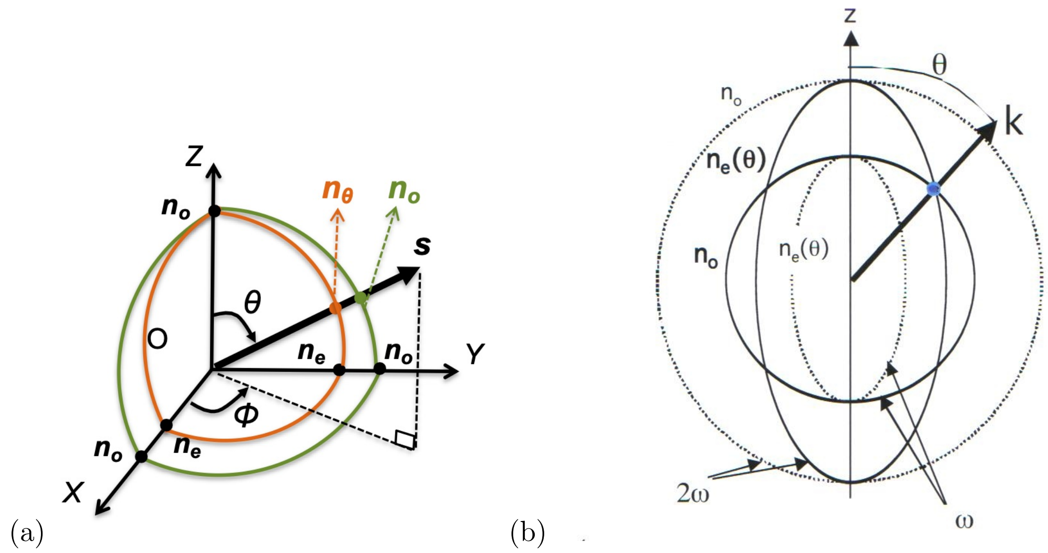

Figure 1: (a) Surface of indices for a negative uniaxial crystal and (b) their projections at ω and 2ω in a plane containing the optical axis Z.